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Water Resources Allocation: A Cooperative Game Theoretic Approach

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ABSTRACT. Water allocation is essentially an exercise in allocating available water to various demanding users. Water allocations merely based on a water rights approach usually do not make efficient use of water for the whole river basin. Meanwhile, an economic efficient water allocation plan cannot be well implemented if the involved participants or stakeholders do not regard it as being fair. A cooperative game theoretic approach is proposed to solve water allocation problems in two steps: (1) initial allocation of water rights to water users or stakeholders based on existing water rights systems or agreements, and (2) reallocation of water to achieve efficient use of water through water transfers. An illustrative example is utilized to show the effectiveness and potential advantages of this approach.

Keywords: Cooperative game theory, water allocation, water rights, water transfers

1. Introduction

Water allocation is central to the management of water resources. Due to geographically and temporally unevenly distributed precipitation (Al Radif, 1999), rapidly increasing water demands driven by the world population and other stresses, and degradation of the water environment (UN-CSD, 1994), there are increasing scarcities of water resources in countries. Conflicts often arise when different water users (including the environment) compete for limited water supply. The need to establish appropriate water allocation methodologies and associated management institutions and policies is recognized by researchers, water planners and governments. Many studies have been carried out in this domain, but there are still many obstacles to reaching equitable, efficient and sustainable water allocations (Dinar et al., 1997; Syme et al., 1999; UN-ESCAP, 2000). Recent concerns about increasing the efficiency of water resources utilization have centered on economical optimal water allocation at the river basin level (McKinney et al., 1999; Mahan et al., 2002). Due to the different production abilities of water users in the real world, an economical efficient water allocation plan is generally not an equitable one for all water users or stakeholders.

Water resources and environmental management problems often engage multiple stakeholders with conflicting interests (Fang et al., 1988, 2002; Hipel et al., 1997, 2003). To achieve equitable and efficient water allocation requires the cooperation of all stakeholders in sharing water resources. Cooperative game theory can be utilized to study the fair allocation of common pool resources (Owen, 1995), and has been applied to the following types of problems in water resources management: (1) cost allocation of water resources development projects, including joint waste water treatment and disposal facilities (Giglio and Wrightington, 1972; Dinar and Howitt, 1997), and water supply development projects (Young et al., 1982; Driessen & Tijs, 1985; Dufournaud and Harrington, 1990, 1991; Dinar et al., 1992; Lejano & Davos, 1995; Lippai & Heaney, 2000); (2) equitable allocation of waste loads to a common receiving medium (Kilgour et al., 1988; Okada and Mikami, 1992); and (3) allocation of water rights (Tisdell & Harrison, 1992). There are only a limited number of models employing cooperative game theory in water allocation, and these models have none or else simple hydrological constraints. Tisdell and Harrison (1992) use a number of different cooperative games to model the efficient and socially equitable reallocation of water among six representative farms in Queensland, Australia. Rogers (1969) uses linear programming to compute the optimum benefits of six strategies of India and East Pakistan (acting singly or in cooperation) in the international Ganges-Brahmaputra river basin, and then analyzes the strategies by a nonzero-sum game for the two countries. Incorporating Nepal into his analysis, Rogers (1993a, b) outlines the applicability of cooperative game theory and Pareto frontier analyses to water resources allocation problems. Okada and Sakakibara (1997) also apply a hierarchical cooperative game model to analyze cost/benefit allocation in a basin-wide reservoir redevelopment as part of water resources reallocation.

In this paper, an equitable and efficient cooperative allocation approach is proposed to solve water allocation problems in two steps. Water rights are initially allocated to water stakeholders and users based on existing water rights systems or agreements, and then water is reallocated to achieve efficient use of water through water transfers. The associated net benefit reallocation is carried out by the application of cooperative game theory. The integrated cooperative water allocation modeling approach is designed to promote and guide equitable water transfers and

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cooperation of relevant stakeholders to achieve optimal economic and environmental values of water, subject to hydrological and other constraints.

2. Perspectives on Water Allocation Problems

2.1. Principles of Water Allocation

What is water allocation? “The simplest definition of water allocation is the sharing of water among users. A useful working definition would be that water allocation is the combination of actions which enable water users and water uses to take or to receive water for beneficial purposes according to a recognized system of rights and priorities” (UN-ESCAP, 2000). Because of water’s fugitive and uneven distribution characteristics, its vital importance to human beings and society, and the complex relationships among climate, hydrology, the environment, society, economics and sustainable development, water allocation is a complex task.

The overall objective of water allocation is to maximize the benefits of water to society, which can be further classified as social, economic and environmental in nature. For each classification, there is a corresponding principle: equity, efficiency and sustainability, respectively. Equity means the fair sharing of water resources within river basins, at local, national, and international levels. Equity needs to be applied amongst water users, existing and potential users, and consumers of water and the environment. It is important to have pre-agreed rules or processes for the allocation of water, especially in situations where water is scarce. Such agreements and methodologies should reflect the wishes of those affected sufficiently to be seen to be equitably and accountably applied. Efficiency is the economical use of water resources, with particular attention to economical activities, demand management, financially sustainable uses of water resources, and fair compensation for water transfers at all geographical levels. Efficiency is not so easy to achieve, because the allocation of water to users relates to physical delivery or transport of water to the demanding points of use. Many factors are involved in water transfers, one of which is the conflict with equitable water rights. For example, a group of farmers should have permits to use certain amounts of water for agricultural irrigation. Some water for irrigation might be transferred to some industrial uses if policy makers decide to try to achieve an efficiency-based allocation of water. In this case, farmers should receive fair compensation for their losses. Sustainability advocates the environmentally sustainable use of land and water resources. This implies that today’s use of water resources should not expand to such an extent that water resources may not be available in the future (Savenije & van der Zaag, 2000).

2.2. Hydrological Cycle, Water Demand and Water Allocation

Two obvious major sources of supply are surface water and groundwater. Although the complete hydrological cycle is global in nature, a rational and suitable water resources modeling and management unit is needed at the river basin level (McKinney et al., 1999). In order to make good operational decisions regarding solutions to sharing water in a watershed, a fundamental scientific understanding of hydrologic constraints and conditions is required.

The need for a water allocation activity arises from demands for water. Where the resources are restricted compared to demands, as is the case for irrigation in some regions, conflicts arise among competing users. In general, water uses can be grouped into various demand groups: water supply for a city, agriculture, hydropower, navigation and other demands including flood storage, recreation, ecological uses and even bulk water export. An operational water allocation plan should be based on the hydrological constraints and linkages between demanding uses and water sources.

2.3. Institutional Mechanisms for Water Allocation

Dinar et al. (1997) discuss four basic institutional mechanisms for water allocation: user-based allocation, marginal cost pricing, public allocation and water markets allocation. “User-based allocation requires collective action institutions with authority to make decisions on water rights” (Dinar et al., 1997). The effectiveness of user-based allocation depends on local norms and the strength of local institutions, but such institutions are not always in place or strong enough to allocate water efficiently. Marginal cost pricing (MCP) sets a price for water equal to the marginal cost of supplying the last unit of that water. MCP is theoretically efficient but it is difficult to collect sufficient information for correct estimation and subsequent monitoring of benefits and costs. User-based allocation and MCP are hard to implement at a river basin level.

Public (administrative) allocation of water resources is broadly employed in countries where water is viewed as a public good and the governments allocate and distribute water permits as water use rights to users based on physical norms and political influence. Public allocation promotes equitable water use, can protect the poor, and can sustain environmental needs, but often leads to inefficient use of water and failure to create incentives for water users to conserve water, improve use efficiency and allow tradable water transfers to achieve maximum benefits in a whole river basin. Water markets allocate water by means of tradable water use rights and promote efficient water usage through allowing users to sell and buy their water rights freely. It requires intervention of government to create necessary conditions before markets become operational, including: (a) defining the original allocation of water rights, (b) creating the institutional and legal frameworks for trade, and (c) investing basic necessary infrastructure to allow water transfers (Holden and Thobani, 1996). Water markets are only concepts in many countries, but water markets do exist in Australia (Pigram et al., 1992), Spain (Reidinger, 1994), California (Howe & Goodman, 1995), Chile (Hearne & Easter, 1995), and India (Saleth, 1996).

Water markets have attractive potential benefits such as distributing secure water rights to users, providing incentives to use water efficiently and gain additional income through the sale of conserved water. There are some challenges in the

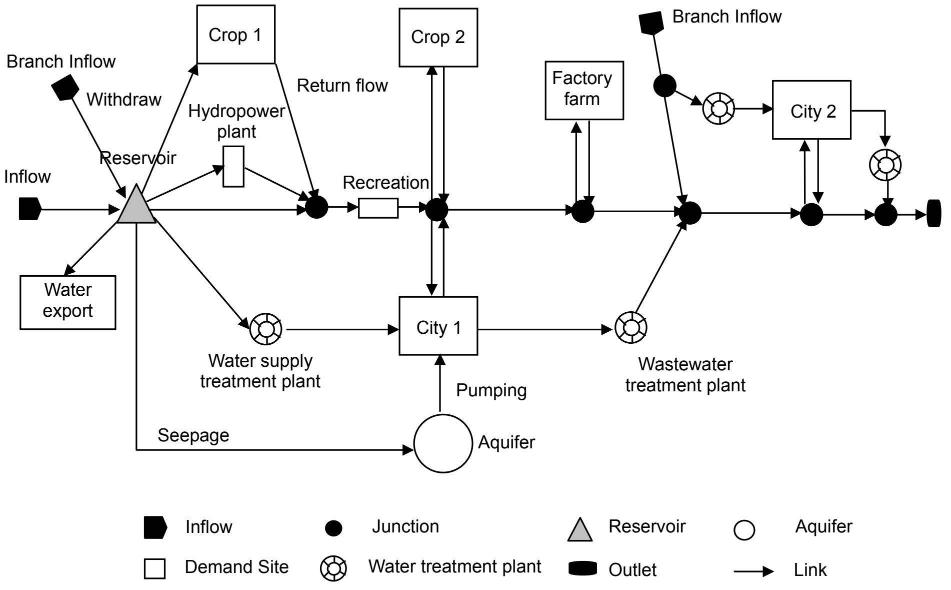

Figure 1. An example of a river basin network.

design of a well-functioning water market. The difficulties include measuring water uses and well-defined water rights, taking into account the variable flows and hydrological constraints, sale-for-cash by poor farmers, externality and third party effects. Furthermore, it can be argued that water is a public good and markets cannot work for raw water, just as Dellapenna (2000) argues that water markets are rare in reality and are not true free markets.

3. Basin Wide Water Allocation: Definitions and Preliminaries

3.1. Node-link River Basin Network

The hydrological characteristics of a river basin form the structure for operational water allocations. A node-link river basin network is an abstract graphical model for describing a real world river basin or watershed. A node is symbolized as a dot, circle, triangle, or rectangle, representing a physical component of interest such as inflow, natural or man-made junction, intake structure, water treatment plant, aquifer, reservoir, natural lake, dam, weir, or water demand site. The links represent the natural or man-made water conduits such as river channels, canals and pipelines between the different nodes. The links include river reaches, diversions, transmission and return flow links (DHI, 2001; SEI, 2001). An example of a river basin network is shown in Figure 1.

Let $G(V, A)$ be the directed network of a river basin, where $V = \{1, 2, ..., v\}$ is the set of nodes, (k_1, k_2) denotes the arc or link from node k_1 to k_2, and $A = \{(k_1, k_2)\text{: } k_1, k_2 \in V$ and $k_1 \neq k_2\}$ is the set of links of the network. Water users are grouped into stakeholders whose set is defined by $N = \{1, 2, ..., n\}$. A number of water use sites which take water and discharge return flows, including offstream and instream economic uses, minimum environmental flow requirements as well as reservoirs, are abstracted as a set of water demand nodes in the node-link river basin network model, where $U = \{j \in V\text{: } j \text{ is a water demand node}\}$. The demand set of stakeholder i, $i \in N$, can be defined as $U_i = \{j \in V\text{: } j \text{ is a water demand node of stakeholder } i\}$. Thus, $U_i \subseteq U \subseteq V$. Note that a stakeholder may have several water demand nodes, and a water demand node may be associated with a number of water uses and users, such as agricultural farms and farmers.

3.2. Water Balance and Constraints

For a general node k, the water and pollutant balance equations for time step t can be written as:

$$\begin{aligned}&S(k,t)\text{-}S(k,t\text{-}1) = \sum_{(k_1,k)\in A} Q(k_1,k,t) - \sum_{(k_1,k)\in A} Q_l(k_1,k,t)\\&+Q_0(k,t) - Q_c(k,t) - \sum_{(k,k_2)\in A} Q(k,k_2,t),\\&\forall k \in V\end{aligned} \tag{1}$$

$$\begin{aligned} & C_p(k,t)\cdot S(k,t)-C_p(k,t-1)\cdot S(k,t-1)= \\ & \sum_{(k_1,k)\in A} C_p(k_1,k,t)\cdot Q(k_1,k,t) \\ & -\sum_{(k_1,k)\in A} Z_{pl}(k_1,k,t)+Z_{p0}(k,t)+Z_{pd}(k,t) \\ & -\sum_{(k,k_2)\in A} C_p(k,k_2,t)\cdot Q(k,k_2,t), \end{aligned} \qquad (2)$$

$$\forall k\in V$$

where t is the index of time steps (step length is Δt), $t\in T=\{1, 2, ..., \tau\}$ (τ is the largest index of time step); $S(k, t)$ is the storage variable for node $k\in$ {aquifers and reservoirs} during step t; $Q_0(k, t)$ is inflow adjustment of node k during step t for the recharge from the local catchment's rainfall drainage or small tributaries; $Q(k_1, k, t)$ is the flow from node k_1 to k; $Q_l(k_1, k, t)$ is the conveyance losses because of evaporation, leakage and seepage of the flow from node k_1 to k; $Q_c(k, t)$ is the water consumed at node k because of economic activities and evaporation; $C_p(k, t)$ is the concentration of pollutant p at node k; $Z_{p0}(k, t)$ is total amount of pollutant p added to node k during time step t through inflow adjustment $Q_0(k, t)$; $C_p(k_1, k, t)$ is the concentration of pollutant p in the water flow from node k_1 to k; $Z_{pl}(k_1, k, t)$ is the conveyance losses of pollutant p in the water flow from node k_1 to k; $Z_{pd}(k, t)$ is the net discharge of pollutant p to node k because of economic activities. Note that $S(k, t) = 0$ for river and demand nodes, except for large storage nodes such as reservoirs and aquifers.

In addition to water and pollutant balance equations, there are some capacity limits for storage nodes and links. The capacity limits, together with typical policy constraints, form the lower and upper bounds for storages and flows, such as:

minimum water volume for a storage node k:

$$S(k,t)\geq S_{\min}(k,t)$$

maximum water volume of a storage node k:

$$S(k,t)\leq S_{\max}(k,t)$$

minimum flow from k_1 to k:

$$Q(k_1,\ k,\ t)\geq Q_{\min}(k_1,\ k,\ t)$$

maximum flow from k_1 to k:

$$Q(k_1,\ k,\ t)\geq Q_{\max}(k_1,\ k,\ t) \qquad (3)$$

Policy constraints may also be complex social, economic and other constraints governing water allocation, which should be specified case by case.

If a node k is simplified to provide water supplies to demand sites and receive the corresponding return flows from them, the total available water volume at node k before diversions should exceed the total supply outflows from k to the demand sites. Let j be a water demand node, $j\in U$, hence,

$$\begin{aligned} & S(k,t-1)-S(k,t)+Q_0(k,t)+\sum_{\substack{(k_1,k)\in A \\ and\ k_1\neq j}} Q(k_1,k,t) \\ & -\sum_{\substack{(k_1,k)\in A \\ and\ k_1\neq j}} Q_l(k_1,k,t)-Q_c(k,t)\geq \sum_{\substack{(k,j)\in A \\ and\ j\in U}} Q(k,j,t), \\ & k\in V \end{aligned} \qquad (4)$$

3.3. Water Allocation Problem

Consider a node-link network where each stakeholder generally has several water demand sites and each demand site may have several inflow or return flow links. Furthermore, water has characteristics of both quantity and quality. So, a perfect definition of water rights allocated to each stakeholder under certain hydrological conditions should be in terms of the water volume, water quality for all inflows, storage and return flows from its every use within each specific time step t.

For a general water demand node $j\in U$ of stakeholder i, the allocated water rights can be defined as a set of volumes and pollutant concentrations for its inflows, storages and return flows, $\{Q_R(k_1, j, t), C_{pR}(k_1, j, t), S_R(j, t), C_{pR}(j, t), Q_R(j, k_2, t), C_{pR}(j, k_2, t)\}$, where $k_1\in(k_1\to j)$, $k_2\in(j\to k_2)$, at every time step t.

Note that the subscript R means the corresponding variables are allocated as water rights. Generally, when there is not enough water at node k_1, $Q_R(k_1, j, t)$ and $S_R(j, t)$ may be less than the demand volume $Q_D(k_1, j, t)$ and $S_D(j, t)$, while $C_{pR}(k_1, j, t)$ and $C_{pR}(j, t)$ may be greater than the demand concentration $C_{pD}(k_1, j, t)$ and $C_{pD}(j, t)$, respectively. Since $Q_R(j, k_2, t)$ and $C_{pR}(j, k_2, t)$ are dependent variables on $Q_R(k_1, j, t)$, $S_R(j, t)$, $C_{pR}(k_1, j, t)$ and $C_{pR}(j, t)$, available best technology and normal technical standards of production and pollution control; $C_{pR}(k_1, j, t)$ and $C_{pR}(j, t)$ are also dependent variables on the quantity and quality of inflows to the river basin and the water volumes allocated to upstream uses, the water rights $\{Q_R(k_1, j, t), C_{pR}(k_1, j, t), S_R(j, t), C_{pR}(j, t), Q_R(j, k_2, t), C_{pR}(j, k_2, t)\}$ can be allocated in such a way: water volumes $Q_R(k_1, j, t)$ and $S_R(j, t)$ are firstly allocated to each use j according to an equitable water volume allocation method, then the dependent variables $C_{pR}(k_1, j, t)$, $C_{pR}(j, t)$, $Q_R(j, k_2, t)$ and $C_{pR}(j, k_2, t)$ are calculated by water and pollutant balance equations.

4. Cooperative Water Allocation: A Cooperative Game Theoretic Approach

Water allocations merely based on a water rights approach usually do not make efficient use of water for the whole river basin. Meanwhile, an economic water allocation plan cannot be implemented if the participants or stakeholders who are involved do not regard it as being fair. Hence, cooperative water allocation is needed to deal with the above problem. Cooperative water allocation is a coordinated effort to allocate water resources among the multiple stakeholders in a river basin under intra-country water rights systems or inter-country agreements in order to promote maximum economical use of water and equitable allocation of the associated costs and benefits. In this section, a cooperative game theoretic approach is proposed to solve water allocation problems in two steps: (1) initial allocation of water rights to water users or stakeholders based on existing water rights systems or agreements, and (2) reallocation of water to achieve efficient use of water through water transfers.

4.1. Initial Water Rights Allocation

Countries have developed their own specific ways of solving the issues of planning, developing, allocating, distributing and protecting their water resources. The various systems of water rights can be grouped into three basic doctrines: riparian rights, prior (appropriative) rights and public allocation (Savenije & van der Zaag, 2000). Water rights allocation systems award users rights to abstract a certain volume of water in a time period. Under a traditional riparian system, water demands in every period are allocated as fully as possible to the transmission links to competing demand sites and flow links to instream uses from upstream to downstream.

Under the prior rights system, water demands in every period are allocated as fully as possible according to the supply priorities assigned to all the transmission links to competing demand sites and flow links to instream uses. The public allocation regime treats water as a public property, so water rights are administratively allocated to users through water permits issued by governments.

Generalized transboundary water allocation principles for sharing the water resources of international river basins between countries include: (1) absolute sovereignty, (2) absolute riverine integrity, (3) limited territorial sovereignty, and (4) economic criteria (Wolf, 1999; Giordano & Wolf, 2001). The seemingly fair and simple principles or guidelines of reasonable and equitable use are difficult to be applied in practice, especially for an inter-country river basin. Measurable criteria and models for water allocation need to be constructed and used to achieve fair apportionment of water (Seyam et al., 2000; van der Zaag et al., 2002).

In a water allocation problem, resource users have heterogeneities arising from physical resource characteristics, users' technologies and skill levels, and institutional arrangements. An institution can cause heterogeneities in pricing, property rights and political power (Schlager & Blomquist, 1998). The water rights are allocated according to a legal intra-country water rights system and water policies or inter-country agreements before moving to the second gaming stage of the cooperative water allocation model.

4.2. Cooperative Water Allocation Game

Recall that $N = \{1, 2, \ldots, n\}$ is the set of water stakeholders or players competing for water allocations in the concerned river basin or sub-watershed, and $i \in N$ a typical stakeholder. A group of stakeholders $S \subseteq N$ entering a cooperative agreement and working together is called a coalition. N itself is called the grand coalition, the coalition consisting of all stakeholders.

A coalition structure is a partition $\pi = \{S_1, S_2, \ldots, S_m\}$ of the n stakeholders, in which $\bigcup_i^n S_i = N$ and for all $i \neq j$, $S_i \bigcap S_j = \phi$. For a game with n players, 2^n coalitions are possible, or $2^n - 1$ if the null coalition is excluded. The expression $v(S)$ is used to represent the aggregate payoff to the members of coalition S, while the payoffs to individual stakeholders acting in isolation are represented as $v(\{1\})$, $v(\{2\})$, …, $v(\{n\})$. In a cooperative water allocation game, the generic notations of payoffs $v(\{i\})$ and $v(S)$ are interpreted specifically as the net benefits by the following definitions.

The payoff $v(\{i\})$ of a stakeholder i is the maximum total net benefit $NB(i)$ that stakeholder i can gain based on its water rights over the entire planning period, subject to not decreasing the water flows and not increasing the pollutant concentrations in the flows to other stakeholders. Thus, $v(\{i\})$ is normally greater than the total net benefit $NB(i)$ gained with the initial water rights since there is additional value for the internal cooperation among the uses and users within stakeholder i.

$$v(\{i\}) = \max NB(i) = \max\left(\sum_{t \in T} NB_{i,t}\right) = \max\left(\sum_{t \in T}\sum_{j \in U_i} NB_{i,j,t}\right)$$

subject to:

water balance and hydrological constraints

$$\begin{aligned}
&Q(k,j,t) \geq Q_R(k,j,t),\ \forall k \in V, \forall j \in U \text{ and } j \notin U_i \\
&S(j,t) \geq S_R(j,t),\ \forall k \in V, \forall j \in RES \text{ and } j \notin U_i \\
&C_p(k,j,t) \leq C_{pR}(k,j,t),\ \forall k \in V, \forall j \in U \text{ and } j \notin U_i \\
&C_p(j,t) \leq C_{pR}(j,t),\ \forall k \in V, \forall j \in RES \text{ and } j \notin U_i
\end{aligned} \tag{5}$$

where, RES is the set of reservoirs.

The payoff $v(S)$ of a coalition S is the maximum total net benefit $NB(S)$ that coalition S can gain based on coalition members' water rights over the entire planning period, subject to not decreasing the water flows and not increasing the pollutant concentrations in the flows to other stakeholders not taking part in coalition S.

$$v(S) = \max NB(S) = \max\left(\sum_{t \in T}\sum_{i \in S} NB_{i,t}\right)$$

$$= \max\left(\sum_{t \in T}\sum_{i \in S}\sum_{j \in U_i} NB_{i,j,t}\right)$$

subject to:

water balance and hydrological constraints

$$\begin{aligned}
&Q(k,j,t) \ge Q_R(k,j,t),\ \forall k \in V, \forall j \in U \text{ and } j \notin U_S\\
&S(j,t) \ge S_R(j,t),\ \forall k \in V, \forall j \in RES \text{ and } j \notin U_S\\
&C_p(k,j,t) \le C_{pR}(k,j,t),\ \forall k \in V, \forall j \in U \text{ and } j \notin U_S\\
&C_p(j,t) \le C_{pR}(j,t),\ \forall k \in V, \forall j \in RES \text{ and } j \notin U_S
\end{aligned} \tag{6}$$

where, $U_S = \bigcup_{i \in S} U_i$.

In the above definitions, $NB_{i,t}$ is the net benefit function of stakeholder *i* during time step *t*, calculated as:

$$NB_{i,t} = \sum_{j \in U_i} NB_{i,j,t} \tag{7}$$

In Equations (6) and (7), $NB_{i,j,t}$ is the net benefit function of stakeholder *i*'s water demand node *j* during time step *t*, given by:

$$\begin{aligned}
&NB_{i,j,t} = f_{i,j,t}(Q(k_1,j,t), C_p(k_1,j,t), S(j,t),\\
&C_p(j,t), Q(j,k_2,t), C(j,k_2,t)),\\
&(k_1,j) \in A,\ (j,k_2) \in A
\end{aligned}$$

The net benefit function of demand node *j*, $f_{i,j,t}(\cdot)$, is determined by $f_{i,j,t}(\cdot) = B_{i,j,t}(\cdot) - C_{i,j,t}(\cdot)$, where, $B_{i,j,t}(\cdot)$ and $C_{i,j,t}(\cdot)$ are the benefit function and cost function for demand node *j*, respectively.

$f_{i,j,t}(\cdot)$ can be estimated from historical data statistics and simulation or obtained through optimization with control variables such as use type, area, user's technology and skill level, price, and other economic and policy factors. Note that in the latter case, $Q(k_1,j,t), C_p(k_1,j,t), S(j,t), C_p(j,t)$, $Q(j,k_2,t)$ and $C_p(j,k_2,t)$ are the control variables used in searching for *v*(*S*), but parameters in searching for the optimal value of $f_{i,j,t}(\cdot)$.

A "solution" to a game is a vector of the payoffs received by each stakeholder. This payoff or reward vector after a trade can be written as $x = \{x_1, x_2, ..., x_n\}$. This trade process to achieve a cooperative water allocation under certain water balance and hydrological constraints is essentially a cooperative water allocation game. The payoff vector is called an imputation to the cooperative game, and meets the conditions of individual rationality, group rationality and joint efficiency (Young et al., 1982; Tisdell & Harrison, 1992):

Individual rationality:

$$x_i \ge \upsilon(\{i\}) \tag{8}$$

Group rationality

$$\sum_{i \in S} x_i \ge \upsilon(S) \tag{9}$$

Joint efficiency

$$\sum_{i \in N} x_i = \upsilon(N) \tag{10}$$

Let $x(S) = \sum_{i \in S} x_i$, then the above three conditions can be reduced to:

Individual and group rationality

$$x(S) \ge \upsilon(S) \quad \text{for all} \quad S \subset N$$

Joint efficiency $\quad x(N) = \upsilon(N)$

The set of reward payoff vectors that satisfy the conditions of individual rationality, group rationality and joint efficiency forms the core to a cooperative game. The core of a cooperative game may not always exist. If it exists, there is no guarantee that it has a unique feasible solution. Core-based and non-core-based allocation concepts may be applied to reduce it to a unique one (Dinar et al., 1986). Nucleolus and related solutions are listed in Table 1. The nucleolus minimizes the maximum excess of any coalition S lexicographically (Schmeidler, 1969).

Let excess $e(S,X) = v(S) - x(S) = v(S) - \sum_{i \in S} x_i$, then the nucleolus is found by

$$\min \quad e$$

subject to:

$$\begin{aligned}
&x(S) + e \ge v(S) \quad \text{for all} \quad S \subset N\\
&x(N) = v(N)
\end{aligned} \tag{11}$$

Table 1. Nucleolus and Variation Solution Concepts

Solution concepts	*Net benefit excess*	*Individual and group rationalities*
Nucleolus	*e = v(S) - x(S)*	min e subject to $x(S) + e \ge v(S)$ for all $S \subset N$
Weak Nucleolus	$e_w = (v(S) - x(S))/\lvert S\rvert$	min e_w subject to $x(S) + e_w\lvert S\rvert \ge v(S)$ for all $S \subset N$
Proportion Nucleolus	$e_p = (v(S)-x(S))/v(S)$	min e_p subject to $x(S) + e_p v(S) \ge v(S)$ for all $S \subset N$
Normalized Nucleolus	$e_n = (v(S)-x(S))/x(S)$	min e_n subject to $x(S)(1+e_n) \ge v(S)$ for all $S \subset N$

Application of this optimizing algorithm narrows the solution space of the core. Successive applications of the algorithm involve setting aside coalitions for which *e(S, X)* equals the critical value of *e* found at each step and running the optimization program for remaining coalitions. Each iteration further constrains the solution space until a unique point is ultimately reached. The excess e can be interpreted as subsidies ($e \geq 0$) or tax ($e < 0$) to the water stakeholders. Variations of the nucleolus are obtained by changing the definition of the excess function, individual and group rationalities while the optimization algorithm remains the same as shown in Table 1 (Lejano & Davos, 1999). The weak nucleolus concept (Young et al., 1982) replaces the excess e with the average excess where $|S|$ is the cardinality of coalition S; proportional nucleolus (Young et al., 1982) replaces e with the ratio of excess to net benefit of coalition S; and normalized nucleolus replaces e with the ratio of excess to imputation of coalition S. The nucleolus and related variation approaches can reduce or expand the core to obtain a unique solution in both cases of large core and empty core (Dinar et al., 1986).

With the Shapley value solution concept, each stakeholder's reward or value to the game should equal a weighted average of the contributions the stakeholder makes to each coalition of which he or she is a member. The weighting depends on the number of total stakeholders and the number of stakeholders in each coalition. The Shapley value gives the payoff to i^{th} stakeholder (Shapley, 1971) such that:

$$\begin{aligned} x_i &= \sum_{\substack{S \subseteq N \\ i \in S}} \frac{(|S|-1)!(|N|-|S|)!}{|N|!}\left[v(S)-v(S-\{i\})\right] \\ &= \sum_{s=1}^{n} \frac{(s-1)!(n-s)!}{n!}\left[v(S)-v(S-\{i\})\right] \quad (i \in N) \end{aligned} \tag{12}$$

5. An Illustrative Example

Suppose there are three stakeholders, Irrigation Water Association (IWA), City 1 and City 2, along a river as shown in the node-link flow network in Figure 2. The IWA has two crop areas located upstream. The return flow coefficients of both crop areas are 20%. The return flow coefficients from both cities are 90%. The minimum demands from the Crop 1, Crop 2, City 1, and City 2 are 40, 50, 20 and 25 million m^3/year, respectively, while the maximum demands are 100, 120, 40 and 50 million m^3/year, respectively, as given in Table 2. The statistical functions of net benefits and salinity in return flows given in Table 2 are designed according to experience published in the literature (Booker & Young, 1994). Now we develop the water allocation plan for a hypothetical drought period with a series of annual upstream flows $Q(1, 2, t)$ and corresponding average salinity concentrations $C_p(1, 2, t)$ shown in Table 3. Inflow adjustment and evaporation losses are not considered in the following analysis.

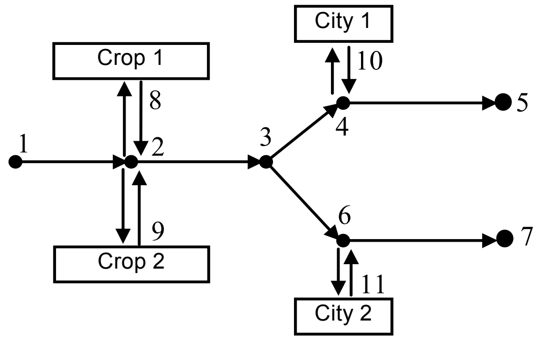

Figure 2. Flow network and water uses.

5.1. Initial Water Rights Allocation

The problem stated above is modeled as a 5-year plan by the cooperative water allocation model with an annual time step. Water flows listed in Table 3 are allocated using a modified dual-priority riparian water rights allocation method, in which initial water rights allocation is solved as a multiperiod maximal network flow programming problem (Wang et al., 2003). In each of the five years, water is firstly allocated to meet all the minimum annual demands of the four uses from upstream to downstream, and then the remaining water is allocated to the maximum demands of these riparian uses along the river. The water flows at node 2 are allocated

Table 2. Annual Net Benefit and Return Flow Salinity Functions

Stakeholder	*Water use*	$NB_{j,t}$ *(10^3\$)*	*Return flow salinity* $Z(j, k, t)$ *(10^6 kg)*	*Minimum demand ($10^6 m^3$/year)*	*Maximum demand ($10^6 m^3$/year)*	*Return flow ratio*
1: IWA	1: Crop 1	$-1000 + 60Q(2,8,t) - 0.2Q(2,8,t)^2$	$0.3Q(2,8,t) -0.0008Q(2,8,t)^2$	40	100	0.2
	2: Crop 2	$-1100 + 60Q(2,9,t) - 0.2Q(2,9,t)^2$	$0.3Q(2,9,t) -0.0008Q(2,9,t)^2$	50	120	0.2
2: City 1	3: City 1	$700Q(4,10,t) - 0.3Q(4,10,t)^2 - 0.25Q(4,10,t) \times max(C_p(4,10,t) - 400, 0)$	$2.5Q(4,10,t) -0.0008Q(4,10,t)^2$	20	40	0.9
3: City 2	4: City 2	$680Q(6,11,t) - 0.3Q(6,11,t)^2 - 0.25Q(6,11,t) \times max(C_p(6,11,t) - 400, 0)$	$2.5Q(6,11,t) -0.0008Q(6,11,t)^2$	25	50	0.9

* Units of flow and salinity are $10^6 m^3$ and *mg/L*, respectively.

Table 3. Total Upstream Inflows and Initial Water Rights Allocations

Time step	*Year 1*	*Year 2*	*Year 3*	*Year 4*	*Year 5*
$Q(1, 2, t)$ (10^6m^3)	280.00	260.00	240.00	240.00	260.00
$Cp(1,2,t)$ (mg/L)	400.00	410.00	420.00	430.00	410.00
$Q(2,8,t)$ (10^6m^3)	100.00	100.00	100.00	100.00	100.00
$Cp(2,8,t)$ (mg/L)	400.00	410.00	420.00	430.00	410.00
$Q(2,9,t)$ (10^6m^3)	120.00	120.00	120.00	120.00	120.00
$Cp(2,9,t)$ (mg/L)	400.00	410.00	420.00	430.00	410.00
$Q(4,10,t)$ (10^6m^3)	40.00	37.33	28.44	28.44	37.33
$Cp(4,10,t)$ (mg/L)	677.69	748.57	857.50	860.63	748.57
$Q(6,11,t)$ (10^6m^3)	50.00	46.67	35.56	35.56	46.67
$Cp(6,11,t)$ (mg/L)	677.69	748.57	857.50	860.63	748.57
$Q(4, 5, t)$ (10^6m^3)	42.22	33.60	25.60	25.60	33.60
$Cp(4,5,t)$ (mg/L)	2437.98	2744.59	2752.49	2752.49	2744.59
$Q(6, 7, t)$ (10^6m^3)	52.78	42.00	32.00	32.00	42.00
$Cp(6,7,t)$ (mg/L)	2430.40	2736.30	2746.17	2746.17	2736.30
$NB_{1,t}$(10^3\$)	6220.00	6220.00	6220.00	6220.00	6220.00
$NB_{2,t}$(10^3\$)	24743.08	22461.87	16415.05	16392.83	22461.87
$NB_{3,t}$(10^3\$)	29778.85	27013.33	19731.85	19704.07	27013.33
Total Net Benefit (10^3\$)	60741.92	55695.20	42366.90	42316.90	55695.20

proportionally to the annual demands of both crops, and the water flows at node 3 are allocated proportionally to the annual demands of both cities, respectively. Note that since there is no storage node and carry-over flow in the flow network of this example, the results of the multi-year maximal network flow program are the same as those of a sequence of single year network flow programs.

The results show that the minimum water demands of all uses and the maximum demands of both crops are met in each time step, but the maximum demands of cities cannot be satisfied during drought years. The salinity concentrations are calculated according to the water and pollutant balance equations after the water flows are allocated. Then, the net benefits for individual stakeholders are calculated by the benefit functions given in Table 2.

5.2. Estimation of Coalition Payoffs

Based on the initially allocated water flows, salinity concentrations and net benefits of individual stakeholders, the water flows are allocated to gain maximum net benefit $v(S)$ for individual stakeholders and every possible coalition S. The values in every time step (in this case, year) under the situation of the grand coalition are listed in Table 4. Table 5 summarizes the overall net benefits during the 5-year period for all independent stakeholders and all possible coalitions. Since irrigation has lower marginal net benefits than the cities, water is transferred from IWA to City 1 and City 2 when they form coalitions. For example, at the last points of abstracting water according to their water rights during the drought year 2, Crop 1 and Crop 2 have very low marginal net benefits given as 0.02 and 0.01 \$/m^3, respectively, while City 1 and City 2 have higher marginal net benefits calculated as 0.59 and 0.56 \$/m^3, respectively. Therefore, water is transferred from crops to cities when they form coalitions. In the grand coalition situation, Crop 1, Crop 2, City 1 and City 2 have marginal net benefits of 0.04, 0.03, 0.64 and 0.62 \$/m^3, respectively. The reason that cities' marginal net benefits increase as the amount of the water received increases is that the water quality improves when Crop 1 and Crop 2 use less water. For the same reason, although only additional 2.67×10^6 m^3 and 3.33×10^6 m^3 of water are received by City 1 and City 2 to obtain the maximum grand coalition net benefit, the amount of water received by Crop 1 and Crop 2 is reduced by 59.42×10^6 m^3 and 34.78×10^6 m^3, respectively. This implies that the hydrology-based cooperative water allocation model can be applied to allocate water flows as well as pollutant trading.

5.3. Reallocation of the Grand Coalition Net Benefit

In Figure 3, the triangle slope plane shows the set of all possible nonnegative allocations of the total net benefit of the grand coalition (305940.11 × 10^3\$) among competing stakeholders. For each point in the triangle, the perpendicular distances from three edges indicate allocated benefit to each stakeholder. The distance from the lower edge gives allocation

Table 4. Optimal Water Allocations and Payoffs for the Grand Coalition

Time step	*Year 1*	*Year 2*	*Year 3*	*Year 4*	*Year 5*
$Q(1, 2, t)$ (10^6m^3)	280.00	260.00	240.00	240.00	260.00
$Cp(1,2,t)$ (mg/L)	400.00	410.00	420.00	430.00	410.00
$Q(2,8,t)$ (10^6m^3)	40.94	40.58	40.34	40.34	40.48
Cp(2,8,t) (mg/L)	400.00	410.00	420.00	430.00	410.00
$Q(2,9,t)$ (10^6m^3)	94.91	85.22	74.21	75.23	85.29
$Cp(2,9,t)$ (mg/L)	400.00	410.00	420.00	430.00	410.00
$Q(4,10,t)$ (10^6m^3)	40.00	40.00	40.00	40.00	40.00
$Cp(4,10,t)$ (mg/L)	524.55	537.36	548.30	558.12	537.28
$Q(6,11,t)$ (10^6m^3)	50.00	50.00	50.00	50.00	50.00
$Cp(6,11,t)$ (mg/L)	524.55	537.36	548.30	558.12	537.28
$Q(4, 5, t)$ (10^6m^3)	53.56	50.47	48.38	47.56	50.46
$Cp(4,5,t)$ (mg/L)	2015.19	2110.08	2180.77	2211.21	2110.48
$Q(6, 7, t)$ (10^6m^3)	108.77	99.90	90.98	90.98	99.93
$Cp(6,7,t)$ (mg/L)	1438.39	1526.58	1629.05	1634.01	1526.17
$NB_{1,t}(10^3\$)$	3913.96	3665.95	3346.12	3376.89	3663.37
$NB_{2,t}(10^3\$)$	26274.47	26146.43	26036.96	25938.84	26147.22
$NB_{3,t}(10^3\$)$	31693.09	31533.04	31396.20	31273.55	31534.02
$v(1, 2,3)_t(10^3\$)$	61881.53	61345.41	60779.28	60589.28	61344.62
Total Net Benefit (10^3\$)	61881.53	61345.41	60779.28	60589.28	61344.62

Table 5. Overall Net Benefits in the 5-year Planning Period

Net Benefit $(10^3\$)$	*Initial allocation*	*Internal cooperation {i}*	*Coalition {1,2}*	*Coalition {1,3}*	*Coalition {2,3}*	*Grand coalition {1,2,3}*
NB_1	31100.00	31260.66	27415.48	25943.71	31100.00	17966.29
NB_2	102474.69	102474.69	125063.89	108725.59	117547.33	130543.92
NB_3	123241.44	123241.44	129661.24	152222.51	108675.39	157429.90
v(1,2)			152479.37			
v(1,3)				178166.22		
v(2,3)					226222.72	
v(1,2,3)						305940.11
Total	256816.13	256976.79	282140.61	286891.82	257322.72	305940.11

to IWA, the perpendicular distances from upper-left and upper-right edges provide allocations to Cities 1 and 2, respectively. Only the shaded area, the core, is the subset of allocations satisfying individual and group rationality. Note that the triangle can be used to explain the allocation only in three-player cooperative games. For games with more than three players, the core cannot be drawn on paper and the existence of the core can be checked using the nucleolus solution concept.

By solving with nucleolus and variation solution concepts and Shapley value, we get the overall 5-year and subsequent annual schedules of equitable and efficient allocation of net benefits in this cooperative water allocation project as shown in Table 6. These Pareto optimal schedules provide the alternatives needed for further negotiation or reaching a final decision.

Table 6. Equitable Allocations of the Net Benefits of the Grand Coalition (10^3 $)

Period	*Stakeholder*	*Nucleolus*	*Weak nucleolus*	*Proportional nucleolus*	*Normalized nucleolus*	*Shapley value*
Overall 5 years	IWA	53666.91	61034.30	57881.75	62299.75	54480.93
	City 1	113410.39	109765.16	110364.56	110910.16	114116.19
	City 2	138862.82	135140.66	137693.81	132730.21	137342.99
Year 1	IWA	10855.03	12345.21	11707.56	12601.17	11019.68
	City 1	22939.16	22201.85	22323.09	22433.44	23081.92
	City 2	28087.34	27334.47	27850.89	26846.92	27779.93
Year 2	IWA	10760.99	12238.26	11606.13	12492.00	10924.21
	City 1	22740.42	22009.50	22129.69	22239.09	22881.94
	City 2	27844.00	27097.65	27609.60	26614.32	27539.25
Year 3	IWA	10661.68	12125.32	11499.02	12376.72	10823.40
	City 1	22530.56	21806.38	21925.46	22033.85	22670.78
	City 2	27587.04	26847.58	27354.80	26368.71	27285.11
Year 4	IWA	10628.35	12087.41	11463.07	12338.02	10789.56
	City 1	22460.13	21738.21	21856.92	21964.97	22599.90
	City 2	27500.80	26763.65	27269.28	26286.28	27199.81
Year 5	IWA	10760.85	12238.10	11605.98	12491.84	10924.07
	City 1	22740.13	22009.21	22129.40	22238.80	22881.65
	City 2	27843.64	27097.30	27609.24	26613.98	27538.90

6. Conclusions

A two-step cooperative water allocation approach is formulated, which consists of an initial water rights allocation and a cooperative water reallocation game. Water rights are initially allocated based on existing water rights systems or agreements, while the cooperative water reallocation game is formulated by using net benefits as a stakeholder's payoff.

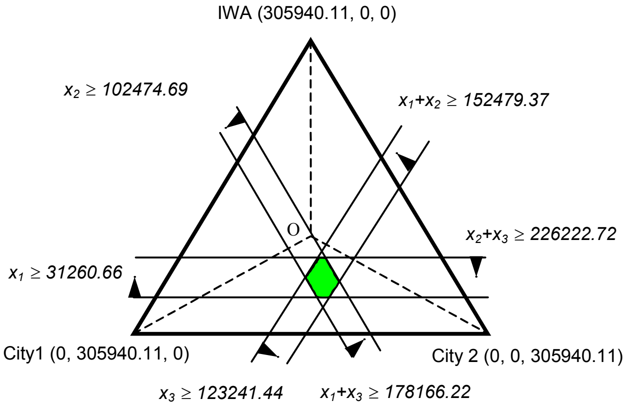

Figure 3. The core of a cooperative water allocation game.

The cooperative water reallocation game can be solved by solution concepts such as the nucleolus, weak nucleolus, proportional nucleolus, normalized nucleolus and Shapley value. Since the model performs initial water rights allocation and subsequent reallocation based on existing water rights systems or agreements, and it utilizes the node-link river basin network, water balance and hydrological constraints, with a time step length of Δt during a planning period, the model realistically takes into account knowledge and sub-models from hydrology, economics and cooperative game theory. This makes it possible to reach fair and efficient water allocation among competing uses with multiple stakeholders in an operational way. The methodology can be applied to an entire river basin or a sub-watershed.

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References

Al Radif, A. (1999). Integrated water resources management (IWRM): An approach to face the challenges of the next century and to avert future crises. *Desalination*, 124 (1-3), 145-153.

Booker, J.F. and Young, R.A. (1994). Modeling intrastate and interstate markets for Colorado River water resources. *J. Environ. Econ. Manage.*, 26, 66-87.

Dellapenna, J.W. (2000). The importance of getting names right: The myth of markets for water. *William and Mary Environ. Law Policy Rev.*, 25, 317-377.

DHI (Danish Hydraulic Institute) (2001). MIKE BASIN 2001: *A versatile decision support tool for integrated water resources management planning* (guide to getting started tutorial), Danish Hydraulic Institute, Horsholm, Denmark.

Dinar, A. and Howitt, R.E. (1997). Mechanisms for allocation of environmental control cost: Empirical test of acceptability and stability. *J. Environ. Manage.*, 49, 183-203.

Dinar, A., Ratner, A. and Yaron, D. (1992). Evaluating cooperative game theory in water resources. *Theory and Decis.*, 32, 1-20.

Dinar A., Rosegrant, M.W. and Meinzen-Dick, R. (1997). *Water allocation mechanisms: Principles and examples*, World Bank Working Paper, World Bank, Washington, DC, USA.

Dinar, A., Yaron, D. and Kannai, Y. (1986). Sharing regional cooperation gains from reusing effluent for irrigation. *Water Resour. Res.*, 22 (3), 339-344.

Driessen, T. and Tijs, S.H. (1985). The cost gap method and other cost allocation methods for multipurpose water projects. *Water Resour. Res.*, 21, 1469-1475.

Dufournaud, C.M. and Harrington, J.J. (1990). Temporal and spatial distribution of benefits and costs in river-basin schemes: A cooperative game approach. *Environ. Plann. A*, 22, 615-628.

Dufournaud, C.M. and Harrington, J.J. (1991). A linear constraint formulation for spatial and temporal cost imputations in accord with Shapley values. *Environ. Plann. A*, 23, 1517-1521.

Fang, L., Hipel, K.W. and Kilgour, D.M. (1988). The graph model approach to environmental conflict resolution. *J. Environ. Manage.*, 27 (2), 195-212.

Fang, L., Hipel, K.W. and Wang, L.Z. (2002). Gisborne water export conflict study, in G.H. Schmitz (Ed.), *Proc. of the Third International Conference on Water Resources and Environment Research*, Dresden, Germany, Vol. 1, pp. 432-436, 2002.

Giglio, R.J. and Wrightington, R. (1972). Methods for apportioning the costs of a water resources project. *Water Resour. Res.*, 8, 1133-1144.

Giordano, M.A. and Wolf, A.T. (2001). Incorporating equity into international water agreements. *Soc. Justice Res.*, 14 (4), 349-366.

Hearne, R.R. and Easter, K.W. (1995). *Water allocation and water markets: An analysis of gains-from-trade in Chile*, Technical Paper Series Number 315, World Bank, Washington, DC, USA.

Hipel, K.W., Kilgour, D.M., Fang, L. and Peng, X. (1997). The decision support system GMCR II in environmental conflict management. *Appl. Math. Comput.*, 83, 117-152.

Hipel, K.W., Fang, L. and Wang, L.Z. (2003). Bulk water export issues in Canada and the Lake Gisborne conflict, in *Proc. of the 2nd International Symposium on Integrated Water Resources Management*, Stellenbosch, South Africa, January 22-24th, 2003.

Holden, P. and Thobani, M. (1996). *Tradable water rights: A property rights approach to improving water use and promoting investment*, Policy Research Working Paper 1627, World Bank, Washington, DC. http://econ.worldbank.org/files/13369_wps1627.pdf.

Howe, C.W. and Goodman, D.J. (1995). Resolving water transfer conflicts through changes in water market process, in A. Dinar and E.T. Loehman (Eds.), *Water Quantity/Quality Management and Conflict Resolution: Institutions, Processes and Economic Analysis*, Preager, Westport, Connecticut, pp. 119-129.

Kilgour, D.M., Okada, N. and Nishikori, A. (1988). Load control regulation of water pollution: An analysis using game theory. *J. Environ. Manage.*, 27, 179-194.

Lejano, R.P. and Davos, C.A. (1995). Cost allocation of multiagency water resource projects: Game-theoretic approaches and case study. *Water Resour. Res.*, 31, 1387-1393.

Lejano, R.P. and Davos, C.A. (1999). Cooperative solutions for sustainable resource management. *Environ. Manage.*, 24(2), 167–175

Lippai, I. and Heaney, J.P. (2000). Efficient and equitable impact fees for urban water systems. *J. Water Resour. Plann. Manage.*, 126 (2), 75-84.

Mahan, R.C., Horbulyk, T.M. and Rowse, J.G. (2002). Market mechanisms and the efficient allocation of surface water resources in southern Alberta. *Socio-Econ. Plann. Sci.*, 36, 25–49.

McKinney, D.C., Cai, X., Rosegrant, M.W., Ringler, C. and Scott, C.A. (1999). *Modeling water resources management at the basin level, Review and future directions*, International Water Management Institute, Colombo, Sri Lanka.

Okada, N. and Mikami, Y. (1992). A game-theoretic approach to acid rain abatement: Conflict analysis of environmental load allocation. *Water Resour. Bull.*, 28 (1), 155-162.

Okada, N. and Sakakibara, H. (1997). Modeling a cost/benefit allocation game in a basin wide reservoir redevelopment as a part of water resources reallocation, in *Proc. of the 1997 IEEE International Conference on Systems, Man, and Cybernetics*, Orlando, Florida, USA, Vol. 1, pp. 791-796, October 12-15th, 1997.

Owen, G. (1995). *Game Theory*, 3rd Edition, Academic Press, New York, NY, USA.

Pigram, J.J., Delforce, R.J., Cowell, M.L., Norris, V., Anthony, G., Anderson, R.L. and Musgrave, W.F. (1992). *Transferable water entitlements in Australia*, Center for Water Policy Research, University of Armidale, Australia.

Reidinger, R. (1994). Observations on water markets for irrigation systems, in G. Le Moigne, K.W. Easter, W.J. Ochs and S. Giltner (Eds.), *Water Policy and Water Markets*, Technical Paper 249, World Bank, Washington, DC, USA.

Rogers, P. (1969). A game theory approach to the problems of international river basin. *Water Resour. Res.*, 5, 749-760.

Rogers, P. (1993a). A model to generate Pareto-admissible outcomes for international river basin negotiations, in *the International Workshop on Economic Aspects of International Water Resources Utilization in the Mediterranean Basin*, Fondazione ENI Enrico Mattei, Milan, Italy.

Rogers, P. (1993b). The value of cooperation in resolving international river basin disputes. *Nat. Resour. Forum*, 17(2), 117-131.

Saleth, R.M. (1996). *Water Institutions in India*. Commonwealth Publishers, New Delhi.

Savenije, H.H.G. and van der Zaag, P. (2000). Conceptual framework for the management of shared river basins; with special reference to the SADC and EU. *Water Policy*, 2 (1), 9-45.

Schlager, E. and Blomquist, W. (1998). Heterogeneity and common pool resource management, in E.T. Loehman and D.M. Kilgour (Eds.), *Designing Institutions for Environmental and Resource Management*, Edward Elgar, Northampton, MA, USA, pp. 101-112.

Schmeidler, D. (1969). The nucleolus of a characteristic function game. *J. Appl. Math.*, 17, 1163-1170.

SEI (Stockholm Environment Institute) (2001). *WEAP: Water evaluation and planning system user guide*, Stockholm Environment Institute-Boston, Boston, MA, USA.

Seyam, I., Savenije, H.H.G., Aerts, J. and Schepel, M. (2000). Algorithms for water resources distribution in international river basins. *Phys. Chem. Earth*, B, 25(3), 309-314.

Shapley, L.S. (1971). Cores of convex games. *Int. J. Game Theory*, 1, 11-26.

Syme, G.J., Nancarrow, B.E. and McCreddin, J.A. (1999). Defining the components of fairness in the allocation of water to environmental and human uses. *J. Environ. Manage.*, 57, 51-70.

Tisdell, J.G. and Harrison, S.R. (1992). Estimating an optimal distribution of water entitlements. *Water Resour. Res.*, 28, 3111-3117.

UN-CSD (United Nations, Commission on Sustainable Development) (1994). *Review of Sectoral Clusters, First Phase: Human Settlement and Fresh Water, Fresh Water Resources*, Report of the Secretary General of the UN, General Overview, Paragraphs 3-10.

UN-ESCAP (United Nations, Economic and Social Commission for Asia and the Pacific) (2000). *Principles and Practices of Water Allocation among Water-Use Sectors*, ESCAP Water Resources Series Number 80, Bangkok, Thailand.

Van der Zaag, P., Seyam, I.M. and Savenije, H.H.G. (2002). Towards measurable criteria for the equitable sharing of international water resources. *Water Policy*, 4(1), 19-32.

Wang, L.Z., Fang, L. and Hipel, K.W. (2003). Cooperative water resources allocation based on equitable water rights, in *Proc. of the 2003 IEEE International Conference on Systems, Man and Cybernetics*, Hyatt Regency, Washington, DC, USA, pp. 4425-4430, October 5–8th, 2003.

Wolf, A.T. (1999). Criteria for equitable allocations: The heart of international water conflict. *Nat. Resour. Forum*, 23, 3-30.

Young, H.P., Okada, N. and Hashimoto, T. (1982). Cost allocation in water resources development. *Water Resour. Res.*, 18, 463-475.