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The Interval Linear Programming: A Revisit

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ABSTRACT. Interval linear programming (ILP) was developed by Huang and Moore (1993) and was widely applied in environmental and resources management. However, the feasibility of optimal solutions for affect directly in generating several decision alternatives, thus a modified interval linear programming (MILP) model is developed to assure its solution space could be absolutely feasible, and its solution algorithm is proposed to incorporates the associated extra constraints into the upper- and lower- bounds submodels. Moreover, the proofs for determining A and B in corresponding constraints are refined in this study. The results of numeric example and its application in water-quality management of Lake Qionghai Basin (China) further indicated the feasibility and effectiveness of the developed MILP model.

Keywords: linear programming, optimization, uncertainty, feasibility, Lake Qionghai

1. Introduction

Many of the input parameters in real-world problems exhibit some level of uncertainty due to the scarcity of data (Dantzig, 1955; Huang et al., 1995; Chinneck and Ramadan, 2000). This includes objective function costs (**C**), constraint coefficients (**A**), and right-hand sides (**B**). Thus, it is important find ways of using LP methods with intrinsic uncertainties in probabilistic, possibilistic, and/or interval formats (Huang and Chang, 2003; Maqsood et al., 2005). The methods developed to do this could be grouped into stochastic linear programming (SLP), fuzzy linear programming (FLP), interval linear programming (ILP), the best/worst case (BWC) method, and their hybrid models (Tong, 1994; Liu, 1997; Sahinidis, 2004).

ILP model, as a potential alternative to SLP or FLP models, could incorporate uncertainty into the LP model without any assumption of probabilistic or possibilistic distributions and was widely applied in environmental management under uncertainty (Maqsood et al., 2005; Li et al., 2007a; Qin et al, 2007). Ben-Israel and Robers (1970) first introduced a preliminary ILP model for solving a specific LP model whose constraints were the upper and lower bounds ($Max\ \mathbf{CX}$, $s.t.\ \mathbf{X} \in \{\mathbf{X} \mid \mathbf{B}_1 \le \mathbf{AX} \le \mathbf{B}_2, \mathbf{X} \ge 0\}$). Rommelfanger et al. (1989) and Inuiguchi and Sakawa (1995) proposed a LP method using only the independent upper and lower bounds of an internal objective function (IOF) ($Z^+ = \mathbf{C}^+\mathbf{X}$ and $Z^- = \mathbf{C}^-\mathbf{X}$). Chanas and Kuchta (1996) and Sengupta et al. (2001) obtained a satisfactory equivalent system for the ILP problem by considering the surrogate objective functions $Max\ Z^\pm = \lambda(\sum_{j=1}^{n} [c_j^- + \varphi_0(c_j^+ - c_j^-)]x_j) + (1 - \lambda)(\sum_{j=1}^{n} [c_j^- + \varphi_1(c_j^+ - c_j^-)]x_j)$ and $Z^- = 0.5(\mathbf{C}^- + \mathbf{C}^-)\mathbf{X}$ instead of the original ones. Huang and Moore (1993) and Tong (1994) proposed a new ILP model and BWC method, respectively. BWC could convert $Max\ Z^\pm = \mathbf{C}^\pm\mathbf{X}$ $s.t.\ \mathbf{X} \in \{\mathbf{X} \mid \mathbf{A}^\pm\mathbf{X} \le \mathbf{B}^\pm, \mathbf{X} \ge 0\}$ into two submodels: $Z^+ = \mathbf{C}^+\mathbf{X}$ $s.t.\ \mathbf{X} \in \{\mathbf{X} \mid \mathbf{A}^-\mathbf{X} \le \mathbf{B}^+, \mathbf{X} \ge 0\}$ and $Z^- = \mathbf{C}^-\mathbf{X}$ $s.t.\ \mathbf{X} \in \{\mathbf{X} \mid \mathbf{A}^+\mathbf{X} \le \mathbf{B}^-, \mathbf{X} \ge 0\}$. Chinneck and Ramadan (2000) extended the BWC method to include nonnegative variables and equality constraints. Although the BWC method does produce the best and worst optimal values, it may result in infeasible decision variable spaces (Huang et al., 1995). Unlike BWC, Huang and Moore's (1993) method is defined generally as $Max\ Z^\pm = \mathbf{C}^\pm\mathbf{X}^\pm$ $s.t.\ \mathbf{X}^\pm \in \{\mathbf{X}^\pm \mid \mathbf{A}^\pm\mathbf{X}^\pm \le \mathbf{B}^\pm, \mathbf{X}^\pm \ge 0\}$, which could provide the solutions of IOF $Z_{opt}^\pm = [Z_{opt}^-, Z_{opt}^+]$ and interval decision variables $x_{j\,opt}^\pm = [x_{jopt}^-, x_{jopt}^+]$ for $\forall j$ (Huang et al., 1995; Huang, 1998; Li et al., 2007b). It has three major advantages. First, the ILP model could incorporate interval information directly into the optimization process. Second, its solution algorithm has lower computational requirements than the SLP and FLP models, and third, the interval solutions can produce several alternatives that reflect different decisions (Huang et al., 1995; Chinneck and Ramadan, 2000). However, the major problems of ILP model include that: its proof in Huang et al. (1995) can not clearly uncover the relationships between decision variables $\mathbf{X}^\pm$ and parameters $\mathbf{A}^\pm$ (or $\mathbf{B}^\pm$); the feasibility of $x_{j\,opt}^\pm = [x_{jopt}^-, x_{jopt}^+]$ for $\forall j$ obtained by two-step solution algori-

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thm did not proved, but it has a direct impact on generating different decision alternatives, thus the determined constraints in the solution algorithm should ensure that its solution space is feasible.

The major objectives are to update the proof ILP model and to find extra constraint for obtaining feasible solution space. We also explain the modified ILP model (MILP) and demonstrate its feasibility analysis using a numeric example, and compare MILP with ILP model. Finally, EILP is applied in water-quality management of the Lake Qionghai Basin, China.

2. Review of Interval Linear Programming

According to Huang and Moore (1993), the ILP model could be generally expressed as follows:

$$Max\ Z^{\pm} = \mathbf{C}^{\pm}\mathbf{X}^{\pm} \tag{1a}$$

$$s.t.\ \ \mathbf{A}^{\pm}\mathbf{X}^{\pm} \le \mathbf{B}^{\pm} \tag{1b}$$

$$\mathbf{X}^{\pm} \ge 0 \tag{1c}$$

where $\mathbf{X}^{\pm} \in \{\Re^{\pm}\}^{n\times 1}$, $\mathbf{A}^{\pm} \in \{\Re^{\pm}\}^{m\times n}$, $\mathbf{B}^{\pm} \in \{\Re^{\pm}\}^{m\times 1}$, and $\mathbf{C}^{\pm} \in \{\Re^{\pm}\}^{1\times n}$, and $\{\Re^{\pm}\}$ denotes a vector set of EI variables. Equations (1) could be transformed into two submodels based on two-step solution algorithm, the first submodel (Z^{+} and feasible region $\mathbf{Q}^{+}$) for maximizing objective function is written as (Huang and Moore, 1993):

$$Max\ Z^{+} = \sum_{j=1}^{k} c_j^{+}x_j^{+} + \sum_{j=k+1}^{n} c_j^{+}x_j^{-} \tag{2a}$$

$$s.t.\ \ \sum_{j=1}^{k} \left|a_{ij}^{\pm}\right|^{-} Sign(a_{ij}^{\pm})x_j^{+} + \sum_{j=k+1}^{n} \left|a_{ij}^{\pm}\right|^{+} Sign(a_{ij}^{\pm})x_j^{-} \le b_i^{+},\ \ \forall i \tag{2b}$$

$$x_j^{\pm} \ge 0,\ \forall\ \text{j=1, 2, …,}\ n \tag{2c}$$

and the second submodel (Z^{-} and $\mathbf{Q}^{-}$) is given by:

$$Max\ Z^{-} = \sum_{j=1}^{k} c_j^{-}x_j^{-} + \sum_{j=k+1}^{n} c_j^{-}x_j^{+} \tag{3a}$$

$$s.t.\ \ \sum_{j=1}^{k} \left|a_{ij}^{\pm}\right|^{+} Sign(a_{ij}^{\pm})x_j^{-} + \sum_{j=k+1}^{n} \left|a_{ij}^{\pm}\right|^{-} Sign(a_{ij}^{\pm})x_j^{+} \le b_i^{-},\ \ \forall i \tag{3b}$$

$$x_j^{-} \le x_{jopt}^{+},\ \forall j = 1, 2, \ldots, k \tag{3c}$$

$$x_j^{+} \ge x_{jopt}^{-},\ \forall j = k+1, k+2, \ldots, n \tag{3d}$$

$$x_j^{\pm} \ge 0,\ \forall j\text{=1, 2, …, } n \tag{3e}$$

When the objective function is to be minimized, the first submodel (Z^{-} and $\mathbf{P}^{+}$) is given as follows:

$$Min\ Z^{-} = \sum_{j=1}^{k} c_j^{-}x_j^{-} + \sum_{j=k+1}^{n} c_j^{-}x_j^{+} \tag{4a}$$

$$s.t.\ \ \sum_{j=1}^{k} \left|a_{ij}^{\pm}\right|^{+} Sign(a_{ij}^{\pm})x_j^{-} + \sum_{j=k+1}^{n} \left|a_{ij}^{\pm}\right|^{-} Sign(a_{ij}^{\pm})x_j^{+} \le b_i^{+},\ \ \forall i \tag{4b}$$

$$x_j^{\pm} \ge 0,\ \forall j = 1, 2, \ldots, n \tag{4c}$$

while the second submodel (Z^{+} and $\mathbf{P}^{-}$) could be written as:

$$Min\ Z^{+} = \sum_{j=1}^{k} c_j^{+}x_j^{+} + \sum_{j=k+1}^{n} c_j^{+}x_j^{-} \tag{5a}$$

$$s.t.\ \ \sum_{j=1}^{k} \left|a_{ij}^{\pm}\right|^{-} Sign(a_{ij}^{\pm})x_j^{+} + \sum_{j=k+1}^{n} \left|a_{ij}^{\pm}\right|^{+} Sign(a_{ij}^{\pm})x_j^{-} \le b_i^{-},\ \ \forall i \tag{5b}$$

$$x_j^{+} \ge x_{jopt}^{-},\ \ j = 1,\ 2,\ \ldots,\ k \tag{5c}$$

$$x_j^{-} \le x_{jopt}^{+},\ \ j = k+1,\ k+2,\ \ldots,\ n \tag{5d}$$

$$x_j^{\pm} \ge 0,\ \forall j = 1, 2, \ldots, n \tag{5e}$$

ILP model has the optimal solution $Z_{opt}^{\pm} = [Z_{opt}^{-}, Z_{opt}^{+}]$ and $\mathbf{X}_{opt}^{\pm} = \{x_{jopt}^{\pm} = [x_{jopt}^{-}, x_{jopt}^{+}] \mid \forall j = 1,\ 2,\ \ldots,\ n\}$ (Huang et al., 1995), the two extreme decisions (Z_{opt}^{-} and Z_{opt}^{+}) simply represent the tradeoff between system benefits and risk of $\mathbf{A}^{\pm}$, $\mathbf{B}^{\pm}$ and $\mathbf{C}^{\pm}$'s violations. In the example of municipal waste management in which the objective is to minimize system costs, the decision maker choosing the lower bound value Z_{opt}^{-} will end up with a lower system cost but also a higher risk of violating the allowable waste-loading levels (Huang and Moore, 1993). Conversely, the decision to accept a higher system cost by selecting the upper bound value Z_{opt}^{+} will correspond to a lower risk. Although decision makers can use interval solution to help generate several alternatives, they cannot assure that any alternative is absolutely feasible. In fact, the first submodel should be determined to achieve the optimal system benefit, while the second could assure the feasibility of final solution space. Therefore, we would first refine the proofs for determining the constraints.

3. Modifications of Interval Linear Programming

3.1. Proof of the Relationship between $\mathbf{X}^{\pm}$ and $\mathbf{A}^{\pm}$

Lemma 1: *To maximize the objective function, the relationships between decision variables* $\mathbf{X}^{\pm}$ *and left-hand sides* $\mathbf{A}^{\pm}$ *should be* $x_j^{+} \to \left|a_{ij}^{\pm}\right|^{-} Sign(a_{ij}^{\pm})$ *(*$\forall j$ *= 1, 2, …, k) and* $x_j^{-} \to \left|a_{ij}^{\pm}\right|^{+} Sign(a_{ij}^{\pm})$ *(*$\forall j$ *= k + 1, k + 2, …, n) for the first submodel* Z^{+}*, and* $x_j^{-} \to \left|a_{ij}^{\pm}\right|^{+} Sign(a_{ij}^{\pm})$ *(*$\forall j$ *= 1, 2, …, k) and* $x_j^{+} \to \left|a_{ij}^{\pm}\right|^{-} Sign(a_{ij}^{\pm})$ *(*$\forall j$ *= k + 1, k + 2, …, n) for the second submodel* Z^{-}*. This leads to optimal solutions for the upper and lower bounds* Z^{+} *and* Z^{-} *with* $x_{jopt}^{-} \le x_{jopt}^{+}$ *for* $\forall j$ *= 1, 2, …, k and* $Z_{opt}^{-} \le Z_{opt}^{+}$.

Proof. (i) The upper-bound should be solved first to achieve the optimal system benefit (Huang et al., 1995) when maximizing the objective function. Here we denote the alternative upper-bound feasible space ($\mathbf{Q}^u$) as $\tilde{\mathbf{Q}}^u$ for AI^+ as follows:

$$\tilde{\mathbf{Q}}^u = \left\{ \mathbf{X}^{\pm} \left| \sum_{j=1}^{k} a_{ij} x_j^+ + \sum_{j=k+1}^{n} a_{ij} x_j^- \le b_i \right. \right\}$$
$$(b_i \in [b_i^-, b_i^+],\ x_j^+, x_j^- \in \mathbf{X}^{\pm},\ \mathbf{X}^{\pm} \ge 0, \forall i) \quad (6)$$

where $a_{ij} \in [a_{ij}^-, a_{ij}^+]$ for $\forall i, j$. There exists at least one of the following: $a_{ij} \ne \left|a_{ij}^{\pm}\right|^- Sign(a_{ij}^{\pm})$ for $\forall j$ = 1, 2, …, k or $a_{ij} \ne \left|a_{ij}^{\pm}\right|^+ Sign(a_{ij}^{\pm})$ for $\forall j = k + 1, k + 2, …, n$.

Assume that $\tilde{x}_{jopt}^+$ for j = 1, 2, …, k, $\tilde{x}_{jopt}^-$ for $j = k + 1$, k + 2, …, n be the optimal solutions of Z^+ subject to $\tilde{\mathbf{Q}}^u$. Let $x_j^+ = a_{ij}\tilde{x}_{jopt}^+ / [\left|a_{ij}^{\pm}\right|^- Sign(a_{ij}^{\pm})]$ for j = 1, 2, …, k and $x_j^- = a_{ij}\tilde{x}_{jopt}^- / [\left|a_{ij}^{\pm}\right|^+ Sign(a_{ij}^{\pm})]$ for $\forall j = k$ + 1, k + 2, …, n, where $x_j^+ \ge \tilde{x}_{jopt}^+$ for j = 1, 2, …, k and $x_j^- \le \tilde{x}_{jopt}^-$ for $j = k + 1$, k + 2, …, n. Therefore, we also have $\sum_{j=1}^{k} \left|a_{ij}^{\pm}\right|^- Sign(a_{ij}^{\pm}) x_j^+ + \sum_{j=k+1}^{n} \left|a_{ij}^{\pm}\right|^+ Sign(a_{ij}^{\pm}) x_j^- = \sum_{j=1}^{k} a_{ij}\tilde{x}_j^+ + \sum_{j=k+1}^{n} a_{ij}\tilde{x}_j^- \le b_i$, while the $\sum_{j=1}^{k} c_j^+ x_j^+ + \sum_{j=k+1}^{n} c_j^+ x_j^-$ is obviously greater than $\sum_{j=1}^{k} c_j^+ \tilde{x}_j^+ + \sum_{j=k+1}^{n} c_j^- \tilde{x}_j^-$. Therefore, $a_{ij} = \left|a_{ij}^{\pm}\right|^- Sign(a_{ij}^{\pm})$ for j = 1, 2, …, k and $a_{ij} = \left|a_{ij}^{\pm}\right|^+ Sign(a_{ij}^{\pm})$ $j = k$ + 1, k + 2, …, n in $\mathbf{Q}^u$ would assist in determining the optimal Z_{opt}^+.

Proof. (ii) Proving Lemma 1 wound ensure that Z^- agrees with $Z_{opt}^- \le Z_{opt}^+$ and $x_j^- \le x_{jopt}^+$ for j = 1, 2, …, k and $x_j^+ \ge x_{jopt}^-$ for $j = k + 1, k + 2,…, n$, where Z_{opt}^+, x_{jopt}^+, and x_{jopt}^- are the optimal solutions of the first submodel. Suppose that $x_j^- = [\left|a_{ij}^{\pm}\right|^- Sign(a_{ij}^{\pm}) / \left|a_{ij}^{\pm}\right|^+ Sign(a_{ij}^{\pm})] \cdot x_{jopt}^+$ for j = 1, 2, …, k and $x_j^+ = [\left|a_{ij}^{\pm}\right|^+ Sign(a_{ij}^{\pm}) / \left|a_{ij}^{\pm}\right|^- Sign(a_{ij}^{\pm})] \cdot x_{jopt}^-$ for $j = k$ + 1, k + 2, …, n in the second submodel, where $x_j^- \le x_{jopt}^+$ exists for j = 1, 2, …, k and $x_j^+ \ge x_{jopt}^-$ for $j = k + 1, k + 2, …, n$. Then we have $\sum_{j=1}^{k} \left|a_{ij}^{\pm}\right|^+ Sign(a_{ij}^{\pm}) x_j^- + \sum_{j=k+1}^{n} \left|a_{ij}^{\pm}\right|^- Sign(a_{ij}^{\pm}) x_j^+ = \sum_{j=1}^{k} \left|a_{ij}^{\pm}\right|^- Sign(a_{ij}^{\pm}) x_j^+ + \sum_{j=k+1}^{n} \left|a_{ij}^{\pm}\right|^+ Sign(a_{ij}^{\pm}) x_j^- \le b_i$. Thus $\mathbf{Q}^l$ would become $\{\mathbf{X}^{\pm} \mid \sum_{j=1}^{k} \left|a_{ij}^{\pm}\right|^+ Sign(a_{ij}^{\pm}) x_j^- + \sum_{j=k+1}^{n} \left|a_{ij}^{\pm}\right|^- Sign(a_{ij}^{\pm}) x_j^+ \le b_i\}$ ($b_i \in [b_i^-, b_i^+]$, $x_{jopt}^+ \ge x_j^-$ for j = 1, 2, …, k; $x_{jopt}^- \le x_j^-$ for $j = k$ + 1, k + 2, …, n; $x_j^{\pm} \in \mathbf{X}^{\pm}$, $\mathbf{X}^{\pm} \ge 0, \forall i$), which would ensure the feasibility of the solution space.

Corollary 1: Similarly, when the objective function is to be minimized, $\mathbf{P}^+$ and $\mathbf{P}^-$ are written as:

$$\mathbf{P}^+ = \left\{ \mathbf{X}^{\pm} \left| \sum_{j=1}^{k} \left|a_{ij}^{\pm}\right|^+ Sign(a_{ij}^{\pm}) x_j^- + \sum_{j=k+1}^{n} \left|a_{ij}^{\pm}\right|^- Sign(a_{ij}^{\pm}) x_j^+ \le b_i \right. \right\}$$
$$(b_i \in [b_i^-, b_i^+],\ x_j^{\pm} \in \mathbf{X}^{\pm},\ \mathbf{X}^{\pm} \ge 0, \forall i) \quad (7a)$$

$$\mathbf{P}^- = \left\{ \mathbf{X}^{\pm} \left| \sum_{j=1}^{k} \left|a_{ij}^{\pm}\right|^- Sign(a_{ij}^{\pm}) x_j^+ + \sum_{j=k+1}^{n} \left|a_{ij}^{\pm}\right|^+ Sign(a_{ij}^{\pm}) x_j^- \le_i b \right. \right\}$$
$$(b_i \in [b_i^-, b_i^+],\ x_{jopt}^+ \ge x_j^- \ for\ j = 1,\ …,\ k;\ x_{jopt}^- \le x_j^-$$
$$for\ j = k+1,\ …,\ n;\ x_j^{\pm} \in \mathbf{X}^{\pm},\ \mathbf{X}^{\pm} \ge 0, \forall i) \quad (7b)$$

3.2. Proof of the Relationship between $\mathbf{X}^{\pm}$ and $\mathbf{B}^{\pm}$

Lemma 2: *To maximize the objective function, the relationships between objective function $Z^{\pm}$ and $B^{\pm}$ on the right-hand side of constraints should be determined as $Z^+ \to b_i^+$ and $Z^- \to b_i^-$.*

Proof. Let $\sum_{j=1}^{k} \left|a_{ij}^{\pm}\right|^- Sign(a_{ij}^{\pm}) x_j^+ + \sum_{j=k+1}^{n} \left|a_{ij}^{\pm}\right|^+ Sign(a_{ij}^{\pm}) x_j^- \le b_i$ ($b_j \in [b_j^-, b_j^+]$) for $\forall i$ = 1, 2, …, m be any feasible alternative of EI inequalities, and the optimal solutions be $\tilde{x}_{jopt}^{\pm} \in \mathbf{X}_{opt}^{\pm}$ ($x_{jopt}^{\pm} \ge 0$, $\forall j$ = 1, 2, …, n). Therefore, $\mathbf{X}_{opt}^{\pm}$ satisfying any feasible alternative of interval inequalities would also meet $\sum_{j=1}^{k} \left|a_{ij}^{\pm}\right|^- Sign(a_{ij}^{\pm}) x_j^+ + \sum_{j=k+1}^{n} \left|a_{ij}^{\pm}\right|^+ Sign(a_{ij}^{\pm}) x_j^- \le b_i^+$, where $x_j^{\pm} = \tilde{x}_{jopt}^{\pm} \cdot b_j^+ / b_j$ for j = 1, 2, …, n. We then have $\sum_{j=1}^{k} \left|a_{ij}^{\pm}\right|^- Sign(a_{ij}^{\pm}) \tilde{x}_{jopt}^+ \cdot b_j^+ / b_j + \sum_{j=k+1}^{n} \left|a_{ij}^{\pm}\right|^+ Sign(a_{ij}^{\pm}) \tilde{x}_{jopt}^- b_j^+ / b_j \le b_i$. Moreover, $\sum_{j=1}^{k} c_j^+ \tilde{x}_{jopt}^+ b_j^+ / b_j + \sum_{j=k+1}^{n} c_j^- \tilde{x}_{jopt}^- b_j^+ / b_j$ is obviously greater than $\sum_{j=1}^{k} c_j^+ \tilde{x}_{jopt}^+ + \sum_{j=k+1}^{n} c_j^- \tilde{x}_{jopt}^-$, which contributes to a more practical solution. Thus, to maximize the objective function, the right-hand side b_i for $\forall i$ should be b_i^+ and b_i^- corresponding to Z^+ and Z^-, respectively.

Moreover, we could convert $\sum_{j=1}^{k} \left|a_{ij}^{\pm}\right|^- Sign(a_{ij}^{\pm}) x_j^+ + \sum_{j=k+1}^{n} \left|a_{ij}^{\pm}\right|^+ Sign(a_{ij}^{\pm}) x_j^- \ge b_i$ into $-[\sum_{j=1}^{k} \left|a_{ij}^{\pm}\right|^- Sign(a_{ij}^{\pm}) x_j^+ + \sum_{j=k+1}^{n} \left|a_{ij}^{\pm}\right|^+ Sign(a_{ij}^{\pm}) x_j^-] \le -b_i = b_i'$, whose right-hand side b_i' for $\forall i$ could be determined based on the above rule. We would transform the EI equality constraints $\sum_{j=1}^{n} a_{ij}^{\pm} x_j^{\pm} = b_i^{\pm}$ for $\forall i$ into two inequalities such that $\sum_{j=1}^{n} a_{ij}^{\pm} x_j^{\pm} \le b_i^+$ and $\sum_{j=1}^{n} a_{ij}^{\pm} x_j^{\pm} \ge b_i^-$ for $\forall i$ (Huang et al., 1995), and the relationship between $a_{ij}^{\pm}$ and $x_j^{\pm}$ could then be determined according to Lemma 1.

Corollary 2: Similarly, to minimize the objective function, the right-hand side b_i for $\forall i$ should be b_i^+ and b_i^- corresponding to Z^- and Z^+, respectively.

Remark 1: Lemmas 1 and 2 present clearly the interactive relationships between the objective function and the constraints, between the upper-bound Z^+ and lower-bound Z^-, and between decision variables ($\mathbf{X}^{\pm}$) and model parameters ($\mathbf{A}^{\pm}$, $\mathbf{B}^{\pm}$ and $\mathbf{C}^{\pm}$). These are essential for developing the ILP/MILP's solution algorithm.

3.3. Proof of Extra Constraints for Feasible Solution Space

Theorem 1: *To ensure that optimal solution $\mathbf{X}_{opt}^{\pm}$ is absolutely feasible, the extra constraints for maximizing $Z^{\pm}$ could be added to Equation (3) as follows:*

$$\sum_{j=k-p+1}^{k} -(\left|a_{\delta j}^{\pm}\right|^+ x_j^- - \left|a_{\delta j}^{\pm}\right|^- x_{jopt}^+) + \sum_{j=n-q+1}^{n} (\left|a_{\delta j}^{\pm}\right|^- x_j^+ - \left|a_{\delta j}^{\pm}\right|^+ x_{jopt}^-) \le 0, \forall \delta \quad (8)$$

where δ is the number of constraints in Equation (2b) that meet $\sum_{j=1}^{k} \left|a_{\delta j}^{\pm}\right|^- Sign(a_{\delta j}^{\pm}) x_{jopt}^+ + \sum_{j=k+1}^{n} \left|a_{\delta j}^{\pm}\right|^+ Sign(a_{\delta j}^{\pm}) x_{jopt}^- = b_\delta^+$ as well as its $a_{\delta j}^{\pm} \le 0$ for $j = k - p + 1$, …, k and $a_{\delta j}^{\pm} \ge 0$ for $j = k - q + 1$, …, n.

Proof. Let us check whether all possible alternatives in solution spaces $\mathbf{X}_{opt}^{\pm}$ are feasible, an important consideration in practical decision making (Huang, 1996).

Since the optimal solutions $\mathbf{X}_{opt}^+$ and $\mathbf{X}_{opt}^-$ occur at a vertex in the polytope or feasible region, then $\exists i = \delta$, such that $\sum_{j=1}^{k} \left|a_{\delta j}^{\pm}\right|^- Sign(a_{\delta j}^{\pm}) x_{jopt}^+ + \sum_{j=k+1}^{n} \left|a_{\delta j}^{\pm}\right|^+ Sign(a_{\delta j}^{\pm}) x_{jopt}^- = b_\delta^+$ for $\forall \delta$. If all of possible alternatives are feasible, the maximum value range inequalities based on the BWC method should be met (Tong, 1994; Chinneck and Ramadan, 2000):

$$\sum_{j=1}^{k} a_{\delta j}^{-} x_j^{\pm} + \sum_{j=k+1}^{n} a_{\delta j}^{-} x_j^{\pm} \le b_{\delta}^{+} \quad ,\forall\ \delta. \tag{9}$$

If we assume that only $a_{\delta j}^{\pm} \le 0$ for $j = k - p + 1, \ldots, k, k + 1, \ldots, k - q$, then Equation (9) could be rewritten as follows:

$$\begin{aligned}&\sum_{j=1}^{k-p} \left|a_{\delta j}^{\pm}\right|^{-} Sign(a_{\delta j}^{\pm}) x_j^{\pm} + \sum_{j=k-p+1}^{k} \left|a_{\delta j}^{\pm}\right|^{+} Sign(a_{\delta j}^{\pm}) x_j^{\pm} \\ &\quad + \sum_{j=k+1}^{n-q} \left|a_{ij}^{\pm}\right|^{+} Sign(a_{\delta j}^{\pm}) x_j^{\pm} + \sum_{j=n-q+1}^{n} \left|a_{\delta j}^{\pm}\right|^{-} Sign(a_{\delta j}^{\pm}) x_j^{\pm} \\ &\quad \le \sum_{j=1}^{k} \left|a_{\delta j}^{\pm}\right|^{-} Sign(a_{\delta j}^{\pm}) x_{jopt}^{+} + \sum_{j=k+1}^{n} \left|a_{\delta j}^{\pm}\right|^{+} Sign(a_{\delta j}^{\pm}) x_{jopt}^{-} \ ,\forall \delta \end{aligned} \tag{10}$$

or

$$\begin{aligned}&\sum_{j=1}^{k-p} \left|a_{\delta j}^{\pm}\right|^{-} Sign(a_{\delta j}^{\pm})(x_j^{\pm} - x_{jopt}^{+}) + \sum_{j=k-p+1}^{k} Sign(a_{\delta j}^{\pm})(\left|a_{\delta j}^{\pm}\right|^{+} x_j^{\pm} - \left|a_{\delta j}^{\pm}\right|^{-} x_{jopt}^{+}) \\ &+ \sum_{j=k+1}^{n-q} \left|a_{\delta j}^{\pm}\right|^{+} Sign(a_{\delta j}^{\pm})(x_j^{\pm} - x_{jopt}^{-}) + \sum_{j=n-q+1}^{n} Sign(a_{\delta j}^{\pm})(\left|a_{\delta j}^{\pm}\right|^{-} x_j^{\pm} - \left|a_{\delta j}^{\pm}\right|^{+} x_{jopt}^{-}) \le 0, \\ &\forall \delta \end{aligned} \tag{11}$$

Let ϕ be the left-hand side of Equation (11). Since $Sign(a_{ij}^{\pm})(x_j^{\pm} - x_{jopt}^{+}) \le 0$ for $j = 1, \ldots, k - p$, $Sign(a_{\delta j}^{\pm}) \le 0$ for $j = k - p + 1, \ldots, k$, $Sign(a_{ij}^{\pm})(x_j^{\pm} - x_{jopt}^{-}) \le 0$ for $j = k + 1, \ldots, n - q$, and $Sign(a_{\delta j}^{\pm}) \ge 0$ for $j = n - q + 1, \ldots, n$, we then have:

$$\begin{aligned}&\sum_{j=k-p+1}^{k} -(\left|a_{\delta j}^{\pm}\right|^{+} x_j^{-} - \left|a_{\delta j}^{\pm}\right|^{-} x_{jopt}^{+}) + \sum_{j=n-q+1}^{n} (\left|a_{\delta j}^{\pm}\right|^{-} x_j^{+} - \left|a_{\delta j}^{\pm}\right|^{+} x_{jopt}^{-}) \ge \phi, \\ &\forall \delta \end{aligned} \tag{12}$$

Until now, the feasibility of Equation (9) depended on the following inequality $\sum_{j=k-p+1}^{k} -(\left|a_{\delta j}^{\pm}\right|^{+} x_j^{-} - \left|a_{\delta j}^{\pm}\right|^{-} x_{jopt}^{+}) + \sum_{j=n-q+1}^{n} (\left|a_{\delta j}^{\pm}\right|^{-} x_j^{+} - \left|a_{\delta j}^{\pm}\right|^{+} x_{jopt}^{-}) \le 0$ for $\forall \delta$. Moreover, the associated extreme point X' is a vertex at the polyhedron of solution space $\mathbf{X}_{opt}^{\pm} = \{x_{jopt}^{\pm} = [x_{jopt}^{-}, x_{jopt}^{+}] \mid \forall j = 1,\ 2,\ \ldots,\ n\}$ as follows:

$$\begin{aligned}X' = \{X' \mid\ & x_j = x_{jopt}^{+} \text{ for } j = 1,\ \ldots,\ k-p;\ x_j = x_{jopt}^{-} \text{ for } j = k-p+1,\ \ldots,\ k; \\ & x_j = x_{jopt}^{-} \text{ for } j = k+1,\ \ldots,\ n-q;\ x_j = x_{jopt}^{+} \text{ for } j = n-q+1,\ \ldots,\ n\}\end{aligned} \tag{13}$$

Obviously, Equation (8) would ensure that the solution space is absolutely feasible.

Corollary 3: When minimizing $Z^{\pm}$, the extra constraints to be embedded in Equation (5) are given by:

$$\sum_{j=1}^{k-p} (\left|a_{\delta j}^{\pm}\right|^{-} x_j^{+} - \left|a_{\delta j}^{\pm}\right|^{+} x_{jopt}^{-}) - \sum_{j=k+1}^{n-q} (\left|a_{\delta j}^{\pm}\right|^{+} x_j^{-} - \left|a_{\delta j}^{\pm}\right|^{-} x_{jopt}^{+}) \le 0\ , \forall \delta \tag{14}$$

where δ is also the number of constraints in Equation (4b) that meet $\sum_{j=1}^{k} \left|a_{\delta j}^{\pm}\right|^{+} Sign(a_{\delta j}^{\pm}) x_{jopt}^{-} + \sum_{j=k+1}^{n} \left|a_{\delta j}^{\pm}\right|^{-} Sign(a_{\delta j}^{\pm}) x_{jopt}^{+} = b_{\delta}^{+}$ as well as its $a_{\delta j}^{\pm} \ge 0$ *for j* = 1, …, *k* – *p and* $a_{\delta j}^{\pm} \le 0$ *for j* = *k* + 1, …, *n* – *q*.

Remark 2: Based on Lemma 1 and 2 and Theorems 1, the MILP model can be solved as two submodels, where the first Z^{+} subject to Equation (2b-c) is calculated when the interval objective function (IOF) is to be maximized, and the Z^{-} subject to Equation (3b-e) and Equation (8) can then be solved according to the results of the upper bound solution. Conversely, when IOF is to be minimized, Z^{-} with its relevant constraints [Equaitons (4b-c)] should be formulated and solved prior to solving Z^{+} with its relevant constraints [Equaitons (3b-e) and (14)]. In addition, Z_{opt}^{+} and Z_{opt}^{-} also represent the appropriate upper and lower bounds for Z_{opt}^{+} whose solution space $\mathbf{X}_{opt}^{\pm}$ is absolutely feasible.

Remark 3: The MILP model is an optimization method that is different in post-optimality analysis than other methods such as sensitivity analysis (SA), Wendell's tolerance approach (WTA; Chinneck and Ramadan, 2000), and parametric programming (PP; Huang et al., 1995). Although the optimal solutions for the SA, WTA, and PP methods can reflect the effects from one or more input coefficients, it is impossible to obtain the complete range of optimum solutions of the objecttive function. Moreover, they assume that variations in the input coefficients occur simultaneously, and could be useful tools in helping interpret the MILP's solutions.

3.4. Modified Solution Algorithm

Corollary 4: When the objective function is to be maximized, the first submodel corresponding to Z^{+} is same as Equation (2), while Z_{opt}^{-}, x_{jopt}^{-} (j = 1, 2, …, k) and x_{jopt}^{+} ($j = k + 1, k + 2, \ldots, n$) could be solved using the following submodel:

$$Max\ Z^{-} = \sum_{j=1}^{k} c_j^{-} x_j^{-} + \sum_{j=k+1}^{n} c_j^{-} x_j^{+} \tag{15a}$$

$$\begin{aligned}s.t.\ \mathbf{X}^{\pm} \in \Big\{x_j^{\pm} \Big|& \sum_{j=k-p+1}^{k} -(\left|a_{\delta j}^{\pm}\right|^{+} x_j^{-} - \left|a_{\delta j}^{\pm}\right|^{-} x_{jopt}^{+}) + \\ & \sum_{j=n-q+1}^{n} (\left|a_{\delta j}^{\pm}\right|^{-} x_j^{+} - \left|a_{\delta j}^{\pm}\right|^{+} x_{jopt}^{-}) \le 0,\ \mathbf{Q}^{-},\ \forall j,\ \delta\Big\}\end{aligned} \tag{15b}$$

Corollary 5: When the objective function is to be minimized, first submodel corresponding to Z^{-} is also same as Equation (4). The second is:

$$Max\ Z^{+} = \sum_{j=1}^{k} c_j^{+} x_j^{+} + \sum_{j=k+1}^{n} c_j^{+} x_j^{-} \tag{16a}$$

$$\begin{aligned}s.t.\ \mathbf{X}^{\pm} \in \Big\{x_j^{\pm} \Big|& \sum_{j=1}^{k-p} (\left|a_{\delta j}^{\pm}\right|^{-} x_j^{+} - \left|a_{\delta j}^{\pm}\right|^{+} x_{jopt}^{-}) \\ & - \sum_{j=k+1}^{n-q} (\left|a_{\delta j}^{\pm}\right|^{+} x_j^{-} - \left|a_{\delta j}^{\pm}\right|^{-} x_{jopt}^{+}) \le 0,\ \mathbf{P}^{-},\ \forall j,\ \delta\Big\}\end{aligned} \tag{16b}$$

Remark 4: Two major differences exist between the MILP

Table 1. Procedures and Solutions of ILP and MILP Models

Method	ILP	MILP
Z^+	$Max = 30x_1^+ - 5.5x_2^-$	$Max = 30x_1^+ - 5.5x_2^-$
s2	$8x_1^+ - 14x_2^- \le 4.2$	$8x_1^+ - 14x_2^- \le 4.2$
s3	$x_1^+ + 0.2x_2^- \le 7$	$x_1^+ + 0.2x_2^- \le 7$
s4	$x_1^+, x_2^- \ge 0$	$x_1^+, x_2^- \ge 0$
Z^-	$Max = 26x_1^- - 6x_2^+$	$Max = 26x_1^- - 6x_2^+$
s2	$10x_1^- - 12x_2^+ \le 3.8$	$10x_1^- - 12x_2^+ \le 3.8$
s3	$1.1x_1^- + 0.19x_2^+ \le 6.5$	$1.1x_1^- + 0.19x_2^+ \le 6.5$
s4	$x_1^-, x_2^+ \ge 0$	$x_1^-, x_2^+ \ge 0$
EC_1 *	$x_1^- \le 6.335897$	$x_1^- \le 6.335897$
EC_2	$x_2^+ \ge 3.320513$	$x_2^+ \ge 3.320513$
EC_3	--	$0.19x_2^+ \le 0.2 \times 3.320513$
Results	$Z_{opt}^{\pm}$ = [111.38, 171.81] $x_{1opt}^{\pm}$ = [5.213, 6.336] $x_{2opt}^{\pm}$ = [3.320, 4.028]	$Z_{opt}^{\pm}$ = [97.96, 171.81] $x_{1opt}^{\pm}$ = [4.574, 6.336] $x_{2opt}^{\pm}$ = [3.320, 3.495]

*Extra constraints (EC) based on the results (x_1^+, x_2^-) of the first submodel

and ILP models. Because of constraints (8) or (14), MILP guarantees the definite feasibility of the solution space while ILP cannot. Furthermore, according to Lemma 1 and 2, the first submodel could be used for achieving the optimal system benefit, while the second could assure the feasibility of final solution space.

3.5. Extensive Discussion with a Numeric Example

We use a simplified numerical example to illustrate the MILP model and its solution algorithm:

$$Max \;\; Z^{\pm} = [26,\ 30]x_1^{\pm} - [5.5,\ 6.0]x_2^{\pm} \tag{17a}$$

$$s.t. \; [8,\ 10]x_1^{\pm} - [12,\ 14]x_2^{\pm} \le [3.8,\ 4.2] \tag{17b}$$

$$[1.0,\ 1.1]x_1^{\pm} + [0.19,\ 0.2]x_2^{\pm} \le [6.5,\ 7] \tag{17c}$$

$$x_1^{\pm}, x_2^{\pm} \ge 0 \tag{17d}$$

where $\mathbf{A}^{\pm} = \{[8,10], [12,14]; [1.0,1.1], [0.19,0.2]\}$, $\mathbf{B}^{\pm} = \{[3.8, 4.2], [6.5, 7]\}$, and $\mathbf{C}^{\pm} = \{[26,30], [-6.0,-5.5]\}^T$. Although Equation (17) is relatively simple with two decision variables, it should be sufficient to show the differences between ILP and MILP. Using the work of Huang et al. (1995), and the above solution algorithm, we solved the ILP and MILP, and the results are summarized in Table 1. Obviously, the ILP's solution space is not absolutely feasible, while that of MILP model is absolutely feasible.

Although the ILP model ensures that $x_{jopt}^- \le x_{jopt}^+$, $\forall j$ = 1, 2, …, n and $Z_{opt}^- \le Z_{opt}^+$, its solutions could not be easily used for generating decision alternatives, since some of them may be absolutely infeasible (i.e., in the gray zone in Figure 1). To investigate this further, let us check whether an extreme point $X' = \{X' |\ x_1 = 6.336,\ x_2 = 4.028\}$ in the solution space meets the inequality $x_1 + 0.19x_2 \le 7$. Putting in the numerical values gives $6.336 + 0.19 \times 4.028 = 7.101 > 7$, indicating this alternative is infeasible.

To avoid the infeasible zone in ILP, MILP with a constraint of $0.19x_2^+ \le 0.2 \times 3.320513$ (Table 1) was proposed, and its optimal solution is $Z_{opt}^{\pm}$ = [97.96, 171.81], $x_{1opt}^{\pm}$ = [4.57, 6.34] and $x_{2opt}^{\pm}$ = [3.32, 3.49]. Moreover, the final solution has no any infeasible zone, which further proves the feasibility of the MILP model.

4. Application in Water-quality Management

4.1. Study Area and its EILP Modeling

Lake Qionghai is the second-largest freshwater lake in Sichuan Province, China, with a high priority for clear water (Figure 1). At normal water levels of 1510.3 m, the lake has an area of 27.88 km^2 and a volume of 2.89 × 10^8 m^3. The Lake Qionghai Basin is located at 27°N and 102°E, with an area of 307.67 km^2 (Zhou et al., 2008). Lake Qionghai currently suffers from pollution that has been induced by point and NPSs and has resulted in lake eutrophication. To protect the regional environment and aquatic ecosystem, it is essential to control the watershed pollutants loads in an effective way. After the discussion of stakeholders and experts, a long-period water pollution management plan during 2005 ~ 2020 was proposed for Lake Qionghai Basin (Zhou et al., 2008). Some alternative strategies were preferred in the management plan. Twenty watersheds were partitioned for better revealing the pollutants

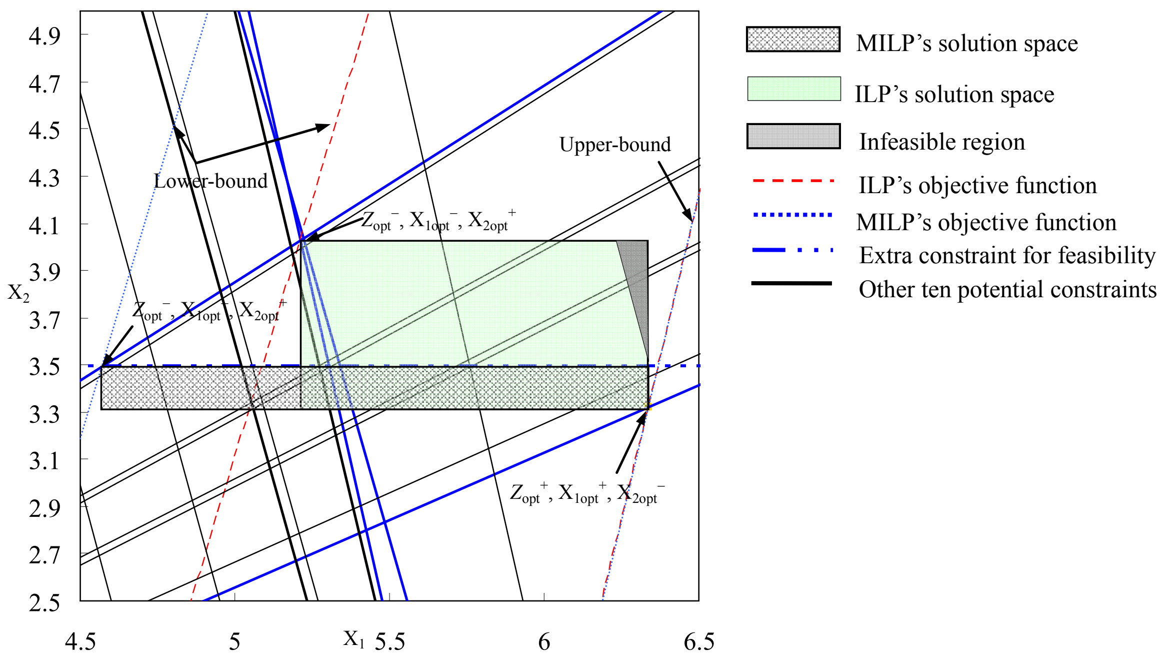

Figure 1. Graphical interpretation for ILP and MILP solutions.

loading and the spatial arrangement of the alternative strategies.

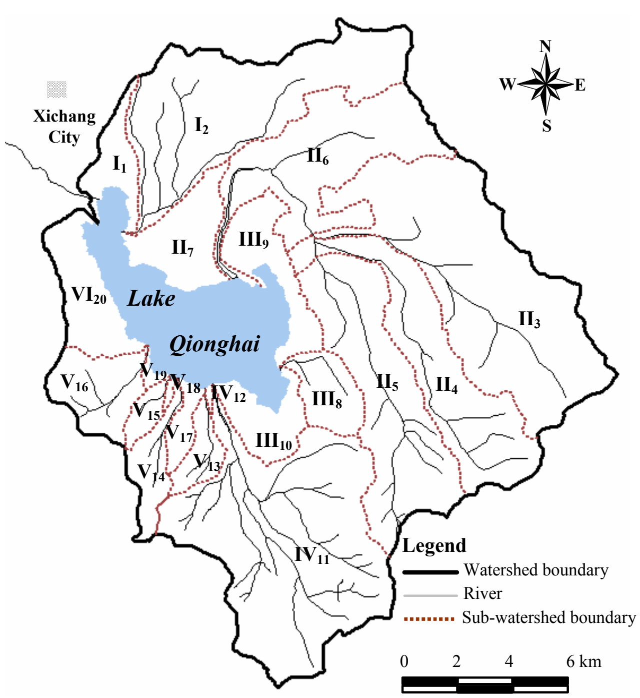

Figure 2. Lake Qionghai Basin and its 20 watersheds.

For the local government, the objective for total phosphorus (TP) control is to minimize the net direct expenditures of pollution abatement, including the initial capital investment and the operating costs, for the alternative strategies. The corresponding constraints consist of TP's total environmental capacity (*TEC*), the governmental minimum requirements on farmland area, land coverage, treatment rate of domestic waste water, and treatment rate of rural wastes, etc. Therefore, the EILP model for water quality management in Lake Qionghai Basin is given by:

$$Min\ f^{\pm} = \sum_{i=1}^{I}\sum_{j=1}^{J}\sum_{k=1}^{K}\left(X_{ijk}^{\pm} IIC_{jk}^{\pm}\right) + \sum_{i=1}^{I}\sum_{j=1}^{J}\sum_{k=1}^{K}\left(X_{ijk}^{\pm} ASC_{jk}^{\pm}\right) \tag{18a}$$

subject to:

$$\sum_{i=1}^{I}\sum_{j=1}^{J}\sum_{k=1}^{k}\left(X_{ijk}^{\pm}\cdot a_{ij}\cdot APR_{j}^{\pm}\right) \geq \sum_{i=1}^{I} TPD_{i}^{\pm} - TEC_{k}^{\pm},\ \forall k \tag{18b}$$

(Environmental capacity constraints)

$$\sum_{i=1}^{I}\sum_{k=1}^{k}\left(X_{i7k}^{\pm}\cdot a_{i7}\right) \leq RML_{k}^{\pm}\cdot TLAQ,\ \forall k \tag{18c}$$

(Farmland constraints)

$$\sum_{j=4}^{5}\sum_{k=1}^{k}\left(X_{ijk}^{\pm}\cdot a_{ij}\right) \geq RWT_{k}^{\pm}\cdot TWD_{ik}^{\pm},\quad i = 8,\ 9,\ 10,\ 20,\ \forall k \tag{18d}$$

(Domestic rural wastes treatment constraints)

$$\sum_{k=1}^{k}\left(X_{i8k}^{\pm}\cdot a_{i8}\right) \geq RRW_{k}^{\pm}\cdot TRW_{ik}^{\pm},\ \forall i,\ k \tag{18e}$$

(Rural wastes treatment constraints)

$$\sum_{k=1}^{k}\left(X_{ijk}^{\pm}\cdot a_{ij}\right)\geq RLS_{jk}^{\pm}\cdot TLS_{ij}^{\pm}, j=1,2,3, \forall i, k \quad (18f)$$

(Soil erosion constraints)

$$\sum_{k=1}^{k}\left(X_{i7k}^{\pm}\cdot a_{i7}\right)\geq RSL_{k}^{\pm}\cdot TSL_{i}, \forall i, k \quad (18g)$$

(Slope lands [> 25°] constraints)

$$\sum_{j=1}^{3}\sum_{k=1}^{K}\left(X_{ijk}^{\pm}\cdot a_{ij}\right)+\sum_{k=1}^{K}\left(X_{i7k}^{\pm}a_{i7}+X_{i9k}^{\pm}a_{i9}\right)+FLA_{i}\geq RFL_{i}^{\pm}\cdot TLA_{i}, \forall i \quad (18h)$$

(Land coverage constraint)

$$\sum_{i}\sum_{j=1}^{J}\sum_{k=1}^{k}\left(X_{ijk}^{\pm}\cdot a_{ij}\cdot APR_{j}^{\pm}\right)\geq\sum_{i}TPD_{i}\cdot RPE_{k}^{\pm}, i=1,2,6,7,20, \forall k$$

(Load reduction constraints for key watersheds) (18i)

$$\sum_{i=1}^{I}\sum_{k=1}^{k}\left(X_{i9k}^{\pm}\cdot a_{i9}\right)\geq RRE_{k}^{\pm}\cdot TRE, \forall k \quad (18j)$$

(River riparian buffer restoration constraints)

$$\sum_{i=1}^{I}\sum_{k=1}^{k}\left(X_{i10k}^{\pm}\cdot a_{i10}\right)\geq RLR_{k}^{\pm}\cdot TLR, \forall k \quad (18k)$$

(Lake riparian buffer constraints)

$$\sum_{j=1}^{J}\sum_{k=1}^{K}\left(X_{ijk}^{\pm}\cdot a_{ij}\cdot APR_{j}^{\pm}\right)\leq TPD_{i}^{\pm}, \forall i \quad (18l)$$

(Load reduction constraints for each watershed)

$$X_{ijk}^{\pm}\geq 0, \forall i, j, k \quad (18m)$$

(Technical constraints)

where $X_{ijk}^{\pm}$ is the newly expanded magnitude of strategies, i is watershed, i = 1, …, 20 for the 20 watersheds, I1, I2, …, VI20 (Figure 2); j is the strategy type, j = 1, …, 10 for moderate erosion restoration, strong erosion restoration, extreme erosion restoration, artificial wetland, waste water treatment plant, NPS controlling measures, reverting slope farmlands to forest, rural wastes treatment, river riparian vegetation buffer, and lake riparian vegetation buffer; k is the time period, k =1, 2, 3 for 2005 ~ 2010, 2011 ~ 2015 and 2016 ~ 2020 respectively; $TEC^{\pm}$ is the total environmental capacity for TP (ton·a^{-1}); $TPDQ$ is the total TP loading to Lake Qionghai based on field investigation (ton·a^{-1}); $IIC_{jk}^{\pm}$ is the unit initial capital cost of strategy j in period k based on field investigation; $ASC_{jk}^{\pm}$ is the annual unit operating cost of strategy j in period k based on field investigation; $TPD_{i}^{\pm}$ is the TP loading in watershed i (ton·a^{-1}); $APR_{j}^{\pm}$ is the unit TP abatement for strategy j; $RML_{k}^{\pm}$ is the governmental requirements on maximum ratio of farmlands being occupied; $TLAQ$ is the current farmland area with the value of 3.19×10^{7}m^{2}; $TWD_{ik}^{\pm}$ is the waste water volume in watershed i for period k (m^{3}·d^{-1}); $RWT_{k}^{\pm}$ is the treatment rate of domestic waste water; $RRW_{k}^{\pm}$ is the treatment rate of rural wastes; $TRW_{ik}^{\pm}$ is the rural wastes in watershed i for period k (m^{3}·d^{-1}); $RLS_{jk}^{\pm}$ is the restoration ratio of soil-erosion type l in period k, where l = 1, 2, 3 for moderate erosion, strong erosion and extreme erosion; a_{ij} is suitability factor of strategy j in watershed i; TLS_{ij} is the area of the soil-erosion type l in watershed i; TSL_{i} is the area of slope lands in watershed i; $RSL_{k}^{\pm}$ is the governmental requirements on the ration of reverting slope farmlands to forests with the slope higher than 25^{o}; FLA_{i} is the current forest area in watershed i (10^{4}m^{2}); $RFL_{i}^{\pm}$ is the anticipative minimum forest coverage in watershed i; TLA_{i} is the total area of watershed i (10^{4}m^{2}); $RPE_{k}^{\pm}$ is the minimum ratio of pollution reduction; $RRE_{k}^{\pm}$ is the minimum area ratio of river riparian vegetation buffer; TRE is the anticipative area of river riparian vegetation buffer, with the calculated value of 1.76×10^{6} m^{2}; $RLR_{k}^{\pm}$ is the minimum area ratio of lake riparian vegetation buffer; TLR is the anticipative area of lake riparian vegetation buffer, with the calculated value of 3.62 × 10^{6} m^{2}.

4.2. Results Analysis

(1) Optimal Costs and Expansion Magnitudes

According to Figure 3, the total cost and initial capital investment are ¥[621, 1148] × 10^{6} and ¥[552, 1034] × 10^{6}, which are contributed in Period 1. In fact, the total cost for

Table 2. Optimal Costs and TP Reduction of Ten Strategies in Three Periods

Strategy	Total cost (¥10^{6})			TP load reduction (ton)		
	k = 1	k = 2	k = 3	k = 1	k = 2	k = 3
j = 1	[12, 25]	[130, 199]	[167, 251]	[1.38, 2.13]	[13.55, 14.68]	[16.07, 16.94]
j = 2	[18, 24]	[0, 0]	[0, 0]	[1.17, 0.98]	[0, 0]	[0, 0]
j = 3	[18, 23]	[0, 0]	[0, 0]	[0.86, 0.85]	[0, 0]	[0, 0]
j = 4	[0.21, 0.42]	[0.33, 0.41]	[3.9, 6]	[0.32, 0.46]	[0.44, 0.37]	[4.77, 4.71]
j = 5	[29, 120]	[28, 100]	[0, 0]	[7.8, 7.44]	[4.82, 4.6]	[0, 0]
j = 6	[0, 0]	[0, 0]	[0, 0]	[0, 0]	[0, 0]	[0, 0]
j = 7	[2.8, 4.2]	[2.5, 3.1]	[0, 0]	[0.29, 0.3]	[0.22, 0.19]	[0, 0]
j = 8	[127, 176]	[0, 0]	[0, 0]	[5.51, 6.03]	[0, 0]	[0, 0]
j = 9	[20, 27]	[0, 0]	[0, 0]	[0.08, 0.08]	[0, 0]	[0, 0]
j = 10	[62, 189]	[0, 0]	[0, 0]	[0.16, 0.18]	[0, 0]	[0, 0]

each strategy is different with each others in three periods and only the total costs of strategies 1, 4, 5 and 7 increase from Period 1 to Period 3 (Table 2). Moreover, rural wastes treatment is the highest invested strategy in Period 1, while moderate erosion restoration is that in other two periods. The total costs of Watersheds 1, 3, 5, 11, 18 and 20 are higher than the others, and the highest invested watersheds for each period are [21.4, 32.3], [30.5, 35.8], and [60.1, 73.8]% of the total cost (Figure 4).

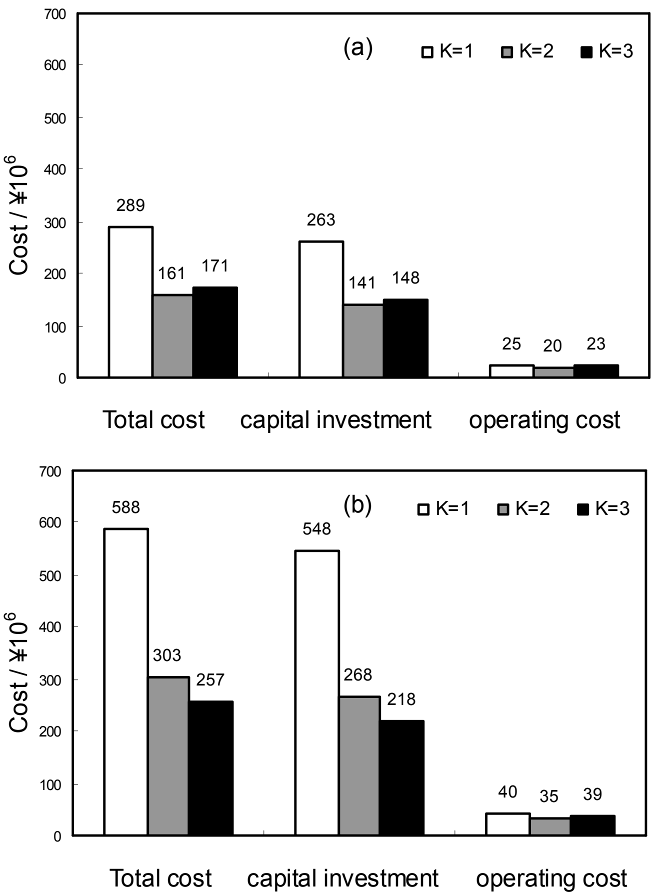

Figure 3. Optimal capital/ operating costs in three periods (a: lower-bound; b: upper-bound).

(2) Optimal TP Load Reduction

According to Figure 5, the TP load reduction is [57.43, 59.95] $t \cdot a^{-1}$ and is increasing from Period 1 to Period 3. In Period 1, waste water treatment plant and rural wastes treatment are the highest invested strategies, while moderate erosion restoration is that in other two periods. The highest invested strategies for each period are [40.3, 44.4], [71.2, 74.0] and [77.1, 78.2]% of total cost (Table 2). For each watershed, the total costs ([77.73, 81.94]% of total cost) of Watershed 3, 20, 11, 1 and 5 are higher than the others, and the highest invested watersheds for each period are [40.7, 44.7]% (Watershed 20), [26.3, 26.9]% (Watershed 1) and [48.2, 58.2]% (Watershed 3; Figure 6).

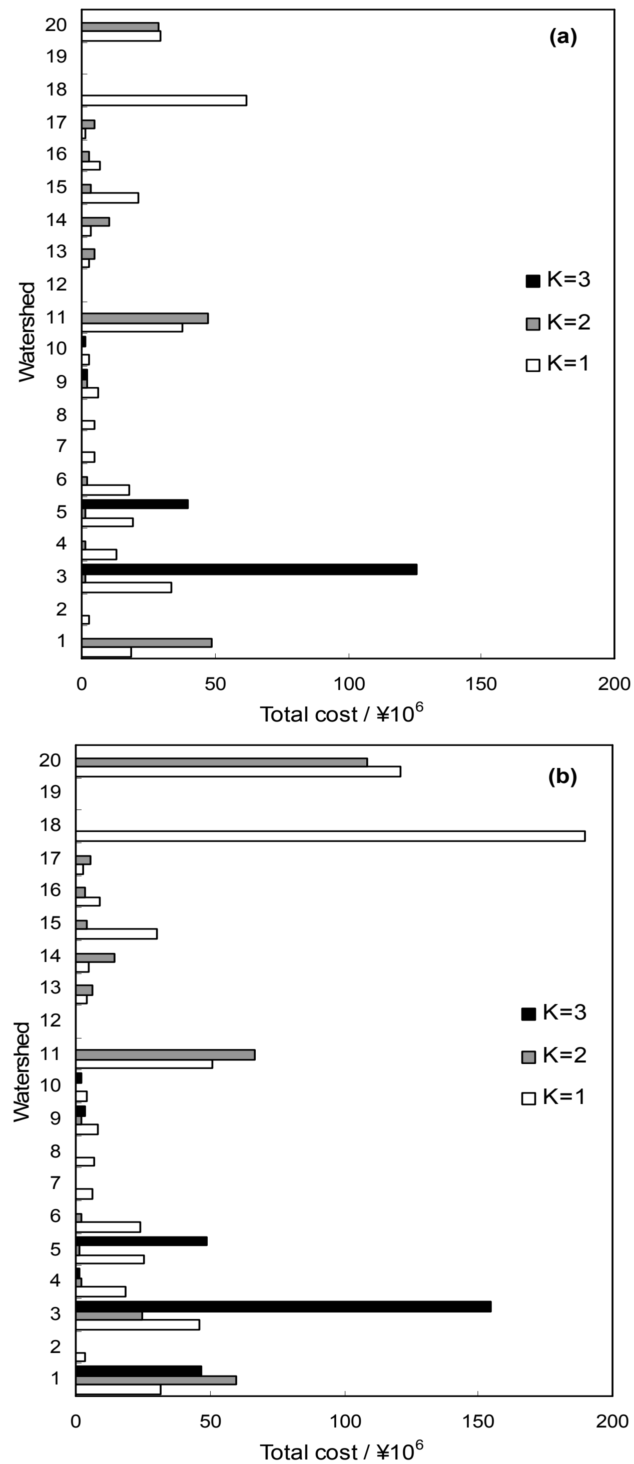

Figure 4. Optimal costs of the watershed in three periods (a: lower-bound; b: upper-bound).

5. Conclusions

An MILP model and its solution algorithm were developed to guarantee its solution space being absolutely feasible. Moreover, the modified proofs for determining constraints could assure that the first submodel achieve the optimal system benefit and the second obtain the feasible solution space. Although the EILP model was first successfully proposed in water-quality management of Lake Qionghai Basin, the results indicate that this model and its solution algorithm could be applied to other optimization problems involving uncertainty.

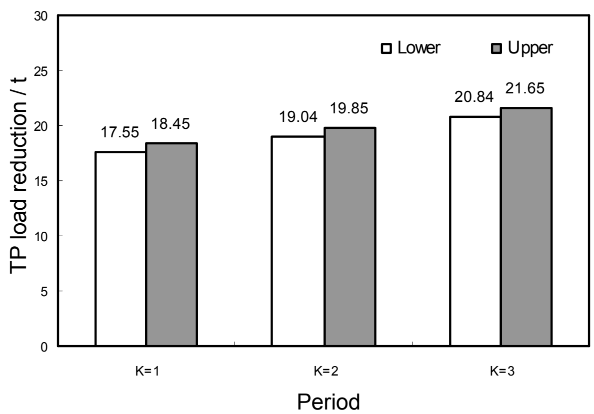

Figure 5. Optimal TP load reductions in three periods.

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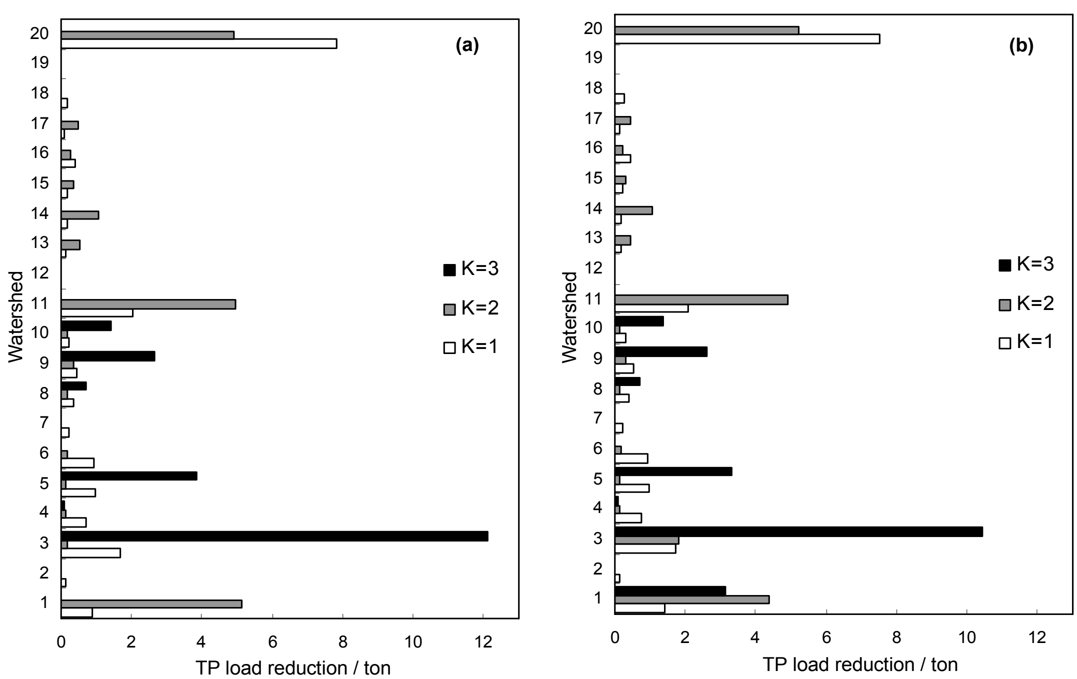

Figure 6. Optimal TP load reduction of the watershed in 3 periods (a: lower-bound; b: upper-bound).

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