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SRFILP: A Stochastic Robust Fuzzy Interval Linear Programming Model for Municipal Solid Waste Management under Uncertainty

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ABSTRACT. A stochastic robust fuzzy interval linear programming (SRFILP) model was proposed for supporting municipal solid waste (MSW) management under multiple uncertainties. The method integrated stochastic robust optimization (SRO), interval linear programming (ILP) and fuzzy possibilistic programming (FPP) methods into a general framework and could simultaneously deal with uncertainties expressed as fuzzy sets, stochastic variables and discrete intervals. The SRFILP model was applied to a hypothetical problem of municipal solid waste management. The results demonstrated that flexible interval solutions under different α-cut levels could be generated, which could help decision makers gain an in-depth insight into system complexities associated with solid waste management. The waste-management alternatives could be generated by adjusting the decision-variable values within their solution intervals. In addition, the proposed method could be used to help evaluate the trade-off between solution robustness and model robustness, and help waste managers identify desired cost-effective policies considering environmental, economic, system-feasibility and system-reliability constraints.

Keywords: stochastic robust optimization, fuzzy possibilistic programming, interval linear programming, solid waste management

1. Introduction

In municipal solid waste (MSW) management, many parameters such as waste-generation rate, facility capacity and operation condition, diversion goal, and waste treatment cost may appear uncertain. These complexities and uncertainties could also be multiplied by dynamic features of the system and interactive feature of the system components (Huang et al., 1993). Over the past decades, a number of inexact optimization methods were used for dealing with the complexities and uncertainties associated with environmental management problems, such as fuzzy, stochastic and interval programming methods (Chang and Wang, 1997; Huang et al., 1992, 1993, 1995a, b; Chanas and Zielinski, 2000; Huang and Loucks, 2000; Maqsood and Huang, 2003, Yeomans et al., 2003; Yeomans and Huang, 2003; Qin et al., 2007; Li et al., 2006, 2009).

Recently, the stochastic robust optimization (SRO), as proposed by Mulvey et al. (1995), Mulvey and Ruszczynski (1995), has received considerable attention. It was widely applied in many real-world problems, such as production planning, power capacity expansion, machine scheduling, telecommunication capacity expansion and logistics (Yu and Li, 2000). However, applications of SRO in the environmental management field were relatively limited. For example, Watkins Jr. and Mckinney (1997) explicitly introduced concept and characteristics of SRO model, and applied SRO model to evaluate tradeoffs among expected cost, cost variability, and risk of violating system constraints in water transfer planning and groundwater quality management. Xu et al. (2009) proposed a hybrid stochastic robust chance-constrained programming (SRCCP) model to support municipal solid waste management under uncertainty. The SRO method could be used to evaluate the trade-off between system economy and stability, and it was especially useful for helping analyze the reliability of satisfying (or risk of violating) system constraints under complex uncertainties. However, as a discrete-time, scenario-based approach, the complexities of SRO would increase significantly as the amount of the designed scenarios increases (Mulvey et al., 1995). Moreover, the solutions obtained by SRO model were fixed values, and could only provide limited information for decision makers.

Fuzzy mathematical programming (FMP) was derived from the incorporation of fuzzy set theory with ordinary mathematical programming framework. FMP could be classified into two categories: fuzzy flexible programming (FFP) and fuzzy possibilistic programming (FPP) (Zimmermann, 1985). In FFP, the flexibility in the constraints and fuzziness in the objective function, which were represented by fuzzy sets and denoted as “fuzzy constraints” and “fuzzy goal”, were introduced into or-

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dinary programming models (Zimmermann, 1985). Applications of FFP methods could be found in Huang et al. (1993, 1995b), Maqsood et al. (2005), and Qin et al. (2007). In FPP, the fuzzy coefficients were regarded as possibility distributions. Compared with FFP, the applications of FPP methods were limited. This was due to the fact that FPP may lead to complicated intermediate models and variables, which were difficult to handle.

Interval linear programming (ILP) is another alternative for handling uncertainties which were expressed as interval numbers (Yeomans and Huang, 2003). Previously, many applications of ILP in dealing with environmental management problems were reported (Huang et al., 1992, 1993, 1995a, b; Maqsood and Huang, 2003; Yeomans et al., 2003; Li and Huang, 2007). From these studies, it was demonstrated that ILP has a relatively low computational requirement and could be easily integrated with other inexact optimization methods (Huang et al., 1995b). However, ILP was incapable of controlling solution deviations under the impacts of uncertainties (i.e. the ability of keeping solution robustness) and might become infeasible when the parameters of the right-hand-side constraints were highly uncertain (Yeomans and Huang, 2003).

Therefore, as the first attempt in the related field, this study aims to develop a stochastic robust fuzzy interval linear programming (SRFILP) model for tackling multiple uncertainties associated with municipal solid waste (MSW) management. As an integration of SRO, FFP and ILP, this method can simultaneously deal with uncertainties expressed as triangular fuzzy numbers, stochastic variables and discrete intervals. Moreover, it is capable of evaluating the trade-offs among the expected costs, cost variability, and risk of violating relax constraints. A hypothetical MSW management case will be used to demonstrate the applicability of the proposed method.

2. Methodology

2.1. Stochastic Robust Optimization

According to the Mulvey et al. (1995b), Mulvey and Ruszczynski (1995), an original stochastic robust optimization (SRO) model can be written as follows:

$$Minimize\ \ Z=\sum_{s\in S}p_s\xi_s+\lambda\sum_{s\in S}p_s\left|\xi_s-\sum_{s'\in S}p_{s'}\xi_{s'}\right|+\omega\sum_{s\in S}p_s\delta_{it}^s \tag{1a}$$

Subject to:

$$Ax\le B \tag{1b}$$

$$C_sx+D_sy_s+\delta_{it}^s=E_s \text{ for all } s\in\Omega \tag{1c}$$

$$x\ge 0,\ \ y\ge 0 \tag{1d}$$

where $\zeta_s=c^Tx+d_s^Ty_s$, and ζ_s is the random objective function value corresponding to scenario s and occurring with probability p_s; p_s is the associated probability of a scenario s, naturally $\sum_{s=1}^{S}p_s=1$; λ and ω are weight coefficients; x and y represent the structural variables and control variables, respectively.

In Equation (1a), $\sum_{s\in S}p_s\left|\xi_s-\sum_{s'\in S}p_{s'}\xi_{s'}\right|$ represents solution robustness, and $\sum_{s\in S}p_s\delta_{it}^s$ represents model robustness. The latter one is used to penalize violations of the control constraints under different scenarios. The trade-off between solution robustness and model robustness can be analyzed by fixing the expected cost at various levels and adjusting the ratio λ/ω (Watkins and Mckinney, 1997). As for the three constraints, Equation (1b) is the structural constraint whose coefficients are fixed and free of noise. Equation (1c) is the control constraint whose coefficients are subject to noise. Equation (1d) ensures non-negative vectors.

To avoid handling of the absolute item in Equation (1a), Yu and Li (2000) proposed an equivalent linear formulation, namely a transformed stochastic robust optimization (SRO) model. It can be written as follows (Yu and Li, 2000):

$$Minimize\ Z=\sum_{s\in S}p_s\xi_s+\lambda\sum_{s\in S}p_s[(\xi_s-\sum_{s'\in S}p_{s'}\xi_{s'})+2\theta_s]+\omega\sum_{s\in S}p_s\delta_{it}^s \tag{2a}$$

Subject to:

$$\xi_s-\sum_{s'\in S}p_{s'}\xi_{s'}+\theta_s\ge 0 \tag{2b}$$

$$Ax\le B \tag{2c}$$

$$C_sx+D_sy+\delta_{it}^s=E_s \text{ for all } s\in\Omega \tag{2d}$$

$$x\ge 0,\ \ y\ge 0 \tag{2e}$$

$$\theta_s\ge 0 \tag{2f}$$

where Equation (2b) is a conversion constraint, and θ_s is a slack variable.

2.2. Fuzzy Possibilistic Programming

According to Lai and Hwang (1992), a general fuzzy possibilistic programming (FPP) can be written as (Lai and Hwang, 1992):

$$Maximize\ \ \sum_{j=1}^{n}\tilde{c}_jx_j \tag{3a}$$

Subject to:

$$\sum_{j=1}^{n}\tilde{a}_{ij}x_j\le\tilde{b}_i\ \ \ i=1,\ \dots,\ m \tag{3b}$$

$$x_j\ge 0\ \ \ j=1,\ \dots,\ n \tag{3c}$$

where $\tilde{c}_j$ (j = 1, …, n), $\tilde{b}$ (i = 1, …, m) and $\tilde{a}_{ij}$ (i = 1, …, m; j = 1, …, n) are imprecise fuzzy variables with possibilistic distribution functions. Among the various expression forms of fuzzy varia- bles, triangular fuzzy numbers are of the most important and are useful in solving FPP problems. Thus, we consider the fu- zzy variables as triangular fuzzy numbers. Let $\tilde{c}_j = (c_j^l, c_j^m, c_j^u)$, where c_j^l is the central value, c_j^m and c_j^u are the left and right spreads, respectively. Similarly, $\tilde{b}_i$ = (b_i^l, b_i^m, b_i^u) and $\tilde{a}_{ij} = (a_{ij}^l,\ a_{ij}^m,\ a_{ij}^u)$ also represented triangular fuzzy numbers respectively. Referring to the (Lai and Hwang, 1992), the concept and algorithm of α-cut can be used to solve model (3). As a result, the transformed FPP model can be written as follows (Lai and Hwang, 1992):

$$Maximize\ \sum_{j=1}^{n}\left[\alpha c_j^l + (1-\alpha)c_j^m, \alpha c_j^l + (1-\alpha)c_j^u\right]x_j \tag{4a}$$

Subject to:

$$\sum_{j=1}^{n}[\alpha a_{ij}^l + (1-\alpha)a_{ij}^m, \alpha a_{ij}^l + (1-\alpha)a_{ij}^u]x_j \le [\alpha b_i^l + (1-\alpha)b_i^m, \alpha b_i^l + (1-\alpha)b_i^u] \quad i = 1, \dots, m; j = 1, \dots, n \tag{4b}$$

$$x_j \ge 0 \quad j = 1, \dots, n \tag{4c}$$

where α-cut of a fuzzy set μ is a crisp subset of X and is deno- ted by:

$$\tilde{A}_\alpha = \left\{x \mid \mu_{\tilde{A}}(x) \ge \alpha,\ x \in X\right\} \tag{5}$$

If X is a collection of objects denoted by x , then a fuzzy set in X is a set of ordered pairs:

$$\tilde{A} = \left\{(x, \mu_{\tilde{A}}(x)) \ \mid x \in X\right\} \tag{6}$$

Obviously, the original fuzzy variables in the model (4) are expressed as intervals. Therefore, based on α-cut algorithm, the original FPP model can be transformed into the interval li- near programming (ILP), such that the interactive algorithm can be used to solve the model.

2.3. Interval linear Programming

Referring to Huang et al. (1992), an ILP model can be wri- tten as follows:

$$Maximize\ f^{\pm} = C^{\pm}X^{\pm} \tag{7a}$$

Subject to:

$$A^{\pm}X^{\pm} \ge B^{\pm} \tag{7b}$$

$$X^{\pm} \ge 0 \tag{7c}$$

$$C^{\pm},\ A^{\pm} \ne 0 \tag{7d}$$

where $A^{\pm} \in \left\{R^{\pm}\right\}^{m\times n}$, $B^{\pm} \in \left\{R^{\pm}\right\}^{m\times l}$, $C^{\pm} \in \left\{R^{\pm}\right\}^{l\times n}$, $X^{\pm} \in \left\{R^{\pm}\right\}^{n\times l}$, and $R^{\pm}$ denotes a set of interval numbers.

As proposed by Huang et al. (1992, 1995b), the model (7) can be solved by an interactive algorithm. A sub-model corres- ponding to f^+ (when the objective function is to be maximized) is first formulated and solved, and then based on the obtained solution of the first sub-model, the relevant sub-model corres- ponding to f^- can be formulated and solved. Finally, the objec- tive value and decision variables as discrete intervals can be obtained as $f_{opt}^{\pm} = \left[f_{opt}^-, f_{opt}^+\right]$ and $x_{j,opt}^{\pm}$ = $\left[x_{j,opt}^-, x_{j,opt}^+\right]$ respect- tively.

2.4. Stochastic Robust Fuzzy Interval Linear Programming for Solid Waste Management

Considering a problem wherein a manager is responsible for allocating solid waste from three municipalities to two treat- ment facilities (including landfill and incinerator) over several time periods. The objective of the study problem is to minimize the system cost with the optimal waste flow patterns (Li et al., 2008). Based on investigation of historical record, public sur- vey, and expert consultation, we assume that the unit transpor- tation costs in three municipalities and the unit operational cos- ts for the two waste-handling facilities are of random natures and specific discrete scenarios are designed for representing possible future conditions.

In a real-world MSW management system, the quality of available uncertain information is normally not good enough; these uncertainties could hardly be described as probability dis- tribution functions, and better be handled by fuzzy sets and/or discrete intervals (Li et al., 2008). For example, the uncertain- ties in revenues from energy sales and capacities of the landfill sites may be presented as discrete intervals. Because the pro- blems of traffic congestion and waste buildup at MSW recei- ving facilities may lead to the existence of the untreated amou- nt, a safe coefficient can be introduced to reflect such a fact. Due to lack of sufficient data, the safe coefficient, treatment ca- pacities of the incinerators, and waste-generate rates could be tackled as triangular fuzzy numbers (Li et al., 2008). Based on the integration of the SRO, FPP and ILP, a SRFILP model for the study case can be formulated as follows:

$$Minimize\ f^{\pm} = \sum_{k=1}^{3}L_k\sum_{s=1}^{S}p_s[\sum_{i=1}^{2}\sum_{j=1}^{3}x_{ijk}^{\pm}\left(TR_{ijks} + OP_{iks}\right) + x_{2jk}^{\pm}FE(FT_k + OP_{1ks} - RE_k)] + \lambda\sum_{k=1}^{3}L_k\sum_{s=1}^{S}p_s\{[\sum_{i=1}^{2}\sum_{j=1}^{3}x_{ijk}^{\pm}(TR_{ijks} + OP_{iks}) + x_{2jk}^{\pm}FE\cdot$$

$$\left(FT_k + OP_{1ks} - RE_k\right)] - \sum_{s'=1}^{S} p_{s'}[\sum_{i=1}^{2}\sum_{j=1}^{3} x_{ijk}^{\pm}\left(TR_{ijks'} + OP_{iks'}\right) + x_{2jk}^{\pm} FE \cdot$$
$$(FT_k + OP_{1ks'} - RE_k)] + 2\theta_s^{\pm}\} + \omega\sum_{k=1}^{3} L_k \sum_{s=1}^{S}\sum_{j=1}^{3} p_s \delta_{jks}^{\pm} \tag{8a}$$

Subject to:

(1) Conversion constraint:

$$[\sum_{i=1}^{2}\sum_{j=1}^{3} x_{ijk}^{\pm}\left(TR_{ijks} + OP_{iks}\right) + x_{2jk}^{\pm} FE\left(FT_k + OP_{1ks} - RE_k\right)] - \sum_{s'=1}^{S} p_{s'} \cdot$$
$$[\sum_{i=1}^{2}\sum_{j=1}^{3} x_{ijk}^{\pm}\left(TR_{ijks'} + OP_{iks'}\right) + x_{2jk}^{\pm} FE\left(FT_k + OP_{1ks'} - RE_k\right)] + \theta_s^{\pm} \geq 0$$
$$s, s' \in S \quad \forall k \tag{8b}$$

(2) Disposal capacity of landfill:

$$\sum_{j=1}^{3}\sum_{k=1}^{3} L_k\left(x_{1jk}^{\pm} + x_{2jk}^{\pm} FE\right) \leq TL_k^{\pm} \qquad i = 1, \ldots, I; \forall k \tag{8c}$$

(3) Treatment capacity of WTE facility:

$$\sum_{j=1}^{3} x_{2jk}^{\pm}(1 + \tilde{\eta}_k) \leq T\tilde{E}_k \qquad i = 1, \ldots, I; \forall k \tag{8d}$$

(4) Disposal demand:

$$\sum_{i=1}^{2} x_{ijk}^{\pm} + \delta_{jks}^{\pm} \geq W\tilde{G}_{jks} \qquad \forall j, k \quad s \in S \tag{8e}$$

(5) Non-negativity constraints:

$$x_{ijk}^{\pm}, \theta_s^{\pm}, \delta_{jks}^{\pm} \geq 0 \qquad \forall i, j, k \quad s \in S \tag{8f}$$

where:

f = net system cost ($/d);
x_{ijk} = waste flow from city j to facility i during period k (ton/day);
FE = residue flow rate from incinerator to landfill (% of incoming mass to waste-to-energy facility);
FT_k = unit transportation cost from waste-to-energy facility to landfill during period k ($/ton);
OP_{iks} = unit operating cost of facility i under scenario s during period k ($/ton);
RE_k = unit revenue from the waste-to-energy facility during period k ($/ton);
TE_k = maximum treatment capacity of the waste-to-energy facility during period k (ton/day);
TL_k = capacity of the disposal landfill during period k (ton);
TR_{ijks} = transportation costs from city j to capacity i under scenario s ($/ton), where i = 1 for the landfill capacity, and i = 2 for the waste-to-energy facility during period k;
WG_{ijk} = waste generation rate in city j under scenario s during period k (ton/day), j = 1, 2, 3;
P_s = fixed probability under each scenario S;
$P_{s'}$ = fixed probability under each scenario S;
η_k = safety coefficient for the incinerator facility during the period k;
δ_{jks} = untreated waste amount in city j during period k under scenario s (ton/day), j = 1, 2, 3;
Λ = weight coefficient, it can be used to reflect system economy;
ω = weight coefficient, it can be used to reflect system stability;
θ_s = slack variable under scenario s;
L_k = length of planning period k (d);
i = index of disposal facility (i = 1 and 2, where i = 1 for the landfill, and 2 for the incinerator);
j = index of city, j = 1, 2, 3;
k = index of planning period, k = 1, 2, 3;
s = index of scenarios, s = 1, 2,…, 9; s' = index of scenarios, s' = 1, 2,⋯ 9; S = set of scenario s.

In Equation (8), λ is the weight coefficient for reflecting the tradeoff between the mean and variance of the system cost. The weight coefficient ω is used to balance feasibility robustness of system cost. Based on the FPP algorithm, the triangular fuzzy numbers can be converted into discrete intervals, such that the original SRFILP model will become a general interval model, such that the transformed SRFILP model can be converted into two sub-models by interactive algorithm (Huang et al., 1992). The sub-model corresponding to the lower bound objective f^- can firstly be formulated as follows:

$$Minimize\ f^- = \sum_{k=1}^{3} L_k \sum_{s=1}^{S} p_s[\sum_{i=1}^{2}\sum_{j=1}^{3} x_{ijk}^{-}\left(TR_{ijks} + OP_{iks}\right) + x_{2jk}^{-} FE(FT_k$$
$$+ OP_{1ks} - RE_k)] + \lambda\sum_{k=1}^{3} L_k \sum_{s=1}^{S} p_s\{[\sum_{i=1}^{2}\sum_{j=1}^{3} x_{ijk}^{-}(TR_{ijks} + OP_{iks}) + x_{2jk}^{-} FE \cdot$$
$$\left(FT_k + OP_{1ks} - RE_k\right)] - \sum_{s'=1}^{S} p_{s'}[\sum_{i=1}^{2}\sum_{j=1}^{3} x_{ijk}^{-}\left(TR_{ijks'} + OP_{iks'}\right) + x_{2jk}^{-} FE \cdot$$
$$(FT_k + OP_{1ks'} - RE_k)] + 2\theta_s^{-}\} + \omega\sum_{k=1}^{3} L_k \sum_{s=1}^{S}\sum_{j=1}^{3} p_s \delta_{jks}^{-} \tag{9a}$$

Subject to:

$$[\sum_{i=1}^{2}\sum_{j=1}^{3} x_{ijk}^{-}\left(TR_{ijks} + OP_{iks}\right) + x_{2jk}^{-} FE\left(FT_k + OP_{1ks} - RE_k\right)] - \sum_{s'=1}^{S} p_{s'} \cdot$$
$$[\sum_{i=1}^{2}\sum_{j=1}^{3} x_{ijk}^{-}\left(TR_{ijks'} + OP_{iks'}\right) + x_{2jk}^{-} FE\left(FT_k + OP_{1ks'} - RE_k\right)] + \theta_s^{-} \geq 0$$
$$s, s' \in S \quad \forall k \tag{9b}$$

$$\sum_{j=1}^{3}\sum_{k=1}^{3} L_k\left(x_{1jk}^{-} + x_{2jk}^{-} FE\right) \leq TL_k^{-} \qquad i = 1, 2, 3, \ldots, I; \forall k \tag{9c}$$

$$\sum_{j=1}^{3} x_{2jk}^{-}(1 + \eta_k^{-}) \leq TE_k^{-} \qquad i = 1, 2, 3, \ldots, I; \forall k \tag{9d}$$

$$\sum_{i=1}^{2} x_{ijk}^{-} + \delta_{jks}^{-} \ge WG_{jks}^{-} \quad \forall j, s \in S \quad \forall k \tag{9e}$$

$$x_{ijk}^{-}, \theta_s^{-}, \delta_{jks}^{-} \ge 0 \quad \forall i, j \quad \forall k \quad s \in S \tag{9f}$$

Then, the sub-model corresponding to the upper bound objective f^+ can be formulated as follows:

$$\begin{aligned} Minimize\ f^{+} = & \sum_{k=1}^{3} L_k \sum_{s=1}^{S} p_s [\sum_{i=1}^{2}\sum_{j=1}^{3} x_{ijk}^{+}\left(TR_{ijks} + OP_{iks}\right) + x_{2jk}^{+} FE(FT_k \\ & + OP_{1ks} - RE_k)] + \lambda \sum_{k=1}^{3} L_k \sum_{s=1}^{S} p_s \{[\sum_{i=1}^{2}\sum_{j=1}^{3} x_{ijk}^{+}(TR_{ijks} + OP_{iks}) + x_{2jk}^{+} FE \cdot \\ & \left(FT_k + OP_{1ks} - RE_k\right)] - \sum_{s'=1}^{S} p_{s'} [\sum_{i=1}^{2}\sum_{j=1}^{3} x_{ijk}^{+}\left(TR_{ijks'} + OP_{iks'}\right) + x_{2jk}^{+} FE \cdot \\ & (FT_k + OP_{1ks'} - RE_k)] + 2\theta_s^{+}\} + \omega \sum_{k=1}^{3} L_k \sum_{s=1}^{S}\sum_{j=1}^{3} p_s \delta_{jks}^{+} \end{aligned} \tag{10a}$$

Subject to:

$$\begin{aligned} & [\sum_{i=1}^{2}\sum_{j=1}^{3} x_{ijk}^{+}\left(TR_{ijks} + OP_{iks}\right) + x_{2jk}^{+} FE\left(FT_k + OP_{1ks} - RE_k\right)] - \sum_{s'=1}^{S} p_{s'} \cdot \\ & [\sum_{i=1}^{2}\sum_{j=1}^{3} x_{ijk}^{+}\left(TR_{ijks'} + OP_{iks'}\right) + x_{2jk}^{+} FE\left(FT_k + OP_{1ks'} - RE_k\right)] + \theta_s^{+} \ge 0 \\ & s, s' \in S \quad \forall k \end{aligned} \tag{10b}$$

$$\sum_{j=1}^{3}\sum_{k=1}^{3} L_k \left(x_{1jk}^{+} + x_{2jk}^{+} FE\right) \le TL_k^{+} \quad i = 1, 2, 3, \ldots, I; \forall k \tag{10c}$$

$$\sum_{j=1}^{3} x_{2jk}^{+}(1 + \eta_k^{+}) \le TE_k^{+} \qquad i = 1, 2, 3, \ldots, I; \forall k \tag{10d}$$

$$\sum_{i=1}^{2} x_{ijk}^{+} + \delta_{jks}^{+} \ge WG_{jks}^{+} \quad \forall j \ \ s \in S \quad \forall k \tag{10e}$$

$$x_{ijk}^{+} \ge x_{ijk}^{-} \qquad \forall i, j, k \tag{10f}$$

$$\theta_s^{+} \ge \theta_s^{-} \qquad s \in S \tag{10g}$$

$$\delta_{jks}^{+} \ge \delta_{jks}^{-} \qquad \forall j, k \quad s \in S \tag{10h}$$

Thus, the final solution can be obtained through solving sub-models (9) and (10). The final solution is $f_{opt}^{\pm} = [f_{opt}^{-}, f_{opt}^{+}]$, $x_{opt}^{\pm} = \left[x_{opt}^{-}, x_{opt}^{+}\right]$, and $\delta_{opts}^{\pm} = \left[\delta_{opts}^{-}, \delta_{opts}^{+}\right]$. Figure1 shows the framework of the SRFILP model. The proposed model can deal with uncertainties expressed as deterministic values under different scenarios (i.e. in discrete probabilistic distributions), triangular fuzzy numbers and discrete intervals.

3. Case Study

In this study, a hypothetical MSW management case, proposed by Xu et al. (2009), will be used for demonstrating the applicability of the proposed method. The study system includes three municipalities and two treatment facilities (consisting of landfill and incineration). The main task of the manager is how to effectively allocate the generated waste from various municipalities to waste treatment facilities in the next fifteen years (with three five-year stages), in order to meet the rising waste disposal needs but spend as little money as possible (Xu et al., 2009).

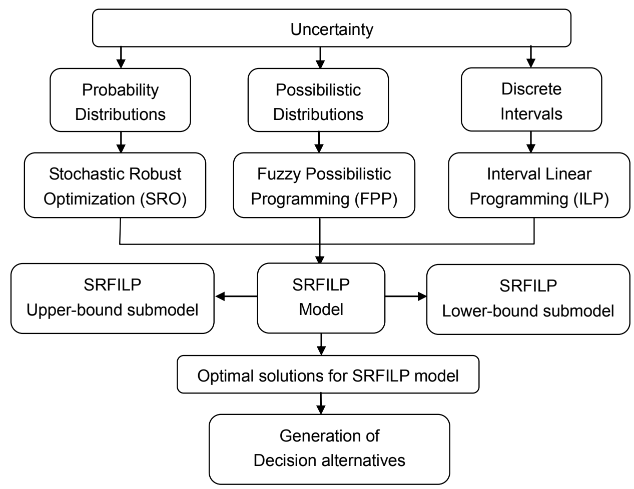

Figure 1. General framework of the SRFILP model.

The multiple uncertainties associated with the waste management system will be expressed in different formats. The transportation costs from the three municipalities to waste-handling facilities and the operational cost for the two facilities in different periods would be described as deterministic values under different scenarios, respectively (Xu et al., 2009). The detailed scenarios and probability information are listed in Tables 1 and 2. The waste-generation rates will be described as triangular fuzzy numbers (as shown in Table 3. The WTE facility generates residues of approximately 30% (on a mass basis) of incoming waste stream, and its revenue from energy sale is \$[15, 25] per ton of waste incinerated. The landfill is used to meet the demand of waste disposal or to receive residues from the incinerator, and it is described as discrete intervals. The landfill has an existing capacity of [3.285, 4.198] × 10^6 t.

For the incinerator, the random arrival and service times of waste delivery vehicles could lead to waste buildup. When the delivery vehicles arrive late in a previous day but early in the following day, a peak will occur such that the wastes cannot be treated timely as normal (Li et al., 2008). Thus, the peak flows originated from the random arrival and service times of waste-delivery vehicles may also raise a risk of contingent in-

Table 1. Transportation Costs from Municipalities to Treatment Facilities under Various Probability Levels

Municipalities	Probability	Unit transportation cost ($/t)		
		k = 1	k = 2	k = 3
To the landfill:				
Municipality 1				
Low	0.2	10.1	11.3	12.6
Medium	0.6	12.1	13.3	14.6
High	0.2	14.1	15.3	16.6
Municipality 2				
Low	0.2	8.5	9.6	10.8
Medium	0.6	10.5	11.6	12.8
High	0.2	12.5	13.6	14.8
Municipality 3				
Low	0.2	10.7	12.0	13.4
Medium	0.6	12.7	14.0	15.4
High	0.2	14.7	16.0	17.4
To the WTE facility:				
Municipality 1				
Low	0.2	7.6	8.6	9.7
Medium	0.6	9.6	10.6	11.7
High	0.2	11.6	12.6	13.7
Municipality 2				
Low	0.2	8.1	9.1	10.2
Medium	0.6	10.1	11.1	12.2
High	0.2	12.1	13.1	14.2
Municipality 3				
Low	0.2	6.8	7.7	8.6
Medium	0.6	8.8	9.7	10.6
High	0.2	10.8	11.7	12.6

Note: The related data are referred to Xu et al. (2009).

Table 2. Operational Costs of Two Facilities under Various Probability Levels

Level of Operation cost	Probability	Unit operational cost ($/t)		
		k = 1	k = 2	k = 3
Landfill:				
Low	0.3	43	50	60
Medium	0.4	50	58	68
High	0.3	57	66	76
Incinerator:				
Low	0.3	67	73	80
Medium	0.4	74	81	88
High	0.3	81	90	96

Note: the related data are referred to Xu et al. (2009).

sufficiency in the receiving facility. The introduction of the safety coefficient would help reflect such uncertainties and assures reliability of the system, even under the worst case due to the variety of uncertainties in waste generation and facility operation (Nie et al., 2006). Usually, this coefficient is estimated empirically and is thus of fuzzy nature. Similarly, the incinerator capacities could also be described as fuzzy sets (Nie et al., 2006). The safe coefficient and incinerator capacities under various α-cut level are provided in Tables 4 and 5.

Table 3. The triangular Fuzzy Sets and Discrete Intervals under Different α-cut Levels of Uncertain Easte-generation Rates for Three Municipalities

Level of waste-generation	Waste-generation rate		
	k = 1	k = 2	k = 3
Municipality1:			
TFS*	(237,282,337)	(287,332,388)	(335,380,435)
α = 0.2	[246,326]	[296,377]	[344,424]
α = 0.5	[260,310]	[310,360]	[358,408]
α = 0.8	[273,293]	[323,343]	[371,391]
Municipality2:			
TFS	(135,180,235)	(158,203,260)	(185,230,285)
α = 0.2	[144,224]	[167,249]	[194,274]
α = 0.5	[158,208]	[181,232]	[208,258]
α = 0.8	[171,191]	[194,214]	[221,241]
Municipality3:			
TFS	(247,292,357)	(282,327,382)	(317,362,417)
α = 0.2	[256,344]	[291,371]	[326,406]
α = 0.5	[270,325]	[305,355]	[340,390]
α = 0.8	[283,305]	[318,338]	[353,373]

TFS: Triangular fuzzy sets.

Table 4. The Triangular Fuzzy Sets and Discrete Intervals under Different α-cut Levels of Safe Coefficients over Three Periods

Level of Safe coefficient	Safe coefficient		
	K = 1	K = 2	K = 3
TFS	(0.15,0.2,0.25)	(0.1,0.15,0.2)	(0.05,0.1,0.15)
α = 0.2	[0.16,0.24]	[0.11,0.19]	[0.06,0.14]
α = 0.5	[0.175,0.225]	[0.125,0.175]	[0.075,0.125]
α = 0.8	[0.19,0.21]	[0.14,0.16]	[0.09,0.11]

Note: The related data are referred to Nie et al. (2006).

Table 5. The Triangular Fuzzy Sets and Discrete Intervals under Different α-cut Levels of Incinerator Capacities

Level of incinerator capacities	Operation capacity of the facilities (t/d)
TFS	(400,580,700)
α = 0.2	[436,676]
α = 0.5	[490,640]
α = 0.8	[544,604]

From above description, it can be seen that uncertainties may exist as discrete probability distributions, triangular fuzzy sets and interval numbers; moreover, due to the importance of the system stability, the tradeoffs among system economy and stability are also desired to be addressed in the management model. To tackle such a problem, the proposed SRFILP will be used.

4. Result Analysis

Table 6 demonstrates the results of the objective function values and the non-zero decision variables at different ω values and α-cut levels. Since SRFILP is an integration of SRO, FPP and ILP, such that the obtained solutions should reflect charac-

Table 6a. Solutions from the SRFILP Model under Different ω_i Levels (α = 0.2)

F	M	P	ω = 120	ω = 160	ω = 200	ω = 240
1	1	1	[246,326]	[246,326]	[246,326]	[246,326]
1	1	2	0	[296,377]	[296,377]	[296,377]
1	1	3	0	0	344	344
1	2	1	[144,224]	[144,224]	[144,224]	[144,224]
1	2	2	0	[167,249]	[167,249]	[167,249]
1	2	3	0	0	[194,203]	[194,274]
1	3	1	[256,344]	[256,344]	[59, 147]	0
1	3	2	0	[291,371]	[291,371]	[210,277]
1	3	3	0	0	0	0
2	1	1	0	0	0	0
2	1	2	0	0	0	0
2	1	3	0	0	0	0
2	2	1	0	0	0	0
2	2	2	0	0	0	0
2	2	3	0	0	0	0
2	3	1	0	0	197	[256,344]
2	3	2	0	0	0	[81, 94]
2	3	3	0	0	0	326
Untreated amount (t/d)			(1618, 2101)	(864, 1104)	(326, 557)	(0, 160)
Total cost ($\times 10^6$ \$)			(73.1, 101.0)	(171.0, 230.5)	(262.6, 324.7)	(338.9, 421.1)

Table 6c. Solutions from the SRFILP Model under Different ω_i Levels (α = 0.8)

F	M	P	ω = 120	ω = 160	ω = 200	ω = 240
1	1	1	[273,293]	[273,293]	[273,293]	[217,237]
1	1	2	0	[323,343]	[323,343]	[323,343]
1	1	3	0	0	[215,235]	[371,391]
1	2	1	[171,191]	[171,191]	[171,191]	[171,191]
1	2	2	0	[194,214]	[194,214]	[194,214]
1	2	3	0	0	[221,241]	[221,241]
1	3	1	[283,305]	[283,305]	[0, 22]	[0, 22]
1	3	2	0	[318,338]	[318,338]	[0, 20]
1	3	3	0	0	[0, 20]	[0, 20]
2	1	1	0	0	0	56
2	1	2	0	0	0	
2	1	3	0	0	0	
2	2	1	0	0	0	
2	2	2	0	0	0	
2	2	3	0	0	0	
2	3	1	0	0	283	283
2	3	2	0	0	0	318
2	3	3	0	0	0	353
Untreated amount (t/d)			(1780, 1900)	(945, 1005)	509	0
Total cost ($\times 10^6$ \$)			(82.2, 89.2)	(190.7, 205.4)	(271.6, 297.3)	(390.9, 421.6)

Table 6b. Solutions from the SRFILP Model under Different ω_i Levels (α = 0.5)

F	M	P	ω = 120	ω = 160	ω = 200	ω = 240
1	1	1	[260,310]	[260,310]	[260,310]	[260,310]
1	1	2	0	[310,360]	[310,360]	[310,360]
1	1	3	0	0	[297,347]	[358,408]
1	2	1	[158,208]	[158,208]	[158,208]	[158,208]
1	2	2	0	[181,232]	[181,232]	[181,232]
1	2	3	0	0	[208,258]	[208,258]
1	3	1	[270,325]	[270,325]	[0,55]	[0,55]
1	3	2	0	[305,355]	[305,355]	[72,122]
1	3	3	0	0	[0,50]	[0,50]
2	1	1	0	0	0	0
2	1	2	0	0	0	0
2	1	3	0	0	0	0
2	2	1	0	0	0	0
2	2	2	0	0	0	0
2	2	3	0	0	0	0
2	3	1	0	0	270	270
2	3	2	0	0	0	233
2	3	3	0	0	0	340
Untreated amount (t/d)			(1702, 2003)	(906, 1056)	401	0
Total cost ($\times 10^6$ \$)			(77.8, 95.3)	(181.2, 218.2)	(271.8, 333.3)	(365.8, 431.2)

teristics of above the three methods. The main procedure of solving SRFILP model is to determine the specific ω value and α-cut level. According to the related reference and the obtained results, we consider ω values of 120, 160, 200 and 240, α-cut levels of 0.2, 0.5 and 0.8 as representative.

Firstly, From Table 6, a majority of solutions would present as discrete intervals. For example, the scheme for the lower bound of the objective function represents an optimal decision scheme with the lowest possible cost (\$338.9 × 10^6, ω = 240 and α = 0.2) for waste treatment; this corresponds to the lower bounds of x_{ijk}, and they are preferable under advantageous conditions. Conversely, the scheme for the upper bound leads to the highest possible cost (\$421.1 × 10^6, ω = 240 and α = 0.2) and this corresponds to the upper bounds of x_{ijk}. The obtained solution intervals reflect the impact of uncertainties and provide a spectrum of decision alternatives for waste managers.

Since FPP method is incorporated into the SRFILP model, the solutions under different α-cut levels can be obtained. When α-cut level increases, the solutions would vary even under fixed values of weight ω. For example, when ω equals 120 in period 1, the waste amount from municipality 1 to the landfill at the significance levels of 0.2, 0.5 and 0.8 are [246, 326], [260, 310] and [273, 293] t/d, respectively. Similarly, the waste amount from municipality 2 at the significance levels of 0.2, 0.5 and 0.8 are [144, 224], [158, 208], and [171, 191], respectively. The results demonstrate that the upper bounds of the treated waste amount would decrease with the increase of significance levels; however, the lower bounds would increase instead. Moreover, the varying trends of both upper and lower bounds reveal that the obtained solution intervals would become narrower as significance levels are higher.

Table 6 also demonstrates that, as the significance level increases, the varying trends of the treated amount under fixed

ω values would be similar to each other. Figure 2 shows the varying trend of the allocated waste amounts from three municipalities to the landfill under different α-cut levels at ω = 200. Obviously, when the significance level increases, the solution intervals would decrease, implying that the system stability would become higher. The introduction of the FPP method could reflect uncertainties that cannot be described by simple intervals; it could help managers to gain an in-depth insight into the complexities of a MSW system.

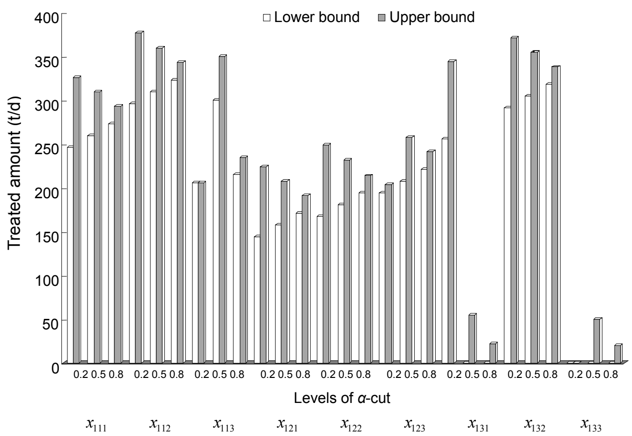

Figure 2. Optimized allocation amounts from the three municipalities to landfill under different α-cut levels (ω = 200).

Due to integration of SRO algorithm, the trade-off between solution robustness and model robustness could be evaluated. From Table 6, it is demonstrated that the solutions under any fixed α-cut level have considerable temporal and spatial variations when the ω level increases. For example, at a significance level of 0.2, the total expected system cost would be [73.1, 101.0], [171.0, 230.5], [262.6, 324.7], and [338.9, 421.1] $\times 10^6$ \$ under ω levels of 120, 160, 200 and 240, respectively. The results demonstrate that the lower and upper bounds of the expected total cost would increase with the increase of the ω value. The expected untreated waste amount would be [1618, 2101], [864, 1104], [326, 557] and [0, 160] t/d, respectively. This implies that the lower and upper bounds of the untreated waste amount would decrease as the weight ω increases; the treated waste amount would increase correspondingly. Similar trends of both the total expected system cost and the untreated waste amount could be observed at significance levels of 0.5 and 0.8, respectively.

Figures 3 and 4 present the varying trends of the expected untreated waste amount and the expected total cost under different ω values and α-cut levels, respectively. As the weight ω increases, the system cost would increase considerably. This demonstrates that both the solution robustness (i.e. closeness to an optimal solution) and system economy would become lower. Conversely, the untreated waste amount would decrease with the increase of ω level; the model robustness (closeness to a feasible solution) and system stability would increase instead. The comparison results demonstrate that the weight ω can be used to help analyze the trade-off between model robustness and solution robustness, and offer waste managers more options to generate final management scheme based on the their preference on system benefit and risk.

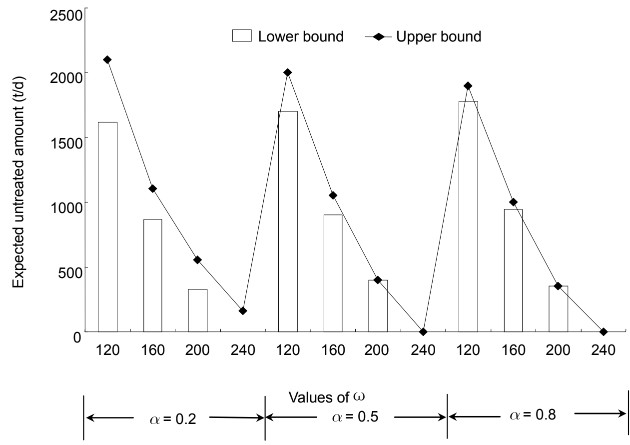

Figure 3. Expected untreated waste amount under different ω and α-cut levels.

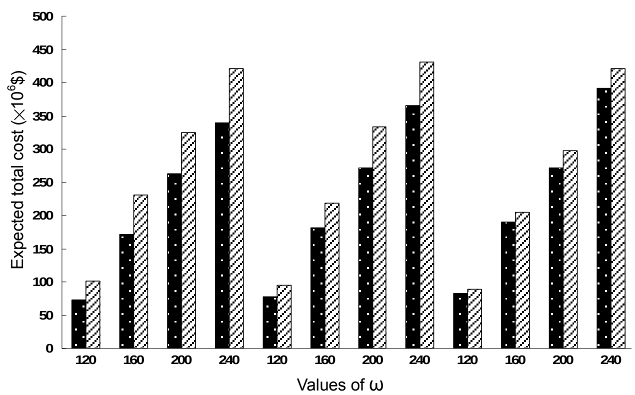

Figure 4. Expected total cost under different ω and α-cut level.

Generally, the study results demonstrate that the proposed SRFILP model owns advantages of SRO, FPP and ILP models. It can be used to: (i) evaluate balances among system economy and stability; (ii) generate solutions under different α-cut levels and offer more decision space to waste managers; (iii) obtain interval solutions where decision alternatives can be generated by adjusting decision variable values within their solution intervals; (iv) help MSW managers to identify desired waste mana- gement strategies under complex uncertainties.

5. Conclusions

In this study, a stochastic robust fuzzy interval linear programming (SRFILP) was developed for supporting municipal solid waste management under multiple uncertainties. As an integration of stochastic robust optimization (SRO), fuzzy possibilistic programming (FPP) and interval linear programming

(ILP), it can simultaneously deal with uncertainties expressed as triangular fuzzy numbers, stochastic variables and discrete intervals. Moreover, it is capable of evaluating trade-offs among the expected costs, cost variability, and risk of violating relax constraints.

The results demonstrated that the model could be used to evaluate tradeoff between system economy and stability; it could help waste managers identify desired policies under various environmental, economic, system-feasibility and system-reliability constraints. Although this study is the first attempt for planning waste-management system through the proposed approach, the study result suggests that such an integrated technique is also applicable to other environmental problems. In addition, the proposed method can be coupled with other uncertainty-analysis techniques (such as two-stage programming and nonlinear programming) for tackling more complicated problems.

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