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# A Mixed-Integer Optimization Model for Compressor Selection in Natural Gas Pipeline Network System Operations

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**ABSTRACT.** This paper presents a Mixed-Integer Linear Programming (MILP) model to optimize the compressor selection operations in natural gas pipeline network system. The objectives of natural gas pipeline network system operations are to minimize the operation costs and provide sufficient gas to the local customers. A pipeline network system is the most cost effective way for moving fluid products over long distances. In this case, it is used for transmitting natural gas from a producer to customers. To ensure demand for natural gas can be met, a dispatcher turns on or off compressor(s) in order to increase or decrease the amount of natural gas in the pipeline system. Compressor selection is one of the most critical operations in the natural gas pipeline network system because the costs associated with turning on or off the compressor make up a large percentage of the total operating costs. In order to minimize the operating costs of the pipeline system, the three most crucial factors that affect the costs are integrated into the MILP model. The three factors include the capacities of compressors, the energy used to turn on the compressors, and the energy used to turn them off. The MILP model provides the decision support in determining the optimal solutions for controlling the compressors. It was developed and verified using the operation data supplied by a gas pipeline company in Saskatchewan, Canada.

*Keywords:* Compressor selection, MILP, natural gas pipeline, optimization

## 1. Introduction

Since pipeline systems are the most cost effective ways for moving fluid products over long distances, they are important for transporting gas, oil, petroleum products as well as water in Canada and North America. Each major city in North America needs pipeline systems for its most basic need of drinking water distribution. In addition, according to Canadian Association of Petroleum Producers, 360,000 cubic meter of crude oil and over 0.5 billion cubic meter of natural gas are transported daily in Canada over 300,000 kilometers of pipeline systems. Automation of gas pipeline operations can potentially optimize the operations. The objective of the project is to construct a Mixed-Integer Linear Programming (MILP) model as part of a decision support system to assist operators in the task of compressor selection in order to satisfy customer demand with minimal operating cost.

In natural gas pipeline operations, a dispatcher is responsible for making two vital decisions: (1) increase or decrease compression in the pipelines, and (2) select individual compressor units to turn on/off. These decisions have a significant impact on effectiveness of the natural gas pipeline operation. When customer demand for natural gas increases, the dispatcher adds compression to the pipeline system by turning on one or more compressors. Alternatively, s/he turns off one or more compressors to reduce compression in the pipeline system if customer demand for natural gas decreases. Incorrect decisions made by the dispatcher on which compressor to turn on/off increases energy cost or may cause customer dissatisfaction. If the pressure of the pipeline drops when customer demand increases, at least one compressor should be opened until the gas pressure resumes an acceptable level. In addition, different compressors provide different compression outputs which increases pressure. Inappropriate operational actions for starting or stopping a compressor as well as the use of different types of compressors can affect the operating cost.

In actual operations, a dispatcher operating the pipeline system often obtains information on pressure and flow from a supervisory control and data acquisition (SCADA) system, and warning signals from a simulation system. For example, the simulation system provides information on when the system runs low on pressure, which affects the dispatcher's decision on selecting one among many compressors to turn on or off. This may be difficult because no formal guidelines have been established on compressor selection in a given situation. The decision can be influenced by many factors such as operation costs (which consist of fuel cost, start-up cost, and maintenance cost), the penalty cost (which is incurred if the compressor does not run for a certain period of time and avoids frequent start/stop actions), and customer demand. Ideally, the pipelines should be operated in the most cost effective manner while ensuring customer demand for natural gas is met. It is important to reduce operation costs because they normally assume more than 60% of the total cost. To

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assist the dispatcher in selecting compressors, a MILP model was developed.

The objectives of this study are 1) to apply the MILP model to the decision making process on compressor selection, 2) to compare the solution generated by the MILP model with the prioritized selection of compressors provided by two senior operators, and 3) to minimize the operation costs for the natural gas pipeline network. The paper proceeds as follows. Section 2 discusses some relevant research work and the problem domain of natural gas pipeline optimization. Section 3 discusses the methodology used to formulate the MILP model. Section 4 presents the application of the MILP model to optimize natural gas pipeline network operations, and specifically for the compressor selection task. Section 5 presents some results and discussions; and Section 6 concludes this paper.

## 2. Background

### 2.1. Problem Domain: Natural Gas Pipeline Operations

This project was conducted in co-operation with a gas pipeline company in Saskatchewan, Canada as a study on applying the MILP model for compressor selection in natural gas pipeline network system operations. The focus of the project was limited to the St. Louis East compressor station in the province of Saskatchewan, Canada. A schematic of the St. Louis East system is shown in Figure 1.

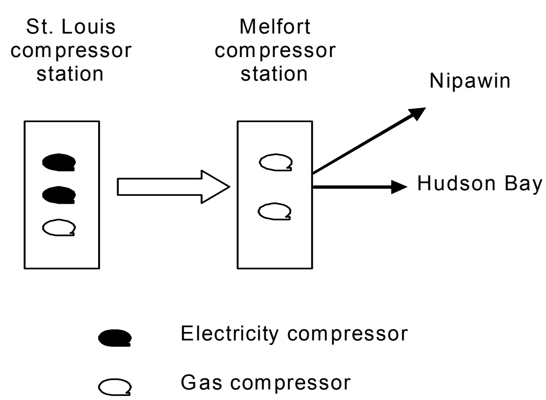


**Figure 1**. A schematic of the St. Louis East natural gas pipeline network system.

The system consists of two compressor stations, Melfort and St. Louis. These compressor stations are used to supply natural gas to two customer locations, Nipawin and Hudson Bay. In St. Louis, there are three compressor units. Two of these units are electric compressor units and the other is a gas compressor unit. In Melfort, there are two gas compressors. An electric compressor unit provides 250 horsepower and a gas unit provides 600 horsepower. The demand for natural gas from customers fluctuates depending on the season. In the winter, the demand for natural gas is usually higher than in the summer. In addition, the demand for natural gas also changes depending on the time of day. For example, in the morning, the demand is higher because the customer begins to use natural gas. In the afternoon, the demand is low since the facilities are already heated up in the morning. The customers can also be grouped into three types: industrial, dehydrator, and heat sensitive customers. Each type of customer reflects a different pattern of natural gas consumption as illustrated in Figure 2.

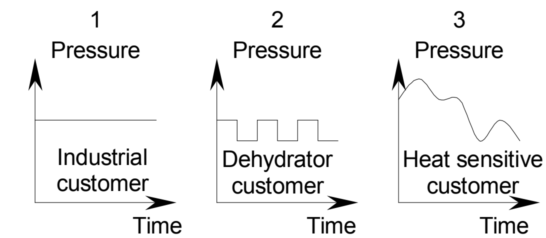


**Figure 2.** Natural gas pipeline customers.

The industrial customer has the same rate of natural gas consumption any time of the day. The dehydrator customer demands two set amounts of natural gas at different periods of time. For example, between 8:00 to 10:00 am, the demand is 240 $10^3$ $m^3$/day while between 10:00 am to 12:00 am, the demand is 200 $10^3$ $m^3$/day. The demand of the heat sensitive customer fluctuates over time and is difficult to predict. In practice, the demand for natural gas can fluctuate from 200 $10^3$ $m^3$/day to over 560 $10^3$ $m^3$/day within one or two hours. A demand of 200 $10^3$ $m^3$/day can be handled from St. Louis with one compressor unit. When the demand goes to over 560 $10^3$ $m^3$/day, all units at St. Louis and both units at Melfort are needed. It is the job of the operator to know ahead of time when the largest volume requirement will occur and to be ready for it. Otherwise, the system pressures at Nipawin and Hudson Bay will be below the required minimum.

The graph showing supply and demand of gas in the pipelines are displayed to the operators. The graphs are generated during operations by a SaskEnergy/Transgas (hereafter referred to as TGL) simulator program. The display includes a compressor discharge pressure curve versus custommer station pressure curve as shown in figure 3. With the graphs, the dispatcher or operator is responsible for moni toring closely the volume of natural gas supplied to the St. Louis East customers. In addition, s/he needs to decide which of the five compressors to use and when to turn on/off compression in order to control the volume of natural gas. Sometimes, the dispatchers might make unnecessary start/stop decisions because they lack experience. A minimum of four months is required to train new dispatchers to operate the system. However, it takes a number of years for them to gain proficiency in operating the system smoothly and cost effectively.

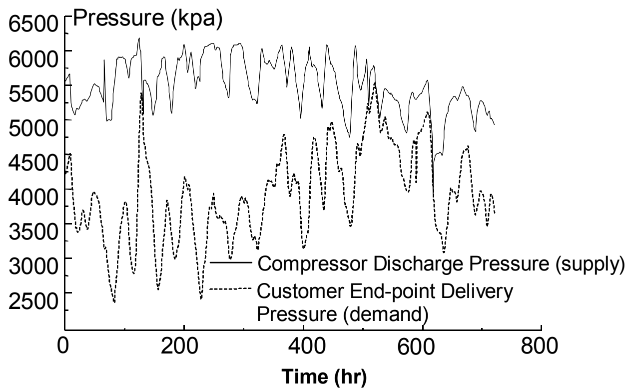


**Figure 3**. Demand versus supply graph.

The typical display shown in Figure 3 provides pressure information to dispatchers. The two curves indicate the relationship between demand and supply and the gap between the two curves is called the comfort zone (CZ). If the gap between these curves is wide, then customer satisfaction is guaranteed but the cost is high. If the comfort zone is too narrow, then the operation is cost effective but future customer demand may not be satisfied. Hence, the dispatcher needs to select a compressor to increase pressure in the pipelines while maintaining the optimum operating pressure in the pipeline system.

### 2.2. Requirements for the Methodology

There are a number of requirements for the methodology for optimization of the natural gas pipeline operations. Firstly, the methodology needs to be robust so as to take into consideration most important aspects of compressor selection for natural gas pipeline operations, and secondly, it should provide satisfactory results. The Mixed-Integer Linear Programming (MILP) model is believed to meet these requirements due to the following considerations. Firstly, the MILP model constrains the decision variables to assume only integer values (i.e. 1 or 0) in the optimal solution. The use of integer variables can optimize the problem where the solution can either be 0 or 1. For example, a decision variable $x_1$, called a 0-1 integer variable, can be used for modeling the on/off decision of whether to turn on or off a compressor. Secondly, the MILP model can also be used to optimize operating costs.

In the domain of gas compressor selection, compressors can be turned either on or off. Thus, a compressor operation can be represented as 0 or 1. Also, compressor selection requires a type of integer programming in which not all the variables to be optimized have integer values. Hence, it is more appropriate to use Mixed-Integer Linear Programming for modeling compressor operations of natural gas pipeline network systems. Due to the linear nature of the operation costs, it can be expressed mathematically in the objective function of the MILP model, but the results must comply with defined constraints such as the demand for natural gas within a specified time horizon.

### 2.3. Relevant Literature

Several researchers used Mixed-Integer Linear Programming (MILP) in solving problems in industry. Vasquez-Alvarez and Pinto [2003] developed a MILP model for the synthesis of protein purification processes by incorporating losses in the target protein along with the purification process. The model provides some guidelines for evaluating the trade-off between products by purity and quantity. Xie et al. [2003] applied a MILP model to manage compiling time for dynamic voltage scaling (DVS) settings. The model was formulated to account for DVS energy switching overhead, by providing control on setting and by considering multiple data categories in the optimization. Jain and Grossmann [2001] combined the MILP approach with a constraint programming (CP) technique to solve problems of job scheduling on parallel machines. Adjiman et al. [1998] formulated a mixed-integer optimization model to solve problems in the process of flowsheets that convert raw materials into desired products. Maulik et al. [1992] presented a new cell-level analog circuit synthesis methodology by applying mixed-integer nonlinear programming. Anbil et al. [1993] suggested that the Mixed-Integer Linear Programming MILP technique had saved $20 million per year over the period of 1985-90 and $3-5 million from 1990-92 at American Airlines. Ciriani [1998] suggested that the MILP technique could be used as a preprocessor while combinatorial problems with a flat objective function can benefit from the use of heuristics. He also concluded that model formulation could provide a better integer approximation. Dillenberger and Wollensak [1993] reported on the successful integration of the MILP technique in the decision support system of IBM's Sindelfingen plant. Ballintjin [1993] described the MILP technique that was used to control mode switching at acceptable levels.

## 3. Methodology

In this paper, the methodology of a MILP model has been tailored to optimize natural gas pipeline network operations, particularly in the compressor selection task. For this task, the model's main functions consist of decision variables, an objective function and constraint equations. The decision variables determine the results of the objective function and constraints. The objective function represents the total operating costs within a specified time horizon. In this study, the operating cost is the sum of the fuel costs, maintenance cost for each compressor, the start-up cost when a compressor is started after being shut down, and penalty cost for not operating a compressor continuously for a certain period of time. The following section will describe the MILP model formulation.

**3.1. MILP Model Formulation**

3.1.1 Decision Variables

The decision variables determine the status of compressors, and represent the model's advice on how to operate the compressors in the natural gas pipeline system. In natural gas pipeline operations, the status of a compressor can either be on or off. In the St. Louis East system, there are two kinds of compressor: gas or electric. The decision variables for this model are defined as $G_{i,p}$ which represents the status of gas compressor $i$ in period $p$ while $E_{i,p}$ denotes the status of electric compressor $i$ in period $p$. The values of $G_{i,p}$ and $E_{i,p}$ can be defined as follows:

$$\left.\begin{matrix} G_{i,p} \\ E_{i,p} \end{matrix}\right\} = \frac{1 \quad \text{if the compressor } i \text{ is turned on in period } p}{0 \quad \text{if the compressor } i \text{ is turned off in period } p}$$

where $G_{i,p}$ = 1 if the natural gas compressor $i$ is selected to operate in period $p$, and 0 otherwise; $E_{i,p}$ = 1 if the electric compressor $i$ is selected to operate in period $p$, and 0 otherwise.

The decision variables are used for determining the operating costs. The objective function of the MILP model can be formulated by incorporating the decision variables.

3.1.2. Objective Functions

The objective function is formulated for determining the optimal operating costs associated with compressor operations within a specified number of operating periods. In this case study, the MILP model is formulated to determine the optimal sets of selected compressors among many possible selected combinations of compressors and to minimize the total operating costs within six operating periods. The reason for using six operating periods as the time horizon is that a dispatcher can have enough time to effectively control the volume of natural gas in the pipeline system if the selected compressors are given within six periods. The total operating cost is the sum of fuel cost, maintenance cost, start-up cost and shutdown cost. The formulae to calculate these costs must be developed before the objective function can be finalized. The equations for calculating the fuel cost, maintenance cost, start up cost and shutdown cost are described in the following.

*Fuel cost*

Fuel cost is the cost associated with the amount of energy used in operating each compressor within a time horizon. In general, compressors have different rates of energy consumption. For example, an electric compressor utilizes more energy in turning on/off than a gas compressor. As a result, the fuel cost for operating electric compressor is normally higher than that of a gas compressor. The fuel cost of electric and gas compressor can be calculated with the following equations.

$$\text{Min} \quad \sum_{i=1}^{n}\sum_{p=1}^{n} Fg_{i,p} \times G_{i,p} \tag{1}$$

$$\text{Min} \quad \sum_{i=1}^{n}\sum_{p=1}^{n} Fe_{i,p} \times E_{i,p} \tag{2}$$

where $Fg$ is the fuel cost of gas compressor, $G_{i,p}$ is gas compressor $i$ in period $p$, $Fe$ is the fuel cost of electric compressor, $E_{i,p}$ is electric compressor $i$ in period $p$.

*Maintenance cost*

The maintenance cost refers to regular maintenance cost of each compressor. Each compressor can only be operated for a limited number of hours, after which it must be shut off for maintenance. For example, in St. Louis East natural gas pipeline system, the total operating hours for each compressor may not exceed 1000 hours. After 1000 hours, a compressor should be serviced and maintained. The following equation is used for calculating the maintenance cost for gas and electric compressor, respectively.

$$\sum_{i=1}^{n}\sum_{p=1}^{n} Mg \times G_{i,p} \tag{3}$$

$$\sum_{i=1}^{n}\sum_{p=1}^{n} Me \times E_{i,p} \tag{4}$$

where $Mg$ and $Me$ are the maintenance costs of gas compressor and an electric compressor, respectively, $G_{i,p}$ is the gas compressor unit $i$ in period $p$, and $E_{i,p}$ is electric compressor $i$ in period $p$.

*Start-up cost*

When a compressor is turned on, there is a cost attached due to the energy spent during its open operations. The start-up cost can be calculated using the following equations:

$$\sum_{i=1}^{n}\sum_{p=1}^{n} Sg \times G_{i,p} - Sg \times G_{i,p} \times G_{i,p-1} \tag{5}$$

$$\sum_{i=1}^{n}\sum_{p=1}^{n} Se \times E_{i,p} - Se \times E_{i,p} \times E_{i,p-1} \tag{6}$$

where $Sg$ is the start up cost of a gas compressor, $Se$ is the start up cost of an electric compressor, $G_{i,p}$ is the gas compressor unit $i$ in period $p$, and $E_{i,p}$ is the electric compressor unit $i$ in period $p$

*Shutdown cost*

Shutdown cost is the cost incorporated in the MILP model to avoid component wear of each compressor. Each compressor should be selected so that the number of times of turning on/off is minimized to reduce component wear. After a compressor is turned on, it should be continuously operating for as long as possible, which is assumed to be a minimum of

3 hours in this example situation. When a compressor is shut off because it is not needed, it should be off for as long as possible. The start up or shut off cost of a compressor is given as a penalty cost, and this cost is added to the total operation costs. For example, compressor 1 and compressor 2 provide the same amount of horsepower. In time interval 1, compressor 1 is selected to turn on to meet the customer demand, while compressor 2 is not needed. In time interval 2, the customer demand remains the same. Therefore, compressor 1 remains on and compressor 2 remains off in order to reduce total operating cost. Based on the expertise provided by two senior operators from a gas transportation company, each compressor should continuously operate for at least 3 hours to avoid component wear. The following equation is used to calculate the penalty cost of the compressor running for less than 3 hours.

$$\begin{aligned}&+PCg\sum_{i=1}^{3}\begin{bmatrix}(3-G_{i,1}-G_{i,2}-G_{i,3})+(3-G_{i,2}-G_{i,3}-G_{i,4})\\+(3-G_{i,3}-G_{i,4}-G_{i,5})+(3-G_{i,4}-G_{i,5}-G_{i,6})\end{bmatrix}\\&+PCe\sum_{i=1}^{2}\begin{bmatrix}(3-E_{i,1}-E_{i,2}-E_{i,3})+(3-E_{i,2}-E_{i,3}-E_{i,4})\\+(3-E_{i,3}-E_{i,4}-E_{i,5})+(3-G_{i,4}-G_{i,5}-G_{i,6})\end{bmatrix}\end{aligned} \tag{7}$$

where $PCg$ is the penalty cost if a gas compressor is running for less than 3 periods, $PCe$ is the penalty cost if an electric compressor is running for less than 3 periods, $G_{i,p}$ is gas compressor $i$ in period $p$, $E_{i,p}$ is electric compressor $i$ in period $p$.

After the equations (1) to (7) for calculating the fuel cost, maintenance cost, start-up and shutdown cost have been formulated, the objective function can be finalized as follows.

$$\begin{aligned}\min Z=&\sum_{i=1}^{n}\sum_{p=1}^{n}G_{i,p}(Fg+Mg+Sg)-\sum_{i=1}^{n}\sum_{p=1}^{n}Sg\times G_{i,p}\times G_{i,p-1}\\&+\sum_{i=1}^{n}\sum_{p=1}^{n}E_{i,p}(Fe+Me+Se)-\sum_{i=1}^{n}\sum_{p=1}^{n}Se\times G_{i,p}\times G_{i,p-1}\\&+PCg\sum_{i=1}^{n}\begin{bmatrix}(3-G_{i,1}-G_{i,2}-G_{i,3})+(3-G_{i,2}-G_{i,3}-G_{i,4})\\+(3-G_{i,3}-G_{i,4}-G_{i,5})+(3-G_{i,4}-G_{i,5}-G_{i,6})\end{bmatrix}\\&+PCe\sum_{i=1}^{n}\begin{bmatrix}(3-E_{i,1}-E_{i,2}-E_{i,3})+(3-E_{i,2}-E_{i,3}-E_{i,4})\\+(3-E_{i,3}-E_{i,4}-E_{i,5})+(3-G_{i,4}-G_{i,5}-G_{i,6})\end{bmatrix}\end{aligned} \tag{8}$$

3.1.3 Constraints

The objective function as finalized in equation (8) is subjected to the following constraints:

*Customer demand*

The primary objective of natural gas pipeline network operations is to deliver natural gas to customers according to their demand. During operations, the dispatcher turns on or off compressors in order to increase pressure in the pipelines so that natural gas can be transported from the compressor stations to the customer locations. Break horsepower (BHP) defines the capacity of a compressor which increases pressure in the pipeline system. According to the senior operators from the St. Louis East natural gas pipeline system, BHP requirement can be calculated using equation (9) (Uraikul et al, 2000), or can be obtained from the horsepower requirement chart shown in Figure 4.

$$\begin{aligned}\text{BHP requirement} = 2.77411\times(&\text{St. Louis flow}\\&+\text{Melfort flow})-1132\end{aligned} \tag{9}$$

From equation (9), the dispatcher can obtain the horse power requirement using St. Louis and Melfort flows. For example, in Figure 4, if the flows are lower than 400 $10^3$/day, no additional horsepower is required.

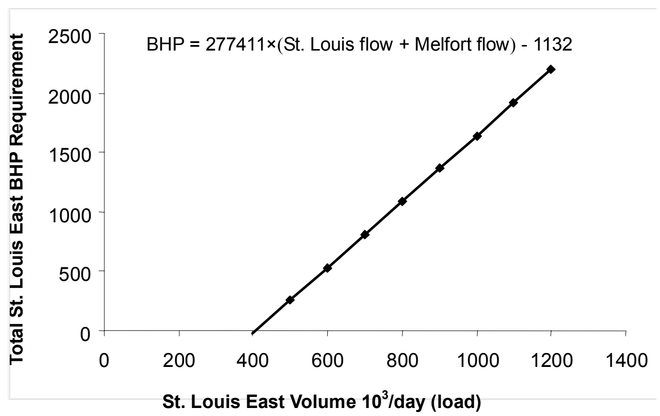


**Figure 4**. Horsepower requirement chart.

where $BHP$ is the break horsepower requirement, $S\_Flow$ is the gas flow measured at St. Louis station, and $M\_Flow$ is the gas flow measured at Melfort station.

The customer-demand constraint can be represented using the following equation.

$$\sum_{i=1}^{n}\sum_{p=1}^{n}Hg_i(G_{i,p})+\sum_{i=1}^{n}\sum_{p=1}^{n}He_i(E_{i,p})\geq D_p\forall p \tag{10}$$

where $Hg_i$ is the horse power of a gas compressor i and $He_i$ is the horse power of an electric compressor $i$, $Dp$ is the customer demand in period $p$, $G_{i,p}$ is gas compressor $i$ in period $p$, $E_{i,p}$ is electric compressor $i$ in period $p$.

*Status of compressors*

The status of a compressor can either be on or off, thus the value of $G_{i,p}$ and $E_{i,p}$ is limited to 0 or 1 (1 if the compressor is on and 0 otherwise). The following equations are formulated in order to limit the value of $G_{i,p}$ and $E_{i,p}$ to be the desired value of 0 or 1.

$$G_{i,p} + G_{i,p-1} - G_{i,p} \times G_{i,p-1} \geq 0 \quad \forall i, p \tag{11}$$

$$G_{i,p} + G_{i,p-1} - G_{i,p} \times G_{i,p-1} \leq 1 \quad \forall i, p \tag{12}$$

$$-G_{i,p} - G_{i,p-1} + 2 \times G_{i,p} \times G_{i,p-1} \leq 0 \quad \forall i, p \tag{13}$$

$$E_{i,p} + E_{i,p-1} - G_{i,p} \times G_{i,p-1} \geq 0 \quad \forall i, p \tag{14}$$

$$E_{i,p} + E_{i,p-1} - E_{i,p} \times E_{i,p-1} \leq 1 \quad \forall i, p \tag{15}$$

$$-E_{i,p} - E_{i,p-1} + 2 \times E_{i,p} \times E_{i,p-1} \leq 0 \quad \forall i, p \tag{16}$$

## 4. Application of the MILP Model to a Natural Gas Pipeline Network System

Having completed the requirements for setting up the MILP model, the model can then be applied to an actual natural gas pipeline network system. In general, each natural gas pipeline system adopts different rules for utilizing the compressors. The assumptions for the St. Louis East system are explicated as follows.

1) Each compressor must be taken into service after it has been operating for 1000 hours. Therefore, the maintenance cost for each compressor is equal to 1/1000 times the maintenance cost of each compressor.

2) Customer demand must be satisfied in every period and the amount of the demand is converted from the load of St. Louis and Melfort flow to BHP requirement based on the BHP requirement chart.

3) A compressor usually requires 1 to 2 days service. This is not modeled since our time horizon is only 6 hours, and at the beginning of each hour, it is reasonable to assume that the availability of each compressor would be known with certainty.

4) A compressor should be operating for as long as 3 hours to avoid component wear, otherwise penalty cost will be applied.

5) The customer demand must be known with certainty for 6-hour periods.

6) The location of compressors does not affect the transmission efficiency of gas from the St. Louis and Melfort stations to Nipawin or Hudson Bay.

Based on these assumptions, the MILP model was applied to the gas pipeline system in the St. Louis East system. There are two compressor stations in this section of the pipeline system: the St. Louis and Melfort stations. There are two electric compressors labeled $E_1$ and $E_2$ and one gas compressor labeled $G_1$ at the St. Louis station, while the Melfort station has two gas compressors labeled $G_2$ and $G_3$. These compressors can provide different capacities of BHP (break horsepower) so as to adjust the gas pressure in the pipeline for different situations. The natural gas is transmitted from the two compressor stations to the two customer locations of Nipawin and Hudson Bay. The important parameters associated with each compressor are listed as follows:

$E_1$ = electric compressor unit 1 (BHP = 250 km$^3$/d)

$E_2$ = electric compressor unit 2 (BHP = 250 km$^3$/d)

$G_1$ = gas compressor unit 1 (BHP = 600 km$^3$/d)

$G_2$ = gas compressor unit 2 (BHP = 600 km$^3$/d)

$G_3$ = gas compressor unit 3 (BHP = 600 km$^3$/d)

Based on the objective function (8), the objective function of the St. Louis East system and its corresponding constraints can be finalized as shown in Table 1.

The model optimizes the natural gas pipeline operations by providing the optimal results of compressor selection based on the required parameters specified in the model. These parameters include the types of compressors, the number of compressors in the pipeline system, and the number of periods to be optimized. Having identified the important parameters, the operating costs that include the fuel cost of each compressor, maintenance cost, start up cost, and shutdown cost can be substituted into the model. However, these costs cannot be fixed because they vary depending on the operating seasons and pricing of gas. The operating costs and results generated by the MILP model will be discussed in following section under “Results and Discussions”.

## 5. Results and Discussions

This section describes the results generated by the MILP model and compares the results with the prioritized orders for running compressors provided by two senior operators from the St. Louis East natural gas pipeline system.

Before the model can be initialized, the following inputs must be identified:

- Horse power of each compressor is known with certainty as mentioned in section 4. In the St. Louis East system, the BHP of each compressor is listed as follows:

  Horse power of gas unit 1 (Hg1) = 600, gas unit 2 (Hg2) = 600, gas unit 3 (Hg3) = 600, electric unit 1 (He1) = 250, electric unit 2 (He2) = 250
- The initial values of compressor status must be provided to the model because the availability of each compressor must be known with certainty before the calculation is initialized. For this scenario, the initial value of each compressor is assumed as 0, meaning that all units are off in the beginning.

  The initial value of gas compressor unit 1 (G10) = 0, gas compressor unit 2 (G20) = 0, gas compressor unit 3 (G30) = 0, electric compressor unit 1 (E10) = 0, electric compressor unit 2 (E20) = 0
- Operating costs are not known with certainty and always varied depending on the operating season and gas pricing. For this scenario, the fuel cost, maintenance cost, start-up

**Table 1.** Objective Function and Constraints of the St. Louis East System

*Min:*

$$
\begin{aligned}
&(Fg+Mg+Sg)\times G11+(Fg+Mg+Sg)\times G12+(Fg+Mg+Sg)\times G13+(Fg+Mg+Sg)\times G14+(Fg+Mg+Sg)\times G15\\
&+(Fg+Mg+Sg)\times G16+(Fg+Mg+Sg)\times G21+(Fg+Mg+Sg)\times G22+(Fg+Mg+Sg)\times G23+(Fg+Mg+Sg)\times G24\\
&+(Fg+Mg+Sg)\times G25+(Fg+Mg+Sg)\times G26+(Fg+Mg+Sg)\times G31+(Fg+Mg+Sg)\times G32+(Fg+Mg+Sg)\times G33\\
&+(Fg+Mg+Sg)\times G34+(Fg+Mg+Sg)\times G35+(Fg+Mg+Sg)\times G36-(Sg)\times G11\times G10-(Sg)\times G12\times G11\\
&-(Sg)\times G13\times G12-(Sg)\times G14\times G13-(Sg)\times G15\times G14-(Sg)\times G16\times G15-(Sg)\times G21\times G20-(Sg)\times G22\times G21\\
&-(Sg)\times G23\times G22-(Sg)\times G24\times G23-(Sg)\times G25\times G24-(Sg)\times G26\times G25-(Sg)\times G31\times G30-(Sg)\times G32\times G31\\
&-(Sg)\times G33\times G32-(Sg)\times G34\times G33-(Sg)\times G35\times G34-(Sg)\times G36\times G35
\end{aligned}
$$

$$
\begin{aligned}
&+(Fe+Me+Se)\times E11+(Fe+Me+Se)\times E12+(Fe+Me+Se)\times E13+(Fe+Me+Se)\times E14+(Fe+Me+Se)\times E15\\
&+(Fe+Me+Se)\times E16+(Fe+Me+Se)\times E21+(Fe+Me+Se)\times E22+(Fe+Me+Se)\times E23+(Fe+Me+Se)\times E24\\
&+(Fe+Me+Se)\times E25+(Fe+Me+Se)\times E26-(Se)\times E11\times E10-(Se)\times E12\times E11-(Se)\times E13\times E12-(Se)\times E14\times E13\\
&-(Se)\times E15\times E14-(Se)\times E16\times E15-(Se)\times E21\times E20-(Se)\times E22\times E21-(Se)\times E23\times E22-(Se)\times E24\times E23\\
&-(Se)\times E25\times E24-(Se)\times E26\times E25+PCg\times(3-G11-G12-G13)+PCg\times(3-G12-G13-G14)
\end{aligned}
$$

$$
\begin{aligned}
&+PCg\times(3-G11-G12-G13)+PCg\times(3-G12-G13-G14)+PCg\times(3-G13-G14-G15)+PCg\times(3-G14-G15-G16)\\
&+PCg\times(3-G21-G22-G13)+PCg\times(3-G22-G23-G24)+PCg\times(3-G23-G24-G25)+PCg\times(3-G24-G25-G26)\\
&+PCg\times(3-G31-G32-G33)+PCg\times(3-G32-G33-G34)+PCg\times(3-G33-G34-G35)+PCg\times(3-G34-G35-G36)
\end{aligned}
$$

$$
\begin{aligned}
&+PCe\times(3-E11-E12-E13)+PCe\times(3-E12-E13-E14)+PCe\times(3-E13-E14-E15)+PCe\times(3-E14-E15-E16)\\
&+PCe\times(3-E21-E22-E23)+PCe\times(3-E22-E23-E24)+PCe\times(3-E23-E24-E25)+PCe\times(3-E24-E25-E26)
\end{aligned}
$$

*Subject to:*

$$
\begin{aligned}
&Hg_1\times(G_{11})+Hg_2\times(G_{21})+Hg_3\times(G_{31})+He_1\times(E_{11})+He_2\times(E_{21}) >= D_1;\\
&Hg_1\times(G_{12})+Hg_2\times(G_{22})+Hg_3\times(G_{32})+He_1\times(E_{12})+He_2\times(E_{22}) >= D_2;\\
&Hg_1\times(G_{13})+Hg_2\times(G_{23})+Hg_3\times(G_{33})+He_1\times(E_{13})+He_2\times(E_{23}) >= D_3;\\
&Hg_1\times(G_{14})+Hg_2\times(G_{24})+Hg_3\times(G_{34})+He_1\times(E_{14})+He_2\times(E_{24}) >= D_4;\\
&Hg_1\times(G_{15})+Hg_2\times(G_{25})+Hg_3\times(G_{35})+He_1\times(E_{15})+He_2\times(E_{25}) >= D_5;\\
&Hg_1\times(G_{16})+Hg_2\times(G_{26})+Hg_3\times(G_{36})+He_1\times(E_{16})+He_2\times(E_{26}) >= D_6;
\end{aligned}
$$

$$
\begin{bmatrix}
G_{11}+G_{10}-(G_{11}\times G_{10}) >= 0;\\
G_{12}+G_{11}-(G_{12}\times G_{11}) >= 0;\\
G_{13}+G_{12}-(G_{13}\times G_{12}) >= 0;\\
G_{14}+G_{13}-(G_{14}\times G_{13}) >= 0;\\
G_{15}+G_{14}-(G_{15}\times G_{14}) >= 0;\\
G_{16}+G_{15}-(G_{16}\times G_{15}) >= 0;\\
G_{21}+G_{20}-(G_{21}\times G_{20}) >= 0;\\
G_{22}+G_{21}-(G_{22}\times G_{21}) >= 0;\\
G_{23}+G_{22}-(G_{23}\times G_{22}) >= 0;\\
G_{24}+G_{23}-(G_{24}\times G_{23}) >= 0;\\
G_{25}+G_{24}-(G_{25}\times G_{24}) >= 0;\\
G_{26}+G_{25}-(G_{26}\times G_{25}) >= 0;\\
G_{31}+G_{30}-(G_{31}\times G_{30}) >= 0;\\
G_{32}+G_{31}-(G_{32}\times G_{31}) >= 0;\\
G_{33}+G_{32}-(G_{33}\times G_{32}) >= 0;\\
G_{34}+G_{33}-(G_{34}\times G_{33}) >= 0;\\
G_{35}+G_{34}-(G_{35}\times G_{34}) >= 0;\\
G_{36}+G_{35}-(G_{36}\times G_{35}) >= 0;
\end{bmatrix}
\begin{bmatrix}
G_{11}+G_{10}-(G_{11}\times G_{10}) <= 1;\\
G_{12}+G_{11}-(G_{12}\times G_{11}) <= 1;\\
G_{13}+G_{12}-(G_{13}\times G_{12}) <= 1;\\
G_{14}+G_{13}-(G_{14}\times G_{13}) <= 1;\\
G_{15}+G_{14}-(G_{15}\times G_{14}) <= 1;\\
G_{16}+G_{15}-(G_{16}\times G_{15}) <= 1;\\
G_{21}+G_{20}-(G_{21}\times G_{20}) <= 1;\\
G_{22}+G_{21}-(G_{22}\times G_{21}) <= 1;\\
G_{23}+G_{22}-(G_{23}\times G_{22}) <= 1;\\
G_{24}+G_{23}-(G_{24}\times G_{23}) <= 1;\\
G_{25}+G_{24}-(G_{25}\times G_{24}) <= 1;\\
G_{26}+G_{25}-(G_{26}\times G_{25}) <= 1;\\
G_{31}+G_{30}-(G_{31}\times G_{30}) <= 1;\\
G_{32}+G_{31}-(G_{32}\times G_{31}) <= 1;\\
G_{33}+G_{32}-(G_{33}\times G_{32}) <= 1;\\
G_{34}+G_{33}-(G_{34}\times G_{33}) <= 1;\\
G_{35}+G_{34}-(G_{35}\times G_{34}) <= 1;\\
G_{36}+G_{35}-(G_{36}\times G_{35}) <= 1;
\end{bmatrix}
\begin{bmatrix}
-G_{11}-G_{10}+2\times(G_{11}\times G_{10}) <= 0;\\
-G_{12}-G_{11}+2\times(G_{12}\times G_{11}) <= 0;\\
-G_{13}-G_{12}+2\times(G_{13}\times G_{12}) <= 0;\\
-G_{14}-G_{13}+2\times(G_{14}\times G_{13}) <= 0;\\
-G_{15}-G_{14}+2\times(G_{15}\times G_{14}) <= 0;\\
-G_{16}-G_{15}+2\times(G_{16}\times G_{15}) <= 0;\\
-G_{21}-G_{20}+2\times(G_{21}\times G_{20}) <= 0;\\
-G_{22}-G_{21}+2\times(G_{22}\times G_{21}) <= 0;\\
-G_{23}-G_{22}+2\times(G_{23}\times G_{22}) <= 0;\\
-G_{24}-G_{23}+2\times(G_{24}\times G_{23}) <= 0;\\
-G_{25}-G_{24}+2\times(G_{25}\times G_{24}) <= 0;\\
-G_{26}-G_{25}+2\times(G_{26}\times G_{25}) <= 0;\\
-G_{31}-G_{30}+2\times(G_{31}\times G_{30}) <= 0;\\
-G_{32}-G_{31}+2\times(G_{32}\times G_{31}) <= 0;\\
-G_{33}-G_{32}+2\times(G_{33}\times G_{32}) <= 0;\\
-G_{34}-G_{33}+2\times(G_{34}\times G_{33}) <= 0;\\
-G_{35}-G_{34}+2\times(G_{35}\times G_{34}) <= 0;\\
-G_{36}-G_{35}+2\times(G_{36}\times G_{35}) <= 0;
\end{bmatrix}
$$

$$
\begin{bmatrix}
E_{11}+E_{10}-(E_{11}\times E_{10}) >= 0;\\
E_{12}+E_{11}-(E_{12}\times E_{11}) >= 0;\\
E_{13}+E_{12}-(E_{13}\times E_{12}) >= 0;\\
E_{14}+E_{13}-(E_{14}\times E_{13}) >= 0;\\
E_{15}+E_{14}-(E_{15}\times E_{14}) >= 0;\\
E_{16}+E_{15}-(E_{16}\times E_{15}) >= 0;\\
E_{21}+E_{20}-(E_{21}\times E_{20}) >= 0;\\
E_{22}+E_{21}-(E_{22}\times E_{21}) >= 0;\\
E_{23}+E_{22}-(E_{23}\times E_{22}) >= 0;\\
E_{24}+E_{23}-(E_{24}\times E_{23}) >= 0;\\
E_{25}+E_{24}-(E_{25}\times E_{24}) >= 0;\\
E_{26}+E_{25}-(E_{26}\times E_{25}) >= 0;
\end{bmatrix}
\begin{bmatrix}
E_{11}+E_{10}-(E_{11}\times E_{10}) <= 1;\\
E_{12}+E_{11}-(E_{12}\times E_{11}) <= 1;\\
E_{13}+E_{12}-(E_{13}\times E_{12}) <= 1;\\
E_{14}+E_{13}-(E_{14}\times E_{13}) <= 1;\\
E_{15}+E_{14}-(E_{15}\times E_{14}) <= 1;\\
E_{16}+E_{15}-(E_{16}\times E_{15}) <= 1;\\
E_{21}+E_{20}-(E_{21}\times E_{20}) <= 1;\\
E_{22}+E_{21}-(E_{22}\times E_{21}) <= 1;\\
E_{23}+E_{22}-(E_{23}\times E_{22}) <= 1;\\
E_{24}+E_{23}-(E_{24}\times E_{23}) <= 1;\\
E_{25}+E_{24}-(E_{25}\times E_{24}) <= 1;\\
E_{26}+E_{25}-(E_{26}\times E_{25}) <= 1;
\end{bmatrix}
\begin{bmatrix}
-E_{11}-E_{10}+2\times(E_{11}\times E_{10}) <= 0;\\
-E_{12}-E_{11}+2\times(E_{12}\times E_{11}) <= 0;\\
-E_{13}-E_{12}+2\times(E_{13}\times E_{12}) <= 0;\\
-E_{14}-E_{13}+2\times(E_{14}\times E_{13}) <= 0;\\
-E_{15}-E_{14}+2\times(E_{15}\times E_{14}) <= 0;\\
-E_{16}-E_{15}+2\times(E_{16}\times E_{15}) <= 0;\\
-E_{21}-E_{20}+2\times(E_{21}\times E_{20}) <= 0;\\
-E_{22}-E_{21}+2\times(E_{22}\times E_{21}) <= 0;\\
-E_{23}-E_{22}+2\times(E_{23}\times E_{22}) <= 0;\\
-E_{24}-E_{23}+2\times(E_{24}\times E_{23}) <= 0;\\
-E_{25}-E_{24}+2\times(E_{25}\times E_{24}) <= 0;\\
-E_{26}-E_{25}+2\times(E_{26}\times E_{25}) <= 0;
\end{bmatrix}
$$

cost, and shutdown cost are assumed as follows.

Fuel cost of gas compressor (Fg) = 50, Maintenance cost of gas compressor (Mg) = 3, Start-up cost of gas compressor (Sg) = 40, Fuel cost of electric compressor (Fe) = 35, Maintenance cost of electric compressor (Me) = 1.5, Start-up cost of electric compressor (Se) = 30, Penalty cost of gas compressor (PCg) = 33, Penalty cost of electric compressor (PCe) = 23.

- The customer demand must be given in 6 operating periods. In the real operation, the demand can be obtained using the company's forecasting facility. For this scenario, the customer demand in 6 periods is assumed as follows:

  Customer demand in period 1 (D1) = 200, period 2 (D2) = 700, period 3 (D3) = 850, period 4 (D4) = 1350, period 5 (D5) = 1280, period 6 (D6) = 1800

Having identified the required inputs which include BHP of each compressor, operating costs, initial status of compressor, and customer demand, the model generated results as follows:

Period 1. G2, E1

Period 2. G1, G2, G3, G4, G5, G6

Period 3. G1, G2, G3, G4, G5, G6

Period 4. G1, G2, G3, G4, G5, G6

Period 5. G1, G2, G3, G4, G5, G6

Period 6. G1, G2, G3

Total operation costs is $1471.500.

The MILP model generated the recommendations on compressor selection operations in 6 periods with the total operating costs of $1471. To investigate the optimal solution based on the parameters provided, the results generated by the model will be compared to the prioritized orders of running compressors provided by two senior operators from the St. Louis East System.

The two expert operators indicated that in real operations, there must be one compressor unit which always operates to maintain the minimum pressure in the pipeline. For the St. Louis East system, the dispatcher will randomly select one compressor among the five compressors to operate all the time, depending on the availability of the compressor and the number of hours before the compressor is taken off for maintenance. For this scenario, an electric compressor at the St .Louis compressor station (E1) will be selected as the all-time-running unit.

For a given amount of horsepower required, a different combination of compressor units can be turned on/off based on the senior operators' knowledge on the prioritized order of running compressors, which is listed as follows.

1. Free flow (no compression) E1 always on
2. (0 < BHP < 800): turn on St. Louis gas compressor G1
3. (800 ≤ BHP < 1200): turn on St. Louis gas compressor G1 and Melfort gas compressor G2
4. (1200 ≤ BHP < 1600): turn on St. Louis gas compressor G1 and Melfort gas compressor G2 and St. Louis electric compressor E2
5. (1600 ≤ BHP < 2200): turn on St. Louis gas compressor G1 and electric compressor E2, and Melfort gas compressor G2 and gas compressor G3.

With the same set of data on customer demand in 6 operating periods as previously provided to the MILP model, the following combinations are the results obtained from the prioritized orders of running compressors:

Period 1: E1 + G1

Period 2: E1 + G1

Period 3: E1 + G1 + G2

Period 4: E1 + G1 + G2 + E2

Period 4: E1 + G1 + G2 + E2

Period 5: G1 + G2 + G3 +E1 +E2

The total operation costs based on these combinations is $1691. In comparison to the results generated by the MILP model, the company can potentially save $219.5, or 13% (219.6/1691) of the operating costs based on this scenario.

The model has also been applied under various scenarios of operating costs and customer demands. It can be observed from the results that the fuel cost per BHP unit has a major impact on the model. For example, if the fuel cost per unit of electric compressor is 150% higher than that of the natural gas compressor, the MILP model would suggest to only operate gas compressors while letting electric compressors stay off. In real operations, however, the cost of operating the electric compressor is relatively higher than that of the gas compressor, but not too high. In short, if the ratio of fuel cost per unit of electric compressor and gas compressor is not high, the model would provide sound advice on the compressor selection to the dispatcher.

## 6. Conclusions

In natural gas pipeline network system operations, compressor selection is one of the most critical decisions because satisfaction of customer demand for natural gas directly depends on turning on or off the compressors. If the dispatcher fails to optimally operate the compressors due to a lack of information, the company will suffer losses. A Mixed-Integer Linear Programming (MILP) model can provide support to this crucial decision making process by suggesting the solution that minimizes operating costs. The four major costs associated with the task of compressor selection are considered in the objective function. These costs include fuel cost, maintenance cost, start-up cost, and shut down cost. The objective of the MILP model is to reduce these costs while ensuring customer demands are satisfied.

A number of advantages of using the MILP model to help solve the compressor selection problem are observed as follows:

- The model can help the dispatcher in deciding which among the five compressors to turn on or off based on the optimal operating costs. In contrast to the prioritized order of running compressors which can only be used in an hourly basis, the model was extended to consider six operating periods and covered a longer duration.

- The developed MILP model can be extended to cover other areas of the natural gas pipeline system in Saskatchewan such as that of the city of Regina, Saskatchewan.
- Users can easily make changes or modify the parameters in the model. Hence, the model can be adapted for variations in season and gas prices, which are important determinants for the operating costs of the compressors.

Some weaknesses in the MILP approach are noted as follows:

- Since the customer demand within six operating periods must be known before the MILP model can be applied, accuracy of the predicted customer demand is important. If the customer demand is not accurately predicted, the model's recommendations on compressor selections may not be accurate.
- The model ignored the consideration of the physical distances between the compressor stations and the customer locations. Hence, the results provided by the MILP model may not be accurate because a compressor located near the customer locations may supply natural gas to the customers faster than a compressor located farther away from the customers.

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