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Integrated Assessment for Setting Greenhouse Gas Emission Targets under the Condition of Great Uncertainty about the Probability and Impact of Abrupt Climate Change

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ABSTRACT. In this paper the mid-21st-century target level for industrial carbon dioxide emissions is analyzed, taking into account the very large uncertainty about abrupt climate change. Following a brief review of integrated assessments of abrupt climate change, this study introduces an extension of DICE-2007, an integrated assessment model for climate policy analysis, which contains a hazard function that connects the rise in air temperature with the probability of abrupt change. The probability of abrupt change under a certain air temperature conditions and the economic impact of abrupt change are treated as widely variable parameters. Graphic indications of the combination of these parameters for several emission targets using the extended model show the necessity of developing adaptation measures to control the economic loss from abrupt change to below 8%, as well as to restrain global industrial carbon emissions in 2055 to the same level as those in 2005, assuming a most likely equilibrium climate sensitivity of 3 °C. Although a more stringent emissions target may be suggested in the spirit of precaution, it may lead to excessive carbon reduction from the viewpoint of cost-benefit balancing.

Keywords: catastrophe, climate change, global warming, hazard function, integrated assessment model, irreversibility, stochastic model

1. Introduction

Integrated assessment models (IAMs) that calculate interactions between economic factors, greenhouse gas (GHG) emissions, and climate change have been used to provide policy makers with optimal GHG emission targets for the future. However, it is difficult to resolve debates on optimal GHG emission control policies, largely because scientific information has not yet sufficiently accumulated, especially regarding the impact of the expected rise in atmospheric air temperature.

With respect to the impact of climate change, uncertainties exist not only concerning rather gradual effects related to the extent of warming, such as heat-related illness and land submergence caused by a rise in sea level, but also regarding abrupt climate change. This paper adopts the definition of abrupt climate change provided by the Committee on Abrupt Climate Change, National Research Council (NRC, 2002): “Technically, an abrupt climate change occurs when the climate system is forced to cross some threshold, triggering a transition to a new state at a rate determined by the climate system itself and faster than the cause.” A slowdown of the thermohaline circulation (THC), the ocean circulation driven by differences in density of sea water, in the North Atlantic Ocean and a collapse of the West Antarctic Ice Sheet (WAIS) are commonly cited as examples of abrupt changes; other scenarios are extreme and persistent droughts in widespread regions and the catastrophic release of methane by the breakdown of frozen gas-ice compounds in permafrost or from the ocean floor. Abrupt changes should be incorporated into IAMs since they can have a substantial impact on optimal GHG emission assessments (Wright and Erickson, 2003).

1.1. Literature Review

Prior to stating the objectives of the present study, let us briefly review the integrated assessment models that have already explicitly dealt with the dangers of these abrupt climate changes.

First, among the DICE (Dynamic Integrated model of Climate and the Economy) models, pioneer work in the integrated assessment of climate change, DICE99 and its multi-regional version RICE99 (Nordhaus and Boyer, 2000) estimated the catastrophic impact of abrupt climate change on the basis of interviews with social and natural scientists (Nordhaus, 1994). In these models, where the objective function is the discounted sum of instantaneous utility, the aggregated economic impact of climate change consisting of abrupt change and other gradual changes in several sectors is given as a steadily increasing function of the rise in global mean air temperature. The same method has

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been adopted by Roughgarden and Schneider (1999), and in DICE-2007 (Nordhaus, 2008), the latest version of DICE. These models applied conventional quantitative cost-benefit analysis to the climate change issue. It should be acknowledged that these models have been used since the early 1990s; however, formulating the climate change impact as an increasing function of temperature rise is not a suitable assumption for incorporating abrupt change, which in many cases is characterized by its irreversibility. That is, while the optimum run of DICE-2007 results in a temperature rise (3.47 °C relative to 1900) up to 2195, after that the temperature is calculated to decline so that the impact will decrease accordingly; this means that the impact is assumed to be reversible.

To introduce irreversibility, the second approach adopts a hazard function as demonstrated by Gjerde et al. (1999) based on a theoretical study by Clarke and Reed (1994). Here, the probability of an abrupt change event increases with the rise in air temperature, and at the time of abrupt change the utility drops and remains lower from that time on compared to the potential utility level if no abrupt change had occurred. Introducing this notion enables an IAM to explicitly represent the irreversible nature of abrupt change, with which the first approach has not dealt. In principle, the objective function of the model that applies this method is not simply the discounted sum of instantaneous utility itself, as in the first approach, but the expectation of it, because the time of abrupt change is a stochastic variable. It should be noted that, for implementing this approach in an IAM, the settings of a function to express the probability of an abrupt change and the impact of abrupt change when it occurs are crucial, while logical determinations of those settings are more difficult. For example, Gjerde et al. (1999) rely on a subjectively determined probability of abrupt change based on the expert opinion adopted in the DICE model, and assume the impact of abrupt change without any scientific basis or expert opinion.

The third, most recently developed type of integrated assessment has been approached by explicitly providing an IAM with specific, scientifically determined thresholds of abrupt change. A model developed by Mastrandrea and Schneider (2001), which couples the DICE model with a climate-ocean model based on a simple climate demonstrator (Schneider and Thompson, 2000), directly introduces the scientifically determined probability of a THC collapse into an IAM. Of the possible abrupt change events, the shutdown of the THC has already been assessed with the help of detailed investigations of paleoclimatic data (NRC, 2002) so that its threshold conditions have been elucidated. Following the threshold concentration of atomspheric carbon dioxide (CO_2) for the shutdown of the THC indicated by Stocker and Schmittner (1997), the model developed by Keller et al. (2004) and its probabilistic extension (McInerney and Keller, 2008) were formulated in order to activate economic damage when the CO_2 concentration exceeds the threshold. Mori et al. (2006) applied the threshold of the THC shutdown to evaluate whether CO_2 emission pathways can avoid the event. Few IAMs incorporate abrupt change events other than the shutdown of the THC, because of the lack of scientific speculation on the disintegration of the WAIS and other abrupt events (apart from the shutdown of the THC); to my knowledge, the only exception is a trial by Nævdal (2006) to include the threshold condition for the disintegration of the WAIS.

All the approaches described above are based on IAMs that embody a quantitative cost-benefit analysis, and are being improved upon by various researchers. However, approaches of this kind have been criticized. Van den Bergh (2004) provided a representative criticism that a qualitative cost-benefit analysis rather than a quantitative one should be applied for the assessment of climate change, since quantitative information is either lacking or unreliable. He pointed out the importance of decision making in conformity with the precautionary principle as well. In a formal sense, he suggested a minimax regret approach, which represents a larger degree of risk aversion than an expected utility approach, of which cost-benefit analysis with risk is a practical elaboration.

1.2. Objectives

The author of the present study agrees that a sound quantitative scientific basis for determinations regarding climate change is still lacking and that it is necessary to take precautionary measures, as stated in Article 3 of the United Nations Framework Convention on Climate Change (United Nations, 1992). Ideally, the most desirable solution is a quantitative cost-benefit analysis with an IAM on the basis of a full picture of the scientific impacts of climatic change, as well as highly accurate estimates of their economic consequences. In reality, however, this ideal presents an insurmountable difficulty: of the many possible abrupt change events, only the shutdown of THC has been explicitly incorporated into IAMs. Although the conditions under which abrupt changes occur have been gradually clarified over time in many studies, they still have not been fully understood; gaining a complete understanding of them will take some time.

Nevertheless, if consensus on a GHG emissions target could be established through discussions among many policy makers, economists and climate experts, some sort of quantitative analysis would still have persuasive power. Furthermore, quantitative discussions are indispensable to set up the emissions target itself. For this reason, while openly admitting that there is insufficient quantitative information on climate change, especially abrupt change, this study intends to seek political insights on the target of CO_2 emissions in the future, especially for the mid-21st century, which has become a recent focal point, under the conditions of extreme uncertainty concerning the probability and impact of abrupt climate change.

For this purpose, the IAM in this study will not follow the third approach described earlier, which tries to reflect specific thresholds of abrupt change, but instead will adopt the second approach, using a hazard function. In addition to Gjerde et al. (1999) introduced earlier, Bosello and Moretto (1999) and Castelnuovo et al. (2003) follow this approach, using a quantitative cost-benefit analysis that takes into account the irreversibility of abrupt climate change. All of these studies set up an ad hoc hazard function assuming a fixed value for the probability of abrupt change under certain air temperature conditions; they also assume that utility is reduced to the utility level of their first evaluation year, i.e., 1990, at the time of abrupt change and after.* Accordingly, the implications regarding carbon emissions control policy derived from these studies depend largely on subjective assumptions. The present study aims at resolving these shortcomings: namely, it seeks to determine what can be implied under a wide variation in the parameters representing these assumptions, thus reflecting their extreme uncertainty.

We also consider that the economic impact of abrupt change like the THC slowdown or WAIS collapse is expected to be controllable at least to a certain extent thanks to an autonomously developed adaptive capacity in the remote future when such events would occur. While, to my knowledge, none of the numerous IAM studies performed so far, except for the two studies by Hope et al. (1993) and De Bruin et al. (2009), have explicitly considered mitigations of the impact of climate change through adaptation measures, this study assesses both the carbon emissions reduction target as well as targeted mitigation levels of the impact of abrupt change through adaptive control.

To ensure transparency and reflect up-to-date information, the study will be based on the open source model DICE-2007, the latest version of Nordhaus's DICE models. Here, we focus on the control targets of industrial CO_2 emissions, i.e., total anthropogenic CO_2 emissions except those due to land-use change, because they are the most important anthropogenic GHG emissions.

2. Modeling Formulation

The DICE-2007 model (Nordhaus, 2008) was modified for use in the present study. The model consists of an economy module based on optimal growth theory and a climate-emissions-damage module representing the relationships between economic activity, GHG emissions, GHG concentrations in the air, and global average air temperature. It operates in time periods of ten years, with the first period centered on 2005. Since the full specifications including formulations and parameter settings of the model have been set forth by its author (Nordhaus, 2007, 2008), the remainder of this section focuses on describing the modifications we made to DICE-2007 to deal with an irreversible abrupt change, while a more detailed description of the model including fundamental modeling assumptions are provided in the Appendix.

2.1. Modeling Abrupt Change by a Hazard Function

The model adopted in this study is that presented by Gjerde et al. (1999), following Clarke and Reed (1994), in which the instantaneous utility function is considered to change discontinuously at the time of an abrupt change. Denoting the time of an abrupt change event by τ, and the instantaneous utility functions before and after the event by U and V, respectively, the objective function is expressed as follows:

$$W = E\left[\int_0^{\tau} e^{-\rho t} U(c(t))dt + \int_{\tau}^{\infty} e^{-\rho t} V(c(t))dt\right] \tag{1}$$

where ρ is the pure rate of social time preference (PRTP). The instantaneous utility function before abrupt change U is given by:

$$U(t) = \begin{cases} L(t)\left[c(t)^{1-\alpha} - 1\right] \big/ (1-\alpha), & \alpha \neq 1, \alpha > 0 \\ L(t)\log\left(c(t)\right), & \alpha = 1 \end{cases} \tag{2}$$

where $c(t)$ is per capita consumption during the 10-year time period t, $L(t)$ is the population, and α is the elasticity of the marginal utility of consumption (EMUC), i.e., relative risk aversion. The utility function after the abrupt change V is assumed to be equivalent to the function U with a consumption drop at a certain rate of Δ, as shown below:

$$V(t) = \begin{cases} L(t)\dfrac{\left[(1-\Delta)c(t)\right]^{1-\alpha} - 1}{1-\alpha}, & \alpha \neq 1, \alpha > 0 \\ L(t)\log\left[(1-\Delta)c(t)\right], & \alpha = 1 \end{cases} \tag{3}$$

This drop is permanent after the time τ, which corresponds to the irreversible nature of the abrupt change. It can be interpreted to be the net permanent damage factor, or the gross damage caused by the abrupt change minus the avoided damage by adaptation measures related to the change. Explicitly introducing this drop Δ as a parameter in the model is a difference from the existing studies.

According to Gjerde et al. (1999), Equation (1) is equivalent to:

$$\begin{aligned} W &= \int_0^{\infty} e^{-\rho t} S(t)U(t)dt + \int_0^{\infty} e^{-\rho t}\left(1 - S(t)\right)V(t)dt \\ &\approx \sum_t e^{-\rho t} S(t)U(t) + \sum_t e^{-\rho t}\left(1 - S(t)\right)V(t) \end{aligned} \tag{4}$$

By replacing Equation (1) with this equation as the objective function, the model is converted from a stochastic (nondeterministic) mathematical programming model to a deterministic one. Here, $S(t)$ is a survival function as shown below, expressing the probability that the event does not occur until the time t:

$$S(t) = \exp\left(-\int_0^t h\left(T(s)\right)ds\right) \approx \exp\left(-\sum_{s=o}^{t} h\left(T(s)\right)\right) \tag{5}$$

where $h(T(t))$ denotes a hazard function that indicates an in-

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* This means that the impact of abrupt change relative to economic scale increases over time, since the utility is formulated as an increasing function of gross domestic product (GDP) in the case of Gjerde et al. (1999), who extended the IAM presented in Kverndokk (1994), and adopted the assumption that the business-as-usual world GDP without climate change would steadily increase approximately 21 times during their planning horizon of 240 years. On the basis of this assumption, the impact of abrupt change can, if it occurs in the 22nd century, be equivalent to a 90% or higher loss of GDP.

crease in the probability of the event with a rise in air temperature $T(t)$. The function is supposed to be an increasing and convex function, and there are a wide variety of functional forms which satisfy the condition. In this paper, we consider the power function of Gjerde et al. (1999):

$$h\left(T(t)\right) = \eta \cdot \left(T(t) - T(2005)\right)^{\beta} \tag{6}$$

where the exponent and intercept of the right-hand side, denoted by β and η, respectively, are the key parameters to deal with the uncertainty in the probability of abrupt change. These two parameters determining the quantitative specification of the hazard function have been, as stated in the previous section, set ad hoc at fixed values. In this study, various cases in wide numerical ranges for these parameters will be considered to take the large uncertainty in this function into account, another feature of our model that distinguishes it from existing studies that have applied the hazard function approach.

2.2. Modeling Gradual (Non-abrupt) Change

In addition to abrupt change, the non-abrupt, gradual impact of climate change is also dealt with simultaneously in the model in this study; it is treated in the same way as in DICE-2007. In DICE-2007, total economic damage due to climate change, denoted by $D(t)$, is expressed by a ratio relative to the gross world product, $Y(t)$, and is given as a quadratic function of the rise in global average air temperature, $T(t)$, by the following equation:

$$D(t)/Y(t) = aT(t)^2 \tag{7}$$

where a is a parameter, and $D(t)$ is the total sum of several kinds of global warming damage including the damage caused by abrupt changes. In this study, gradual and abrupt damages in DICE-2007, denoted respectively by $D_1(t)$ and $D_2(t)$, are separated using parameters a_1 and a_2 ($a_1 + a_2 = a$) divided from parameter a as follows:

$$D_1(t)/Y(t) = a_1T(t)^2 \tag{8}$$

and

$$D_2(t)/Y(t) = a_2T(t)^2 \tag{9}$$

Equation (7) used in DICE-2007 can be replaced by Equation (8).

3. Parameter Setting

Table 1 shows the reference values for the major parameters used in the model, which are set to be basically the same as those used in DICE-2007 (Nordhaus, 2008). The parameters newly introduced in this study, i.e., a_1, a_2 (The parameter a_2 is only for reference purposes and is not used in the model), β, p, η, and Δ, are set as follows. The parameters a_1 and a_2 in Equations (8) and (9) are estimated from the detailed breakdown of climate change damage given by Nordhaus (2007). Nordhaus (2007) provided a sectoral breakdown of damage estimates underlying DICE-2007. The sectors are the same as those considered in his earlier estimates (Nordhaus and Boyer, 2000), and include agriculture, other vulnerable markets, coastal vulnerability, health, non-market time use, settlements (so far, non-catastrophic), and catastrophic impact.

Table 1. Major Parameter Setting

Parameter	Value
Pure rate of social time preference (PRTP), ρ	1.5%/yr (0.1%/yr)*
Elasticity of the marginal utility of consumption (EMUC), i.e. relative risk aversion, α	2.00 (2.87)*
Asymptotic global population	8,600 million
Initial growth rate of total factor productivity	0.92%/yr
Equilibrium climate sensitivity	3.0 °C
Gradual change damage function parameter, a_1	0.000978
Abrupt change damage function parameter, a_2**	0.001861
Probability of an abrupt change in the case of a 2.5 °C rise in 2090, p	1.2% (0 ~ 30%)
Exponent of hazard function, β	2 or 12
Intercept of hazard function, η	1.02×10^{-4} for $\beta = 2$, and 9.18×10^{-7} for $\beta = 12$ ($0 \sim 2.56 \times 10^{-3}$ for $\beta = 2$, and $0 \sim 2.30 \times 10^{-5}$ for $\beta = 12$)***
Impact of an abrupt change, Δ	30% (0 ~ 100%)

Notes: Numbers outside brackets are reference values while those in brackets indicate the assumed values or ranges of values for sensitivity analysis (see Section 4.2).

[*] For the sensitivity analysis, the values of PRTP (ρ) and EMUC (α) were changed simultaneously to 0.1%/yr and 2.87, respectively, to maintain the calibration of the rate of return on capital with empirical estimates. The PRTP of 0.1%/yr follows the assumption by Stern (2007).

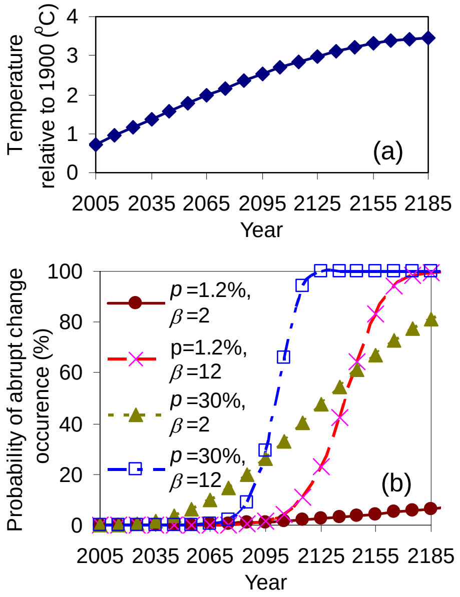
[**] This is not used for the model in this study (see the main text).

[***] The value of η is set consistently according to that of p.

The parameters β, p and η are related to the hazard function expressed in Equation (6). Though Gjerde et al. (1999) have subjectively presumed that exponent β equals 1.5, this value is uncertain and may largely affect the solution of the model. Hence, slightly greater ($\beta = 2$) and much greater ($\beta = 12$) values are considered for the exponent in Equation (6), to represent weaker and stronger convexity of the hazard function; more specifically, quadratic and 12th-order hazard functions are assumed in this analysis. The parameter p is defined as the probability of an abrupt change when the air temperature in 2090 has risen by 2.5 °C relative to that in 1900. The intercept η of the hazard function is so determined as to have the value of the survival function S (2095), the probability that the event does not occur until the time period (decade) centered on 2095 (i.e., 2090 ~ 2100), equal to $1 - p$

when an air temperature path reaching a 2.5 °C rise relative to temperatures in 1900 by the period centered on 2095 is given. The reference values for the parameters p and Δ are set at 1.2% and 30% in reference to their original settings in DICE (Nordhaus and Boyer, 2000).

The following example will help to clarify the role of the parameters β and p. Figure 1(a) shows a sample hypothetical trajectory of average air temperature, which is obtained from the optimum run of the original DICE-2007. Figure 1(b) shows the trajectory of the value of $1 - S(t)$, meaning the cumulative probability of an abrupt change prior to the time period t, calculated ex-post by Equations (5) and (6), and corresponding to the air temperature change given in Figure 1(a). For parameter p, 30% is considered here as a worst-case value, while the reference value of p is 1.2%.

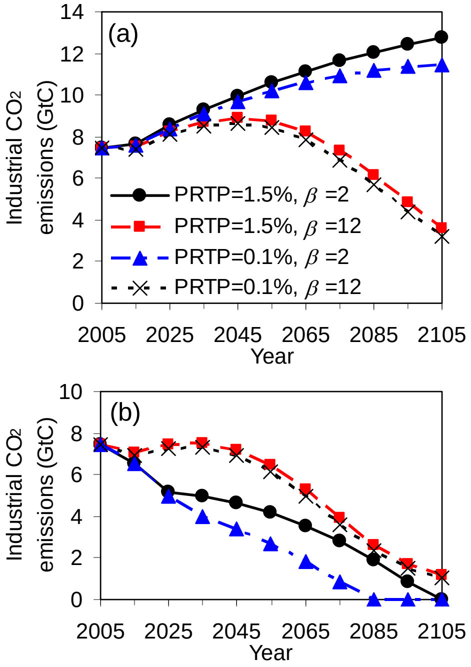


Notes: p: probability of an abrupt change in the case of a 2.5 °C rise in 2009; β : exponent of hazard function.

Figure 1. Average air temperature and cumulative probability of abrupt change: (a) sample profile and (b) probability [= 1 – SUR(t)].

As illustrated in Figure 1, the probability of an abrupt change prior to t is a monotone increasing function with regard to t; while the probability increases rather gradually for a quadratic hazard function, it steeply rises to 90% in a few decades after the probability reaches 10% when the 12th-order hazard function is applied. If p = 1.2%, the probability of an abrupt change is negligibly small during the 21st century. In the case that a very high value of 30% is assumed for p, the probability gradually becomes noticeable from relatively earlier periods in this century for the quadratic hazard function but stays at a negligible level up to a later period in the century and begins to rapidly increase after 2070 when the rise in air temperature exceeds 2 °C relative to 1900 temperatures in the 12th-order hazard function. In sum, the variations of the assumed values of β and p represent a wide variety of dynamics for the probability of abrupt change.

Taking into consideration the great uncertainty in the parameters p and Δ, corresponding to the probability and impact of abrupt change, a sensitivity analysis was performed by treating these parameters as variables. The parameter related to people's risk aversion α, together with the parameter PRTP (ρ), was also changed for the sensitivity analysis, as shown in Table 1.



Notes: PRTP: pure rate of social time preference; β: exponent of hazard function.

Figure 2. Optimal global industrial CO$_2$ emissions: (a) p = 1.2%, and (b) p = 30%.

4. Results

4.1. Basic Analysis

The model was developed using the reference values for the parameters as shown in Table 1. Since the model was transformed to a deterministic model from its essential stochastic form as described in Section 2.1, an anticipated large increase in computation time due to modeling stochasticity was avoided; the computation time for a set of assumed values of parameters is in general virtually equal to or slightly longer than that of the original DICE-2007; i.e., within four seconds when the model is written and solved by GAMS/CONOPT (Brooke et al., 1992; Drud, 1994) with a PC based on the Intel (R) Pentium (R) M processor, 1.2 GHz with 504 MB RAM.

The optimal schedule of total world industrial CO$_2$ emissions determined by our model is shown in Figure 2. Figure

2(a) includes the result for the reference case where PRTP = 1.5% and the probability of abrupt change in the case of a 2.5 °C rise in 2090 relative to 1900 temperatures, p = 1.2%. It shows that the optimal emissions schedule largely depends on the assumption of the exponent β of the hazard function; greater emissions reduction becomes optimal for a larger value of β. This result is understandable from Figure 1, which shows that, when p = 1.2%, the probability of abrupt change reaches about 1% at most up to 2100 for the quadratic hazard function while the probability begins to increase steeply around 2050 for the 12th-order function.

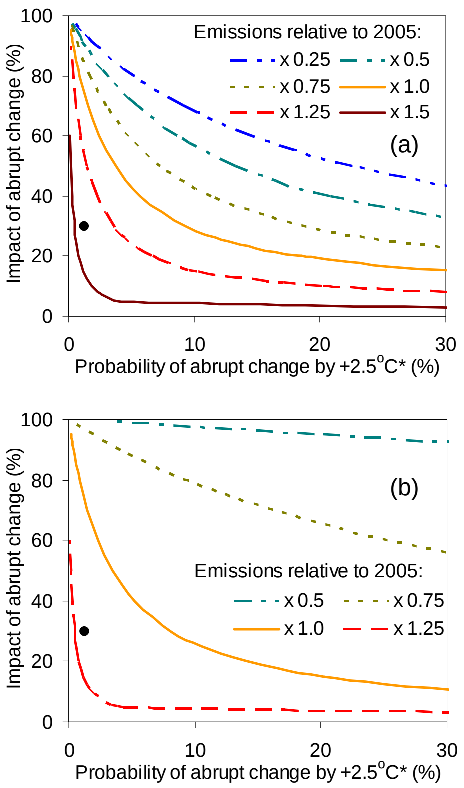
When we assume that people have a higher risk aversion, i.e., a higher EMUC of 2.87 and accordingly a lower PRTP of 0.1% as shown in Table 1, the optimal industrial CO_2 emissions are less than those in the reference case. This result is consistent with general IAM studies which demonstrate that accelerating CO_2 abatement is optimal for a lower PRTP because the impact of climate change occurring at later periods is valued highly. This explanation is commonly applicable to all the results shown later. The difference in the optimal emissions schedule according to the assumption of PRTP is, however, small compared to that affected by the assumption of the exponent β of the hazard function; for example, the relative difference of the optimal 2055 emissions is below 5% regardless of the exponent β.

Figure 2(b) shows the emissions schedule when p takes the worst-case value of 30%. In this case, contrary to the case of p = 1.2%, a more stringent emissions reduction is optimal for a smaller exponent β. This is because, if p = 30% and β = 2, the probability of an abrupt change becomes noticeable at an earlier time period, as shown in Figure 1.

4.2. Analysis of the Uncertainties of the Probability and Impact of Abrupt Change

The basic analysis shown above has been performed on the strong assumptions that the probability of an abrupt change in the case of a 2.5 °C rise in temperature by 2090, p, is fixed at 1.2 or 30% and that the impact of an abrupt change Δ = 30%, which are in reality highly uncertain. In this section, then, we assume that these are widely variable parameters: the values of p and Δ are changed within the range of 0 ~ 30% and 0 ~ 100%, respectively. For this sensitivity analysis, model computations are done for hundreds of combinations of assumed values of these uncertain parameters. This task consists merely of a pile of independent simulation runs that take a few seconds each; thus, the computation time is not a practical obstacle.

Focusing on the optimal industrial CO_2 emissions level in 2055, we investigated the conditions of the combination of p and Δ for the optimal emission levels of 1.5, 1.25, 1.0, 0.75, 0.5, and 0.25 relative to 2005. The results are presented in Figure 3, in which the horizontal and vertical axes indicate the values of p and Δ, respectively. A smooth L-shaped curve is drawn for each optimal emissions level.



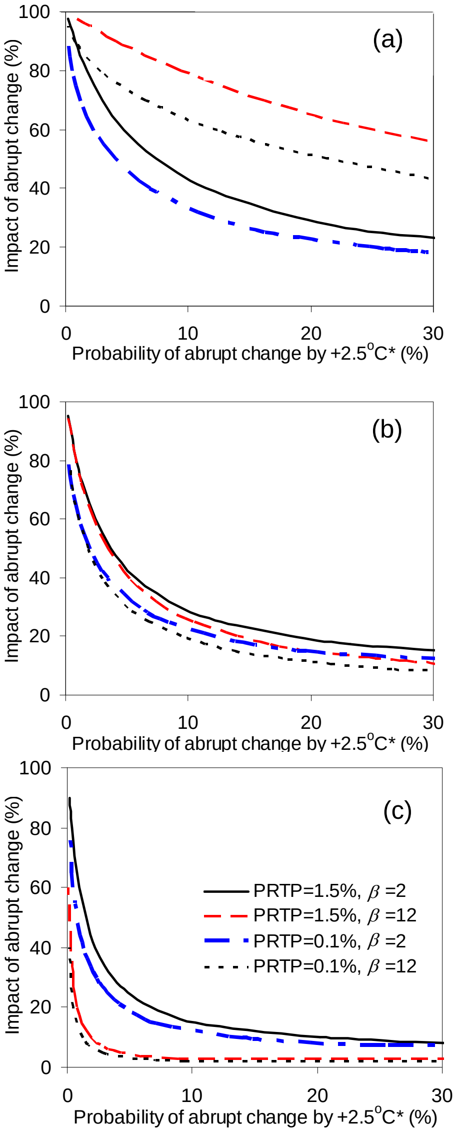
Note: The black dot in each figure corresponds to the reference parameter setting adopted in the original DICE model (Nordhaus and Boyer, 2000).*The probability corresponds to that of abrupt change in the case of a 2.5 °C rise in 2090 relative to temperatures in 1900.

Figure 3. Conditions for optimal global industrial CO_2 emission levels in 2055: (a) quadratic hazard function (β = 2), and (b)12th-order hazard function (β = 12).

When a quadratic hazard function was assumed, as shown in Figure 3(a), while all the curves start at almost the same points close to (p, Δ) = (0, 100%) regardless of the optimal emission levels, the curves are smoother for lower optimal emission levels. This means that the optimal emission levels are highly sensitive to changes in Δ. Each curve asymptotically approaches a horizontal line as the value of p increases to 30%, meaning that the change in p has less effect on the optimal emissions levels for a higher absolute value of p.

Figure 3(b) corresponds to the results when the exponent of the hazard function β = 12; compared to Figure 3(a), the curves representing the conditions where the optimal 2055 emission levels are 50 and 75% relative to 2005 shift upwards while the curve corresponding to the condition of 125% shifts down. Interestingly, the curve for 100% is at almost the same location. These observations are more clearly seen in Figure 4, where the curves are drawn separately by three cases of optimal emissions levels in relation to that of 2005.

In Figure 4(a), showing the condition of an optimal 2055 emissions level of 75% relative to that of 2005, the curve corresponding to the exponent β = 12 is located significantly above that of β = 2. This is because, when β = 12, such stringent reduction is not needed since controlling the temperature rise below a certain level so postpones a surge in the abrupt change probability [Figure 1(b)] that the present value of the abrupt change impact is sufficiently reduced. Focusing on the values of Δ corresponding to p = 30 on the four curves in the figure for the purpose of indicating the variation between the curves, the maximum and minimum corresponding values of Δ were 56% and 18%, respectively, and so the variation of Δ was 56 – 18 = 38% points.



Note: PRTP: pure rate of social time preference; β: exponent of hazard function. *The probability corresponds to that of abrupt change in the case of a 2.5 °C rise in 2090 relative to temperatures in 1900.

Figure 4. Conditions for optimal global industrial CO_2 emission levels of (a) 75%, (b) 100% and (c) 125% in 2055 relative to 2005.

In Figure 4(b), contrary to Figure 4(a), the curve corresponding to the higher exponent of the hazard function, i.e., β = 12, is located lower for the same PRTP. This is because this 2055 emissions control level of 100% relative to 2005 is not expected to sufficiently postpone the time period of an upsurge in the abrupt change probability to reduce the present value impact of abrupt change. The differences in curve location regarding the changes in PRTP and the exponent β were relatively small: the maximum and minimum values of Δ for p = 30 were 15% and 8%, respectively; thus, the variation of Δ was only 7% points.

In the case of the optimal 2055 emissions level of 125% relative to 2005, as shown in Figure 4(c), the curve corresponding to β = 12 is located still lower than that of β = 2 for the same PRTP for basically the same reason shown previously in the case of optimal 2055 emissions of 100% relative to those of 2005[Figure 4(b)]. However, the difference in curve location regarding the assumption of β is especially large in the area where the value of p is small. This is because, as implied from Figure 1(b), the potential rate of increase in the probability of abrupt change is remarkably higher when β = 12 than that in the case of β = 2 where the rise in probability is likely to stay modest for a relatively small value of p, e.g., p = 1.2%, while in a case of larger p, e.g., p = 30%, the probability may begin to noticeably grow even earlier for a smaller β. However, the difference between the maximum and minimum values of Δ for p = 30 was only 6%, which is smaller than in the case of the 2055 emissions control level of 100% relative to that of 2005.

5. Discussion

The implications derived from the above results include the fact that, if we assume a combination of p and Δ values of an optimal 2055 emissions level of 75% relative to those of 2005 in the case of the hazard function exponent β = 2 to be plausible, and accordingly attempt to control the emissions to this level, this attempt may lead to excessive emissions reduction on the basis of cost-benefit analysis, considering that a larger exponent of the hazard function, e.g., β = 12, could possibly be more appropriate in reality.

This kind of problem is small for the cases of the 2055 emission levels of 100% and 125% relative to 2005 since the value for the combination of p and Δ for these emission levels is less sensitive to the assumption of β as indicated in the previous section.

Now, suppose that the impact of an abrupt change Δ can be reduced to a certain extent by artificial measures for adapting to climate change such as disaster-tolerant social systems design while the true value of p is determined by nature and is beyond human control.

On this presumption, controlling the value of Δ can justify the 2055 emissions level of 100 or 125% relative to 2005 admitting the uncertainty of the p value. Although the true value of p may possibly be very large, the optimal emissions level tends to converge with an increase in p for emissions around these levels, as seen in Figures 4(b) and (c). Thus, the following discussion will concentrate on the worst-case value of p = 30 used in the analysis.

When a 2055 emissions level of 100% relative to 2005 is the target, the economic impact of an abrupt change should be a value between 8 and 15%, or the lower limit 8% from the precautionary viewpoint. If there is a potential for the economic damage to exceed this level, adaptation measures should be adopted for damage mitigation. Similarly, when the 2055 emissions level of 125% relative to 2005 is the target, the allowable economic damage of an abrupt change is 2%.

The critical issue here is how great the economic loss of an abrupt change actually will be. Among the very few examples of quantitative economic impact assessment of abrupt change, Tol (1998) estimated the economic damage of the THC collapse to be 3% or less. If we adopt this as a reference value, controling the economic impact of an abrupt change to below 2% seems to pose a potential difficulty.

6. Conclusions

Taking the above discussion into consideration, the analysis in this study yields the following policy suggestion: although growth in global industrial CO_2 emissions is acceptable for the next few decades, the emissions should be cut back to the 2005 level by 2050 ~ 2060. At the same time, introducing adaptation measures to abrupt climate events is recommended to enable the economic impact of abrupt change to stay below 8%. More stringent emissions reduction, e.g., a 2055 emissions level of 75% relative to that of 2005, may be advisable in the spirit of precaution; however, this stringent target can be excessive from the cost-benefit viewpoint. On the other hand, mitigating the 2055 emissions target to 125% relative to that of 2005 requires at the same time reducing the economic loss by an abrupt change to below 2%. Since there seems to be a considerable possibility of failure to control the economic damage to such a very low level, this emissions target should be considered too lax.

The following issues remain for the further extension of this study, although they would not influence the essentials of the conclusion described above: while the hazard function approach adopted in this study deals with a single abrupt change event, it can be extended to treat more than one event; further, it is expected that the model could be disaggregated to several different geographical regions, as was done in the RICE model (Nordhaus and Boyer, 2000), yielding regional policy suggestions.

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Appendix: Detailed Description of the Model

The IAM extended from DICE-2007 (Nordhaus, 2008) in this study consists of a macro-economy module including the calculation of economic impacts due to climate change, a module for computing anthropogenic carbon emissions, carbon-cycle and climate modules. The model is a nonlinear programming model that maximizes its objective function contained in the macro-economy module subject to several constraints. A time period is comprised of 10 years and the initial time period ($t = 2005$) is the 10 years centered at the year 2005; the time horizon ($t = Tmax = 2595$) is the 60th time period.

Fundamental formulations and assumptions of the model are shown below each module. The equations marked with asterisks are those added or modified from the original DICE-2007 model. Readers who wish to know further detailed background information on DICE-2007 are suggested to refer Nordhaus (2007, 2008) in addition to the following explanation.

(1) Macro-economy module

Following the neoclassical optimal economic growth model based on Ramsey (1928), the objective function is the total sum of the instantaneous utility at every period discounted by the pure rate of social time preference (PRTP) from the initial to horizontal future time periods. As shown in Section 2.1 of the main text, in this study where the definition of utility function is changed after abrupt climate change, the objective function to be maximized is as follows, corresponding to Equation (4) in the main text:

$$W = \sum_t e^{-\rho t} S(t) U(t) + \sum_t e^{-\rho t} \left(1 - S(t)\right) V(t) \tag{A.1}$$

where ρ is a parameter representing PRTP (for its assumed value, see Table 1 of the main text). Variables $U(t)$ and $V(t)$ are instantaneous utilities of consumption before and after the abrupt change, respectively, defined as follows:

$$U(t) = L(t)\left[c(t)^{1-\alpha} - 1\right] / \left(1 - \alpha\right) \tag{A.2}$$

$$V(t) = L(t) \frac{\left[\left(1 - \Delta\right) c(t)\right]^{1-\alpha} - 1}{1 - \alpha} \tag{A.3}$$

the same as Equations (2) and (3) in the main text. Here, $L(t)$ and α denote an exogenously given population and the elasticcity of the marginal utility of consumption (EMUC); the former is set to increase up to a saturated level of 8.6 billion referring to Lutz et al. (2008) and the latter is determined, as shown in Table 1 in the main text, so that the interest rate computed from the model result is consistent with recent actual records. For this reason, the assumed values of EMUC differ according to those of ρ.

The variable $c(t)$ denotes the consumption per capita, which equals the consumption at a certain time period t endogenously calculated as described later, and divided by $L(t)$ as indicated above. Equation (A.3) as compared to Equation (A.2) indicates that, after an abrupt change, the utility level is permanently reduced from its potential level assuming a decrease in the contribution of consumption per capita to the utility by a factor of Δ. The parameter setting of this constant depression rate Δ is shown in Table 1 in the main text.

The variable $S(t)$ in Equation (A.1) is called a survival function, and is expressed by using a hazard function $h(T(t))$ representing the extent of the probability of abrupt change occurrence due to a rise in air temperature relative to 1990, denoted by $T(t)$:

$$S(t) = \exp\left(-\sum_{s=o}^{t} h\left(T(s)\right)\right) \tag{A.4}$$

which is the same as Equation (5) in the main text. The hazard function is a strictly concave increasing function with $T(t)$ and is defined as follows using two uncertain parameters β and η:

$$h\left(T(t)\right) = \eta \cdot \left(T(t) - T(2005)\right)^{\beta} \tag{A.5}$$

The settings for the exponent β and intercept η are shown in Table 1 in the main text. The value of η is calibrated, as shown in Section 3 in the main text, according to the probability of abrupt change in 2090 on the assumption of a 2.5 ^{o}C rise in air temperature by that time relative to 1900 temperatures, the probability of which is represented by the uncertain indicator p (see Table 1 in the main text for its parameter setting), so that the value of the survival function $S(t)$ in 2090 is consistent with the value of p in the assumption. The consumption per capita $c(t)$ used in the definition of utility is calculated as:

$$c(t) = C(t) / L(t) \tag{A.6}$$

where the variable $C(t)$ denotes consumption and is determined as follows based on a neoclassical economic growth model incorporating economic impacts by considering climate change in the future. Endogenous variables representing investment, capital stock and production, denoted by $I(t)$, $K(t)$ and $Y(t)$, are related to each other by the following two equations:

$$Y(t) = C(t) + I(t) \tag{A.7}$$

$$K(t) = I(t) + \left(1 - \delta_K\right) K(t-1) \tag{A.8}$$

where δ_K denotes the rate of capital depreciation. The variable $Y(t)$ follows the production function shown below. This is basically a typical Cobb-Douglas production function assuming Hicks-neutral technological change, and the function is multiplied by a damage factor, denoted by $\Omega(t)$, that represents the climate change-related cost comprised of the cost of CO_2 emissions abatement as a countermeasure to climate change and that of gradual (non-abrupt) impacts as a result of climate change:

$$Y(t) = \Omega(t) A(t) K(t)^{\gamma} L(t)^{1-\gamma} \tag{A.9}$$

where $A(t)$ is the exogenously given total factor productivity assumed to increase by 0.92%/yr (Nordhaus and Yohe, 1983; Webster, 1997) in the first 10 years and at a lesser rate thereafter. The elasticity of production with respect to capital, denoted by γ, is calibrated based on the actual capital share in the factor input to the production. The damage factor $\Omega(t)$ is calculated endogenously by:

$$\Omega(t) = \frac{1 - \theta_1(t)\mu(t)^{\theta_2}}{1 + a_1 T(t)^2} \tag{A.10}$$

The numerator contains a cost factor of CO_2 abatement, which is a convex increasing function of the emissions-control rate, denoted by $\mu(t)$; this rate is a non-negative endogenous policy variable, and the industrial CO_2 emissions become zero when this rate reaches unity, which is its upper limit. The CO_2 abatement cost, denoted by $\Lambda(t)$, is calculated by using $\mu(t)$ as:

$$\Lambda(t)/Y(t) = \theta_1(t)\mu(t)^{\theta_2} \tag{A.11}$$

The parameters $\theta_1(t)$ and $\theta_2(t)$ appeared in the above abatement cost function are set referring to an estimated abatement cost with the Mini Climate Assessment Model (MiniCAM) (Edmonds, et al., 2004), which is an IAM incorporating a detailed energy systems module.

The denominator in Equation (A.10) calculates the gradual economic impact in several sectors due to climate change as the following quadratic function regarding the rise in air temperature:

$$D_1(t)/Y(t) = a_1 T(t)^2 \tag{A.12}$$

The intercept of the right side of this equation, a_1, is set based on an estimation by the developers of a DICE model based on their literature survey (Nordhaus and Boyer, 2000). While this term includes the catastrophic impacts of climate change in the original DICE-2007, as described in Section 2.2 of the main text, this portion is removed from our model.

(2) CO_2 emissions module

The anthropogenic emissions of CO_2, denoted by $E(t)$, is comprised of those induced by industrial activity and by artificial land use and land-use change:

$$E(t) = E_{Ind}(t) + E_{Land}(t) \tag{A.13}$$

The second term of the right side of this equation $E_{Land}(t)$ is given exogenously, referring to the future projection of land use-induced CO_2 emissions given in the B2 marker scenario provided by an IPCC's Special Report on Emissions Scenario (Nakićenović and Swart, 2000). The first term, $E_{Ind}(t)$, is the emissions caused by industrial productions and is calculated by:

$$E_{ind}(t) = \sigma(t)\left[1 - \mu(t)\right] A(t) K(t)^{\gamma} L(t)^{1-\gamma} \tag{A.14}$$

where the parameter $\sigma(t)$ denotes the CO_2 emissions per production, so-called CO_2 intensity, and is assumed to gradually decrease in relation to historical ratios of CO_2 emissions to GDP for major economies (Webster, 1997) since an autonomous decarbonization of energy resources is expected even without climate change problems. Namely, industrial CO_2 emissions are primarily from fossil fuel combustion, and the total sum of the future emissions is limited because fossil resources are finite:

$$CCum \geq \sum_{t=0}^{Tmax} E_{Ind}(t) \tag{A.15}$$

This limit $CCum$ is set at 6000 GtC referring to Nordhaus and Yohe (1983).

(3) Carbon-cycle module

Relationships among three reservoirs of CO_2 in the air, upper ocean and lower ocean, denoted respectively by $M_{AT}(t)$, $M_{UP}(t)$, $M_{LO}(t)$, are represented approximately by the following three linear equations. The CO_2 emissions calculated with Equation (A.13) flows into the reservoir in the air at every time period as shown in the first equation:

$$M_{AT}(t) = E(t) + \phi_{11} M_{AT}(t-1) + \phi_{21} M_{UP}(t-1) \tag{A.16}$$

$$M_{UP}(t) = \phi_{12} M_{AT}(t-1) + \phi_{22} M_{UP}(t-1) + \phi_{32} M_{LO}(t-1) \tag{A.17}$$

$$M_{LO}(t) = \phi_{23} M_{UP}(t-1) + \phi_{33} M_{LO}(t-1) \tag{A.18}$$

The seven parameters, ϕ_{11}, ϕ_{12}, ϕ_{21}, ϕ_{22}, ϕ_{23}, ϕ_{32}, and ϕ_{33}, appearing in these equations are calibrated so that estimated carbon-cycle matches with the calculation by the Model for the Assessment of Greenhouse-gas Induced Climate Change (MAGICC) (Wigley and Raper, 2001).

(4) Climate module

The radiative forcing, denoted by $F(t)$, caused by the atmospheric accumulation of greenhouse gases (GHGs) is calculated as follows according to the ratio of the atmospheric CO_2 accumulation estimated in the carbon-cycle module to that before the industrial revolution, and the radiative forcing induced by non-CO_2 GHGs, $F_{EX}(t)$, given exogenously, and based on the IPCC's future projection of non-CO_2 GHGs emissions (Nakićenović and Swart, 2000):

$$F(t) = \lambda\left\{\log_2\left[M_{AT}(t)/M_{AT}(1750)\right]\right\} + F_{EX}(t) \tag{A.19}$$

where λ denotes the estimated forcing of equilibrium CO_2 doubling in the air (= 3.8 W/m^2).

The mean surface air temperature changes in response to a change in the radiative forcing. The air temperature is also influenced mutually by the mean temperature in the deep ocean, denoted by $T_o(t)$. These relationships are expressed approximately by the following two equations:

$$T(t) = T(t-1) + \xi_1\{F(t) - \xi_2 T(t-1) - \xi_3\left[T(t-1) - T_o(t-1)\right]\} \tag{A.20}$$

$$T_o(t) = T_o(t-1) + \xi_4\left[T(t-1) - T_o(t-1)\right] \tag{A.21}$$

The four parameters ξ_1, ξ_2, ξ_3, and ξ_4 are calibrated, assuming the equilibrium rise in global mean atmospheric temperature for a doubling of CO_2 concentrations in the air, i.e., equilibrium climate sensitivity, of 3 ^{o}C, which is the best estimate value according to the latest IPCC assessment report (Meehl et al., 2007), so that the above two equations derive the same atmospheric temperature during the 21st century as calculated with MAGICC under a given trajectory of radiative forcing.

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