

Quantifying Spatio-Temporal Errors in Forest Fire Spread Modelling Explicitly

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ABSTRACT. Forest fire growth models (FGMs) are widely used in both research and operations. FGMs involve modelling complex physical-chemical dynamic processes over large spatially heterogeneous forest landscapes and long periods under changing weather conditions. Because of their complexity, it is difficult to validate these models. A typical approach is to graphically compare predicted boundaries to the corresponding boundaries of actual fires, which provides a visual rather than quantitative evaluation of modelling errors of forest fire spread. In this paper, we propose a method to quantify two-dimensional spread process modelling errors, in this case for forest fire spread modelling. We introduce several indices that can be used to quantify spatio-temporal modelling errors of two-dimensional spread processes explicitly and to evaluate overall modelling errors. We demonstrate the effectiveness of the indices through a case study in which the modelling errors of a forest fire simulated by a FGM are compared with those of a reference fire. The case study illustrates that the spatio-temporally explicit indices do work to quantify modelling errors of forest fire spread compared to a reference model and that this error analysis is not only useful for validating FGMs but also provides a basis for improving them. Because of the similarity of other two-dimensional spread processes to forest fire spread, we suggest potential applications of the method in other spatial spread processes, such as the spread of forest insect and contagious disease. The limitations of the method are presented.

Keywords: spatially explicit, boundary, two-dimensional spread, simulation error index (SEI), shape deviation index (SDI), error analysis, forest fire, fire growth

1. Introduction

Forest fires have occurred for thousands of years as part of natural processes in many forests. The boreal forest of Canada in particular is a fire-driven ecosystem. Today fires that occur both naturally from lightning and through human activities are increasingly affecting forest ecology, especially in the context of climate change, and thus interest in understanding fire processes increases. To address the need to understand and manage forest fires, various forest fire growth models (FGMs) have been developed for both research and operational purposes (e.g., French, 1992; Berjak and Hearne, 2002; Finney, 2004; Trunfio, 2004; Cui and Perera, 2008; Tymstra et al., in press). FGMs model complex physical-chemical dynamic processes over large spatially heterogeneous forest landscapes over durations of days or even months under changing weather conditions. Hence, it is not only difficult to build FGMs to accurately predict fire growth, but also difficult to validate the models. A typical method of validating FGMs is to graphically compare predicted boundaries to the corresponding boundaries of refer-

ence fires, such as actual fires (Feunekes, 1991; Berjak and Hearne, 2002; Trunfio, 2004; Opperman et al., 2006). One problem with this method is that it is a visual rather than quantitative evaluation of modelling errors.

A few studies on error analysis in FGMs are reported, such as those of Feunekes (1991) and French (1992). However, quantitative methods of error analysis of spatio-temporal spread processes were not well studied until Fujioka (2002) first explored a method for two-dimensional error analysis of forest fire spread. He used a polar coordinate system and measured fire spread modelling error using the difference between the radials of the simulated and actual fires. Though this measure is spatially explicit, it does not indicate modelling error in area by fire spread direction. It is more important for FGMs to distinguish the magnitude of error in area and thus identify causes of errors and/or indicate to what degree an error is caused by a given factor (Cui and Perera, 2008).

Although spatially explicit error assessment of predicted fires compared to “real” fires is common in remote sensing (e.g. Rimmel and Perera, 2002), the methods used do not address fire as a dynamic process with a temporal dimension. Quantifying modelling errors associated with spatial spread of ecological processes is also of interest to many disciplines other than fire modelling, for example, spread of tree species (Lischke et al., 2006), infectious diseases of colonizing populations (Bar-David et al., 2006), the foot and mouth epidemic in livestock

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(Keeling et al., 2001), and insect infestation, such as pine beetle (Beukema et al., 1997).

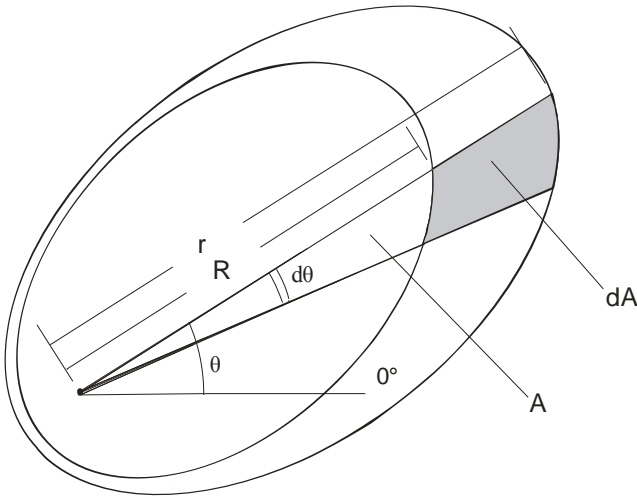
In this paper, we propose a method to quantify two-dimensional spread process modelling errors for forest fire spread models. Specifically, we introduce several indices and demonstrate their effectiveness through a case study in which the modelling error of a forest fire simulated by a FGM was analysed. We suggest the potential applications of this method in other models involving spatial spread in ecological processes, and also discuss its limitations.

2. Methods

2.1. Measuring Spatio- Temporal Fire Modelling Errors Explicitly

The method we propose for the spatial analysis of two-dimensional fire spread modelling errors is similar to that of Fujioka (2002), in that it describes fire simulation errors in a spatially explicit way. Fire boundaries are represented using the polar coordinate system, with the ignition point as the origin. We let $R(\theta, t)$ represent the reference radial and $r(\theta, t)$ represent the corresponding simulated radial at angle θ at time t . When r and R are the same for all θ and t , then the simulated boundary is identical to the reference boundary. Fujioka (2002) focused on fire spread modelling error: the difference, D , of the radials of the reference and simulated fire boundaries:

$$D(\theta, t) = r(\theta, t) - R(\theta, t) \quad (1)$$



Note: a is not labelled in the figure but it is the sum of A and dA .

Figure 1. Schematic illustrating the system used to calculate SEI (shaded area), the size difference at angle θ . a and A are the area of simulated fire size and the reference fire area, respectively, within an increment of angle $d\theta$ at angle θ and time t , and dA is the area difference of a and A .

The difference relative to the radial of the reference fire, D_R , can be expressed as:

$$D_R(\theta, t) = \frac{r(\theta, t) - R(\theta, t)}{R(\theta, t)} \quad (2)$$

This measure reflects the fire spread modelling error in distance. However, it is more important FGMs to measure the modelling error in fire size (area burned) for each angle, θ (Cui and Perera, 2008). Thus, we propose a new measure, the fire spread *Simulation Error Index (SEI)* to quantify the fire spread modelling error in area. This is defined as the difference in fire sizes between reference and simulated fires:

$$SEI(\theta, t) = dA(\theta, t) = a(\theta, t) - A(\theta, t) \quad (3)$$

where a and A are the area of simulated fire size and the reference fire area, respectively, within an increment of angle $d\theta$ at angle θ and time t , and dA is the area difference of a and A (Figure 1). Based on its definition, SEI can be both positive and negative.

Because $d\theta$ is infinitesimally small and if degree is used as the unit of angle, then a and A can be approximated as:

$$a(\theta, t) \approx \frac{d\theta}{360} \times [r^2(\theta, t) \times \pi] \quad (4)$$

$$A(\theta, t) \approx \frac{d\theta}{360} \times [R^2(\theta, t) \times \pi] \quad (5)$$

Thus

$$SEI(\theta, t) = a(\theta, t) - A(\theta, t) \approx \frac{d\theta}{360} \times [r^2(\theta, t) \times \pi] - \frac{d\theta}{360} \times [R^2(\theta, t) \times \pi] = \frac{[r^2(\theta, t) - R^2(\theta, t)] \times \pi}{360} d\theta \quad (6)$$

To see the relative weight of SEI in relation to the reference fire size F , a ratio, T_R , can be used:

$$T_R(\theta, t) = \frac{SEI(\theta, t)}{F(t)} = \frac{[r^2(\theta, t) - R^2(\theta, t)] \times \pi}{360 \times F(t)} d\theta, F(t) > 0 \quad (7)$$

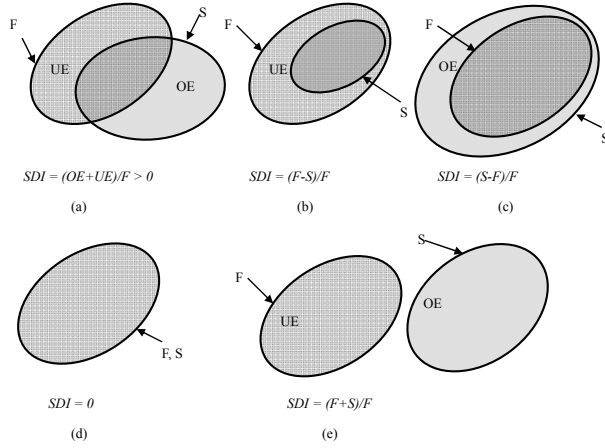
where t is time. This index, T_R , can be used to determine fire modelling errors spatially explicitly over time.

2.2. Measuring Overall Fire Spread Modelling Errors

Although SEI and/or T_R can be used to evaluate fire simulation errors in a spatio-temporal manner, measures are also needed to estimate the overall fire simulation error compared to reference fire boundary or any other boundary. We defined *Shape Deviation Index, SDI*, to evaluate the overall deviation of a fire's boundary from that of the reference fire:

$$SDI(t) = \frac{OE(t) + UE(t)}{F(t)} \quad (8)$$

where t is time, S is the size of simulated fire, F is the size the reference fire, OE is the area of S that is outside of F , and UE is the area of F that is outside of S at time t , respectively, as shown in Figure 2.



Note: OE is the area of S that is outside F , and UE is the area of F that is outside S .

Figure 2. The five topological relationships between the simulated fire boundary, of size S , and the reference fire boundary, of size F .

The larger the SDI , the larger is the deviation between the two boundaries. If $SDI=0$, the boundaries are identical (Figure 2d). When $SDI = (S + F)/F$, its maximum value (when $OE = S$ and $UE = F$), the boundaries only intersect at the ignition point or not at all, as shown in Figure 2e.

As an overall measure of fire spread modelling error, SDI , does not distinguish overestimation ($OE > 0$ and $UE = 0$, as shown in Figure 2c), underestimation ($OE = 0$ and $UE > 0$, as shown in Figure 2b), or a combination of both ($OE > 0$ and $UE > 0$, as shown in Figure 2a). Thus, we introduce two indices that measure the overall overestimation and underestimation:

$$SDI_{over}(t) = \frac{OE(t)}{F(t)} \quad (9a)$$

$$SDI_{under}(t) = \frac{UE(t)}{F(t)} \quad (9b)$$

where SDI_{over} and SDI_{under} are *Shape Overestimation Index* and *Shape Underestimation Index*, respectively. Based on the definitions of the indices:

$$SDI_{over}(t) + SDI_{under}(t) = SDI(t) \quad (10)$$

For calculating these indices, S and F values are from the simulation results; OE and UE can be calculated from the fire

spread modelling error in area, SEI , according to the definition of OE and UE if the simulated and reference boundaries both have continuous boundaries and share the same start point:

$$OE(t) = \int_0^{360} T(\theta, t) d\theta \quad (11a)$$

$$T(\theta, t) = \begin{cases} |SEI(\theta, t)|, & \text{if } SEI(\theta, t) < 0 \\ 0, & \text{if } SEI(\theta, t) \geq 0 \end{cases} \quad (11b)$$

$$UE(t) = \int_0^{360} T(\theta, t) d\theta \quad (12a)$$

$$T(\theta, t) = \begin{cases} SEI(\theta, t), & \text{if } SEI(\theta, t) > 0 \\ 0, & \text{if } SEI(\theta, t) \leq 0 \end{cases} \quad (12b)$$

where t is time, θ is the fire spread direction, $T(\theta, t)$ is an intermediate function, and $SEI(\theta, t)$ is defined in Equation 3.

3. Case Study

3.1. Case Study Background

To demonstrate the use of our method for the spatial analysis of modelling errors of a two-dimensional spread process, we compared a forest fire simulated using a FGM with a reference fire.

The FGM used is contained in BFOLDS, which is a forest fire regime model that simulates forest fires and succession over large boreal forest landscapes (millions of hectares) and long periods (hundreds of years) (Perera et al., 2008). The FGM in BFOLDS is deterministic, based on the Canadian Forest Fire Prediction (FBP) system (Forestry Canada, Fire Danger Group, 1992) and Canadian Fire Weather Index (FWI) system (Van Wagner and Pickett, 1985), and uses Huygens' Principle of wave propagation with the simple ellipse as the underlying template to shape fire growth (Catchpole et al., 1982). It uses a 32-direction raster-based fire spread algorithm and advances time in discrete steps over a continuous time domain. Although the FGM in BFOLDS is deterministic, BFOLDS itself is stochastic. The spatio-temporal variability stems mainly from the method used to simulate forest succession and the stochastic selection of weather data and number and locations of ignitions (Perera et al., 2008).

The size of the case study area was 60 by 60 km (360,000 ha) with homogeneous topography. Fuel was a randomly distributed spatial configuration of two kinds of rasters, black spruce and leafless aspen, defined as C2 and D1, respectively, in the FBP system (as shown in Figure 3a). Each fuel type made up approximately 50% of the total number of rasters. We referred this randomly fuel type configuration as RM. We only simulated one fire whose burn time was 15 hours and a single constant weather scenario was used: temperature 27.6 °C, relative humidity 18%, wind speed 16.0 km/h, wind direction 270° (north-south), 24-hour cumulative rainfall 0 mm, and FFMC, DMC, BUI of 95, 42, and 101, respectively. FFMC, DMC, and BUI are indices of Canadian Fire Weather Index (FWI) system.

FFMC (Fine Fuel Moisture Code) is a numeric rating of the moisture content of litter and other cured fine fuels. This code is an indicator of the relative ease of ignition and the flammability of fine fuel. DMC (Duff Moisture Code) is a numeric rating of the average moisture content of loosely compacted organic layers of moderate depth. This code indicates fuel consumption in moderate duff layers and medium-size woody material. DC (Drought Code) is a numeric rating of the average moisture content of deep, compact organic layers. This code is a useful indicator of seasonal drought effects on forest fuels and the amount of smouldering in deep duff layers and large logs. BUI (Buildup Index) is a numeric rating of the total amount of fuel available for combustion. It combines the DMC and the DC (Van Wagner and Pickett, 1985).

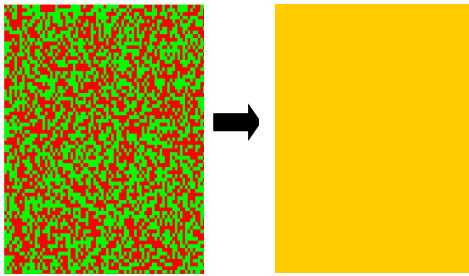


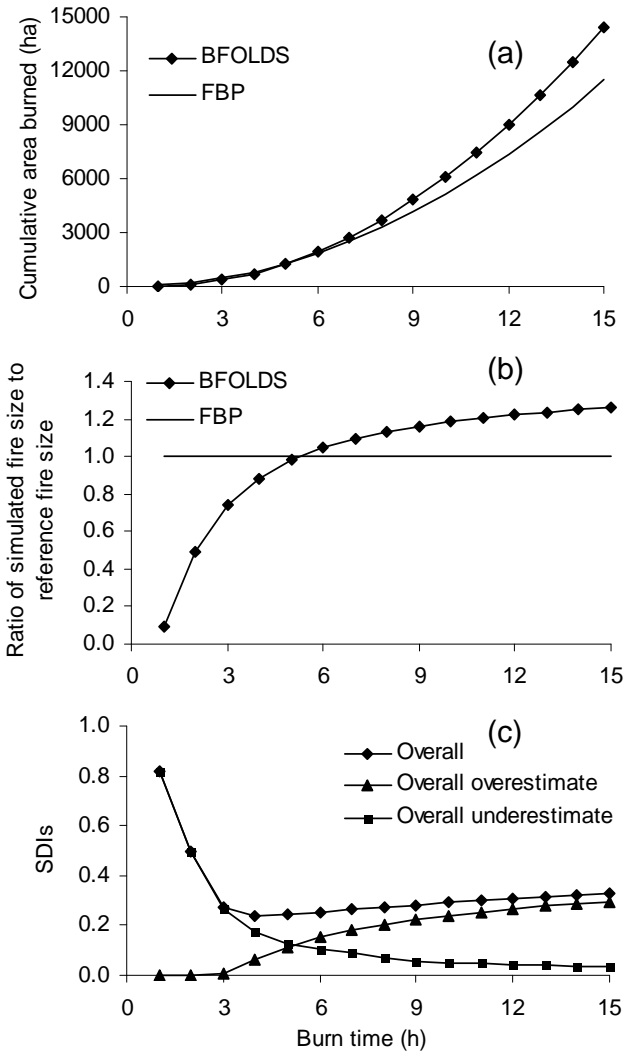
Figure 3. The fuel type spatial configurations used in the case study: (a) RM was a randomly distributed fuel type spatial configuration of two kinds of rasters, black spruce and leafless aspen, defined as C2 and D1, respectively, in the FBP system, with each fuel type comprising 50% of the total number of rasters. (b) M1 configuration consisted entirely of rasters of fuel type defined as M1_50 in the FBP system (a boreal mixedwood of black spruce (50%) and leafless aspen (50%).

The reference fire boundary was calculated using the FBP system directly based on the FBP equations (Forestry Canada Fire Danger Group, 1992) and the same elliptical fire growth model (Catchpole et al., 1982) used by the FGM in BFOLDS. Because the FBP system cannot accommodate multiple fuel types, we calculated FBP fire growth for the uniform fuel type M1_50%, a boreal mixedwood of black spruce (50%) and leafless aspen (50%) (as shown in Figure 3b). Since the M1 and RM fuel type spatial configurations had the same average fuel type composition (50% black spruce and 50% leafless aspen), at an infinitesimally small raster size RM would become M1_50%, as shown in Figure 3. Thus, in theory, predictions by the FGM in BFOLDS using RM fuel type configuration should be similar to those simulated by the FBP system using the uniform fuel type, M1_50% because:

$$ROS(M1_50\%) = \frac{ROS(C2) + ROS(D1)}{2 \times ROS(C2) \times ROS(D1)} \quad (13)$$

where $ROS(M1_50\%)$, $ROS(C2)$, and $ROS(D1)$ are the rates of spread for fuel types M1_50%, C2, and D1, respectively.

One could compare the prediction of a FGM to perimeters of a real fire, if the goal is to assess the validity/veracity/accuracy of the prediction. However, the goal here is to demonstrate how the method works, and consequently we compared predictions by an FGM to a reference fire that is theoretical (generated by FBP system).



Note: In this study, a fire calculated by the FBP system was used as the reference fire.

Figure 4. Overall forest fire spread modelling error indices over a 15-hour period: (a) cumulative fire sizes, (b) ratio of cumulative fire sizes predicted by the FGM in BFOLDS to those predicted by the FBP system, and (c) SDI , SDI_{under} , and SDI_{over} .

3.2. Case Study Results

Cumulative fire sizes (areas burned) of the fire simulated by the FGM in BFOLDS compared with those simulated using the FBP systems are shown in Figure 4a. For comparison, these cumulative fire sizes simulated by the FGM in BFOLDS were also represented using a ratio (Figure 4b):

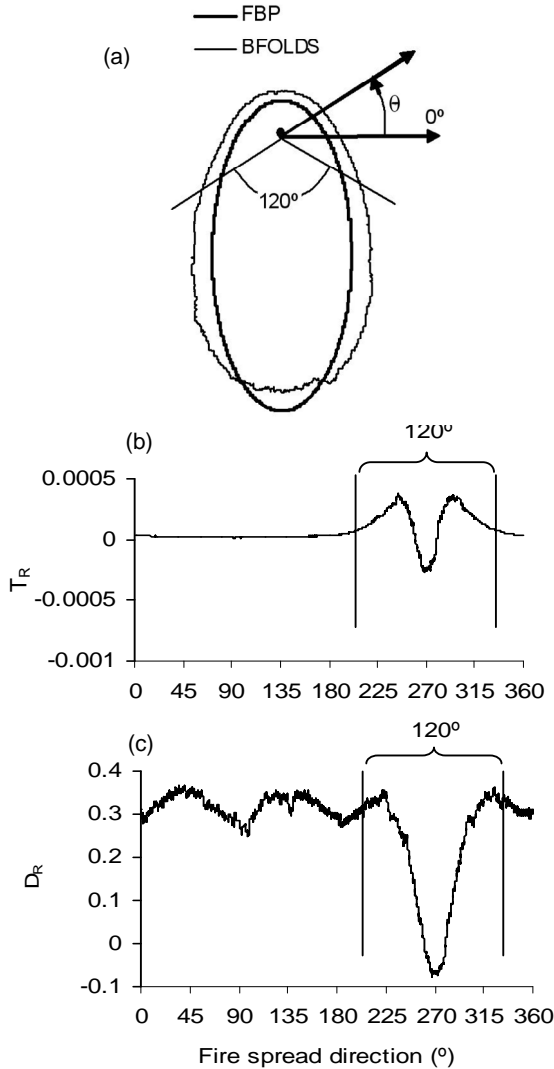


Figure 5. Example of (a) Fire boundaries simulated by the FGM in BFOLDS and the FBP system, respectively, (b) the corresponding SEI relative to the fire size predicted by the FBP system, T_R , for all fire spread directions, (c) the corresponding difference of the radials of the reference and simulated fire boundaries relative to reference radial, D_R , for all fire spread directions.

$$Ratio = S/F, F > 0 \tag{14}$$

where S was the fire size simulated by the FGM in BFOLDS and F was the fire size simulated by the FBP system. $Ratio = 1$ indicates that the FGM fire size equals FBP fire size and $ratio < 1$ or > 1 indicates that the FGM fire size is smaller or larger, respectively, than the corresponding FBP fire size.

Figures 4a and 4b show that the absolute fire size difference increased with burn time, while the relative size difference first decreased. After it reached zero at roughly the fifth hour then it increased but at a decreasing rate.

However, the absolute and relative fire size differences do not reflect the magnitude of fire spread modelling errors. Even though simulated fire sizes were larger than reference fire sizes (calculated by the FBP system) after the fourth hour (Figures 4a and b), the simulated fire boundaries still only covered part of the reference fire boundaries (Figures 4c). The boundaries coincided at two points. The overall modelling errors quantified by the Shape Deviation Index- SDI , overestimation index- SDI_{over} , and underestimation index- SDI_{under} , are shown in Figure 4c. In this case study, overall modelling errors measured by SDI , were large initially, but decreased with burn time. SDI reached the minimum after four hours, and then increased with burn time as predicted cumulative fire sizes increased and became larger than those predicted by the FBP system. This increase was mainly result of overestimation since the underestimation decreased with burn time (Figure 4c).

Figure 5a shows the boundaries simulated by the FGM in BFOLDS and calculated by the FBP system at time $t = 15$ h, while Figures 5b and 5c show the corresponding fire spread modelling error in area and distance, respectively, relative to the boundary calculated by the FBP system. Although the differences in distance (measured by D_R , ratio of D to the radius predicted by the FBP system) were higher over fire spread directions $0^\circ \sim 210^\circ$ and $330^\circ \sim 360^\circ$, the differences in fire size (measured by the T_R , ratio of SEI to the fire size predicted by the FBP system) were relatively small. On the contrary, most of the difference in fire size occurred in the direction of head fire (in this case the same as the wind direction) in an angle range of roughly 120° ($210^\circ \sim 330^\circ$) over which D_R was relatively small, as shown in Figures 5a to 5c.

Combining the graphs of T_R for one hour time steps, the temporal dimension of forest fire spread modelling errors in fire size can be quantified as shown in Figure 6. The figure shows the spatio-temporal trend of relative simulation errors during fire growth

4. Discussion

4.1. Measuring Overall Fire Spread Modelling Errors Using SDI

Based on the results shown in Figure 4, simulated fire sizes cannot reliably be used to measure forest fire spread modelling errors. If the simulated fire size is similar or identical to the reference fire size, it does not necessarily mean that modelling errors are small or non-existent because: (1) the two areas may not match each other, due to different orientation (as shown in Figure 2a), location (as shown in Figure 2e), shape (as shown in Figure 7), or any combination of the these. However, if the simulated fire size is different from the reference fire size, fire spread modelling errors definitely exist, though the magnitudes of the errors are unknown for reasons cited above.

SDI is good indicator of overall forest fire spread modelling errors of area. As shown in the previous section, it measures the difference in fire size between the simulated fire and the reference fire in the form of a ratio, as defined in Equation 8.

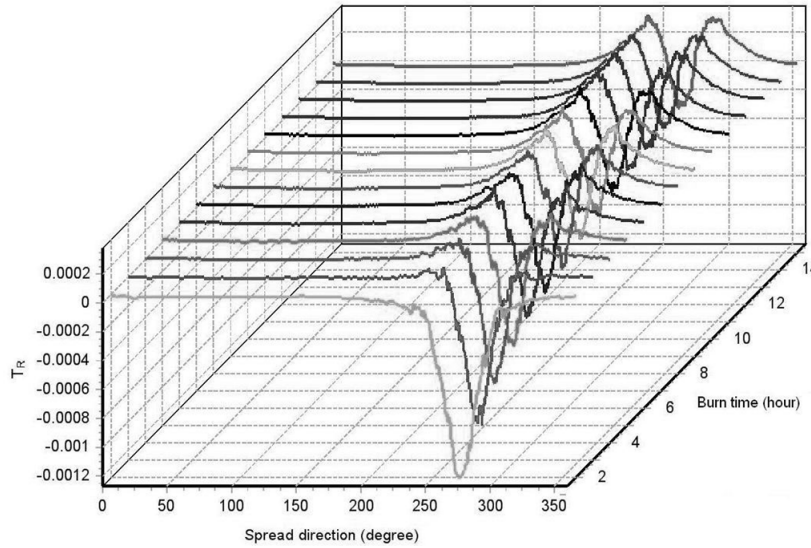


Figure 6. A spatio-temporal illustration of SEI of the FGM in BFOLDS over a 15-hour period in all fire spread directions.

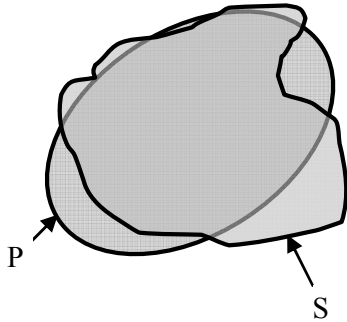


Figure 7. A simulated fire polygon (S) can be the same fire size as a reference fire polygon (F) but have a different shape.

If $SDI = 0$, then the simulated fire is identical to the reference fire: the same fire size, shape, orientation, and location. The simulated fire completely coincides with the reference fire and vice versa. If the simulated fire size is different from the reference fire size, even if the area of one is completely covered by the other (as shown in Figures 2b and 2c), the magnitude of fire spread modelling error can be measured by SDI : $SDI = SDI_{under}$ for the case shown in Figure 2b and $SDI = SDI_{over}$ for the case shown in Figure 2c. SDI is especially valuable for measuring overall fire spread modelling errors when the simulated fire size is identical to the reference fire size: (1) if $SDI = 0$, the simulated fire is identical to the reference fire; (2) if $1 > SDI > 0$, the simulated fire either has different orientation (as shown in Figure 2a), location (as shown in Figure 2e), or shape (as shown in Figure 7) than the reference fire, or any combination of the three; (3) if SDI is at its maximum, which is $(S+F)/F$, the simulated fire and the reference fire share only the ignition point (start point) or do not intersect at all (as shown in Figure 2e).

SDI_{under} and SDI_{over} are useful for assessing fire spread mo-

delling errors caused by overall underestimation and overestimation, respectively, of fire size in some fire spread directions. SDI alone or together with SDI_{under} and SDI_{over} can be used to quantify the overall forest fire spread modelling errors. One important aspect of these overall modelling error indices is that they can be temporal, as defined in Equations 11 and 12 and shown in the Figure 4c. This allows tracking of the temporal trend of the overall forest fire spread modelling errors.

4.2. Measuring Spatio-Temporal Forest Fire Spread Modelling Error Explicitly

In theory, both D_R and T_R can be used to quantify spatio-temporal forest fire spread modelling errors explicitly. However, they emphasize different aspects of error: D_R measures fire spread modelling errors as radial difference relative to the reference fire radial while T_R measures fire size difference relative to the reference fire size. The advantage of T_R is that it indicates directions of fire spread modelling errors for fire size, which better measures the performance of a forest fire growth model than radial difference. As shown in Figures 5a, 5b, and 6, fire spread modelling errors measured by T_R are concentrated in an angle range of about 120° in fire spread directions. However, D_R does not produce this indication. The two indices can be complimentary and they reveal different aspects of modelling errors.

The sensitivity of T_R to fire spread simulation error by spread direction is significant because it not only reveals the directions with the most modelling errors but also identifies an area for possible FGM algorithm improvement. For example, Cui and Perera (2008) proposed that adding more fire spread directions around the direction of the head fire rather equally in all directions would enhance the performance of raster based FGMs without significantly reducing modelling efficiency (Feunekes, 1991).

4.3. Potential Applications for and Limitations of the Two-Dimensional Error Analysis Method

The method suggested for spatial analysis of two-dimensional fire spread modelling errors could be applied to other two-dimensional spread processes, such as the spread of tree species, disease epidemics, and insect infestations, such as pine beetle. These spread processes share the basic characteristics of forest fire spread: they all (1) spread spatially in two dimensions over space and with a temporal dimension, (2) are driven by their intrinsic characteristics (e.g., forest fires spread when fuel is available and weather conditions are favourable) but are also greatly affected by the local conditions (e.g., fuel types for fires) over space and factors that change over time (e.g., weather conditions for fires).

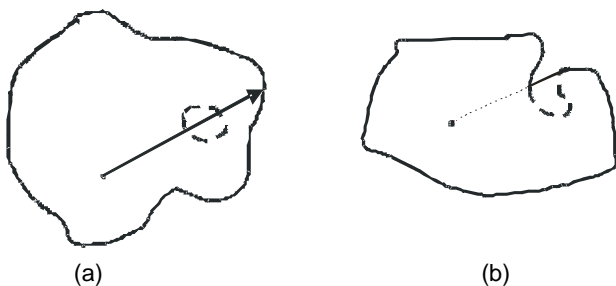


Figure 8. Two examples of assumed boundaries of a certain spread process in which (a) a hole within a boundary is ignored, and (b) part of a concave of a boundary is ignored. I is the start point of a two-dimensional spread process. Areas within hidden lines are ignored.

The overall modelling error indices, SDI , SDI_{under} , and SDI_{over} , can be readily used in these contexts. However, SEI or its relative form T_R has some limitations: it can only be used for processes that have a single start point and one continuous boundary. These indices also cannot deal with boundaries when any radial from start point has more than one intersection point with the boundaries because holes and/or concaves are ignored as if the hole does not exist (as shown in Figure 8a) and/or the concave is smaller (as shown in Figure 8b).

When SEI and SDI are used to quantify temporal errors, interpreting the magnitude of errors must be judicious. For example, when the rate of spread is slow, the temporal errors can be smaller due to the small change perimeters of fire or other two-dimensional spread processes because magnitude of differences may not be linear with time

5. Conclusions

To better verify or validate the ecological models, the modelling errors of two-dimensional spread processes, such as forest fire spread, need to be quantified. Methods for assessing two-dimensional modelling error should include indices for overall modelling errors and indices for assessing error in a spatio-temporally explicit manner.

The fire spread *Simulation Error Index*, SEI , or its relative

form, T_R , can be used to quantify spread modelling errors in fire size by spread directions over time, which measures the performance of a two-dimensional spread model, such as a forest fire growth model. A major limitation of these indices is that they can only be used for processes that have a single start point and continuous boundary.

The *Shape Deviation Index*, SDI , as an overall modelling error measure can capture not only the magnitude of the overall modelling errors but also overall characteristics of modelling errors together with indices, such as size, SDI_{under} , and/or SDI_{over} : overestimation, underestimation, and/or both (if both, overestimation and underestimation occur in different spread directions, respectively, over all spread directions). These overall indices can be used over the whole spread process and thus are useful in tracking overall modelling error trend over time. They also do not share the limitations of the spatio-temporally explicit indices: in theory they can be used for any two-dimensional spread processes with multiple start points and multiple boundaries.

SEI or T_R can spatio-temporally pinpoint the major sources of fire spread modelling errors. It is complimentary with D_R proposed by Fujioka (2002). The two reveal different aspects of modelling errors. For the conditions in the case study, forest fire spread modelling errors of the FGM in BFOLDS mainly happened in an angle range of 120° around the head fire direction, as indicated by T_R over the 15 hour burn time (Figure 6). The overall fire spread modelling errors, measured by SDI , decreased with burn time for first four hours, and then increased gradually over the 15-hour burn time modelling period. Modelling errors in the first four hours were mainly the result of fire size underestimation in some spread directions (measured by SDI_{under}), after which they declined from overestimation in some spread directions (measured by SDI_{over}).

The indices not only quantified forest fire spread modelling errors and but also identified some key areas for further algorithm improvement for some fire growth models. For example, based on the case study, it is evident that modelling errors are mostly come from fire front. Therefore it may be useful to bring special attention to simulating fire fronts during improvements to fire growth models. The indices also have potential for quantifying spread modelling errors in other two-dimensional ecological spread processes.

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