

## Inference on Environmental Population Using Horvitz-Thompson Estimator

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**ABSTRACT.** The purpose of this paper is to propose a method to estimate the total number of the environmental population in a region. Here, we assume that the occurrence of the population in the sample site follows a distributional count pattern. We use the Horvitz-Thompson estimator for the formation of the estimation process. We provide simulated results and an example using tiger census data to illustrate the proposed estimation procedure, and compare it with estimates obtained from a commonly used method known as the simple random sampling (SRS). The results show that the mean squared error (MSE) of the proposed method is lower than that of the SRS and the statistical test also shows a good result in favour of the proposed method. Our method of estimation also shows very good outcome for smaller sample sizes compare to the SRS. As well, this method is flexible in application and also very inexpensive in both time and cost.

*Keywords:* detection probability, Horvitz-thompson, poisson distribution, environmental population, tiger census

### 1. Introduction

Detectability, or detection probability, is a widely used procedure in environmental sampling. Detectability is defined as the probability that an object in a given unit, plot, or site is observed, seen, heard, caught, or detected by some other means. For example, in a survey of environmental populations, the observer is typically unable to detect every individual of a species in the vicinity of a selected sampling site. A number of special techniques (e.g. Thompson, 1992, 2002; Cochran, 1977) have been developed for estimating population when detectability problems are generally high.

This paper emphasizes estimation of the total population size by observing the occurrence pattern of the objects in the plot or site. We use both detectability and Horvitz-Thompson estimator to derive our method. To illustrate the proposed method, we provide both simulated results and a real-life environmental example using male tiger data from the 2004 Sundarban tiger census (Tiger Census, 2004).

Detectability is crucial to the estimation of the actual number of a particular species in an environment. Some approaches use basic methods of detectability to estimate the total environmental population (Cormack, 1979; Seber, 1973, 1986; Gates, 1979; DeVries, 1979; Sirken, 1970; Thompson and Seber, 1996). A number of methods have been developed to address this problem with occurrence estimation based on the observed presen-

ces and absences of a species in replicated samples (MacKenzie et al., 2002; Royle and Nichols, 2003).

A more advanced approach to estimating environmental populations is based on zero-inflated distributions. This approach uses models to describe the spatial distribution of a rare species (Welsh et al., 1996; Ridout et al., 1998; Cunningham and Lindenmayer, 2005; Martin et al., 2005). The double-observer (Nichols et al., 2000), double-sampling (Bart and Earnst, 2002), distance-sampling (Buckland et al., 2001), and removal (Farnsworth et al., 2002) models have been proposed to estimate detection probabilities. Alldredge et al. (2006, 2007) provided an independent-observer point-count method, which is based on closed-population capture-recapture methods. This approach can incorporate detection distance to account for individual differences in detection probabilities associated with measurable sources of variation.

Some recent studies have also examined the inclusion probabilities of the Horvitz-Thompson (HT) estimator, in which modification of the HT estimator was proposed for complex sampling designs. The inclusion probabilities were then estimated by means of independent replications of the sampling scheme (Fattorini, 2006). Pollock et al. (2006) used the generalized HT estimator based on the overall detectability for individual dugongs to generate population estimates. They also developed a new simulation-based method for estimating standard errors and confidence intervals. Sasso et al. (2007) estimated summer abundance of juvenile loggerhead turtles with a HT-type estimator that used count data and recapture probability to estimate population.

Some of these methods have drawbacks in their estimation procedures, such as bias and problems with shape parameters,

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and some of them are expensive and impractical for large-scale applications. Moreover, wildlife biologists need to choose criteria to select among these methods. Recent publications on detectability (e.g. Nichols et al., 2000; Buckland et al., 2001; Bart and Earnst, 2002; Rosenstock et al., 2002; Farnsworth et al., 2002; MacKenzie et al., 2006; Fieberg and Giudice, 2008; Meter et al., 2008; Hotaling et al., 2009; Pagano and Arnold, 2009) suggest that changes in sampling techniques and estimation procedures may be necessary.

In this paper, we use a distributional concept to estimate detectability and use the HT estimator to estimate the population total. Normally, the occurrence of objects in any site follows a Poisson process (Ross, 2008), so in this paper, we estimate the detectability of the object using the Poisson distribution. We use a random sampling approach to select the sites. Most environmental populations are clustered around a particular area (Thompson and Seber, 1996); we consider the areas where the object of interest is normally available.

In Section 2, we discuss the procedure for estimating the detectability of an environmental object in a particular site. In Section 3, we elaborate on our proposed method of estimation of population total, as well as its variance. In Section 4, we give simulated results of the proposed method and compare them with the simple random sampling (SRS) estimation procedure. Then, in Section 5, we estimate the population total of the male tiger using male tiger census data for 2004 and discuss the results. Finally, in Section 6, we draw conclusions and give some ending remarks.

## 2. Estimating Detectability

In this section, we define the detectability of an object that follows a distributional pattern of occurrence in the sample site. If there are  $N$  sites in an area  $A$ , we sample  $v$  sites from that area and observe the environmental population. Again, we assume that the occurrence of the objects in the site follows the Poisson process (Ross, 2008), i.e.,  $M_i \sim Pois(\lambda_i)$ , where  $M_i = 1, 2, \dots, \infty$ , and  $i = 1, 2, \dots, v$ . According to Farnsworth et al. (2002), detectability ( $\delta_i$ ) is potentially a function of multiple factors. It is therefore useful to subdivide  $\delta_i$  into two main components:  $\hat{p}_{i1}$  is the probability of occurrence of objects in the site  $i$ , and  $\hat{p}_{i2}$  is the probability of selection of an object in that site. We consider  $m_1, m_2, \dots, m_v$  objects observed in  $v$  sites.

The probability of the occurrence of objects can be defined as the probability of occurrence of a maximum of  $m_i$  individuals in the  $i$ th site:

$$\hat{p}_{i1} = \sum_{r=0}^{m_i} \frac{\lambda_i^r \exp(-\lambda_i)}{r!}, \quad i = 1, 2, \dots, v \quad (1)$$

This probability is also known as ‘‘availability’’ (Farnsworth et al., 2002).  $\hat{p}_{i2}$  can be defined as the probability that at least one object is observed in the site  $i$ :

$$\hat{p}_{i2} = \Pr(m_i \geq 1) = 1 - \Pr(m_i < 1) = 1 - \Pr(m_i = 0) = 1 - \exp(-\lambda_i)$$

$$i = 1, 2, \dots, v \quad (2)$$

Finally, the overall detectability for  $i$ th site can be obtained by multiplying the availability by the selection probability of the objects:

$$\hat{\delta}_i = \hat{p}_{i1} \times \hat{p}_{i2} = \exp(-\lambda_i) \{1 - \exp(-\lambda_i)\} \sum_{r=0}^{m_i} \frac{\lambda_i^r}{r!} \quad (3)$$

The values  $m_1, m_2, \dots, m_v$  are random, and in this paper, we assume a homogeneous Poisson process to mitigate computational complexity. For the homogeneous process, the parameter  $\lambda_{is}$  ( $i = 1, 2, \dots, v$ ) is constant throughout the area  $A$ . To estimate the detectability from equation (3), we need to know the value of  $\lambda_i$ . Given a homogeneous Poisson process, an estimated value of  $\lambda_i$  can be obtained from the observed  $m_i$  of  $v$  sites.

## 3. Estimation Procedure of the Proposed Method

Let us consider, with any design ‘with’ or ‘without’ replacement, that  $\pi_i$  is the probability that the site  $i$  is included in the sample for  $i = 1, 2, \dots, N$ . An unbiased estimator of the population total  $\tau$  can be obtained using the Horvitz–Thompson (1952, 1992) estimator:

$$\hat{\tau} = \sum_{i=1}^v \frac{y_i}{\pi_i} \quad (4)$$

in which  $v$  is the effective sample size, and  $y_i$  is the variable of interest in site  $i$ . In wildlife inference, Steinhorn and Samuel (1989) have addressed a generalization of this situation in which sites are selected by any sampling design with known inclusion probability  $\pi_i$ , and the detectability  $\delta_i$  may differ for different objects. Here, the variable of interest  $y_{ij}$  of the  $j$ th object in the  $i$ th site may be any type of variable, e.g., continuous, discrete, or indicator. We take  $M_i$  as the number of objects in the  $i$ th site and  $m_i$  as the number of objects observed. Hence, the population total in site  $i$  can be written as  $\tau_i = \sum_{j=1}^{M_i} y_{ij}$ , yielding the population total in the area:

$$\tau = \sum_{i=1}^N \sum_{j=1}^{M_i} y_{ij} = \sum_{i=1}^N \tau_i \quad (5)$$

We assume an indicator function for the variable  $y_{ij}$  because of the count pattern of objects. Therefore,  $y_{ij} = I_{ij} = 1$ , if any  $j$ th object is observed in the  $i$ th site, and  $y_{ij} = I_{ij} = 0$ , otherwise. If all sites have observations, then using the indicator variable, we can rewrite the population total in the  $i$ th site as  $\tau_i = \sum_{j=1}^{M_i} I_{ij} = M_i$ . Thus, the population total in the area can be written as:

$$\tau = \sum_{i=1}^N \sum_{j=1}^{M_i} I_{ij} = \sum_{i=1}^N M_i \quad (6)$$

Because our objective is to estimate the total population,

an unbiased estimator of  $\tau$  based on the Horvitz–Thompson method (Steinhorst and Samuel, 1989) can be defined as:

$$\hat{\tau} = \sum_{i=1}^v \frac{1}{\pi_i} \sum_{j=1}^{m_i} \frac{I_{ij}}{\delta_{ij}} \quad (7)$$

in which  $\delta_{ij}$  is the probability of detection of the  $j$ th object in the  $i$ th site. Now, the estimated total in site  $i$  can be obtained as:

$$\hat{\tau}_i = \sum_{j=1}^{m_i} \frac{I_{ij}}{\delta_{ij}} \quad (8)$$

Given the assumption that the detectability  $\delta_{ij}$  is the same within the site. So the estimated total in site  $i$  is:

$$\hat{\tau}_i = \sum_{j=1}^{m_i} \frac{I_{ij}}{\delta_{ij}} = \frac{m_i}{\delta_i} \quad (9)$$

and the estimated population total of an area is:

$$\hat{\tau} = \sum_{i=1}^v \frac{1}{\pi_i} \frac{m_i}{\delta_i} = \sum_{i=1}^v \frac{\hat{\tau}_i}{\pi_i} \quad (10)$$

Normally, since the value of  $\delta_i$  is not previously known, we replace it with  $\hat{\delta}_i$ , its estimate. We also replace the known inclusion probability  $\pi_i$  with  $\pi_i = v/N$  (Thompson, 1992). Finally, we have our estimator:

$$\hat{\tau}_{est} = \sum_{i=1}^v \frac{1}{\pi_i} \frac{m_i}{\hat{\delta}_i} = \sum_{i=1}^v \frac{N}{v} \frac{m_i \exp(\lambda_i)}{\{1 - \exp(-\lambda_i)\} \sum_{r=0}^{m_i} \frac{\lambda_i^r}{r!}} \quad (11)$$

However, the estimator is biased because of the replacement of the detectability  $\delta_i$  with its estimate  $\hat{\delta}_i$  (Thompson, 1996).

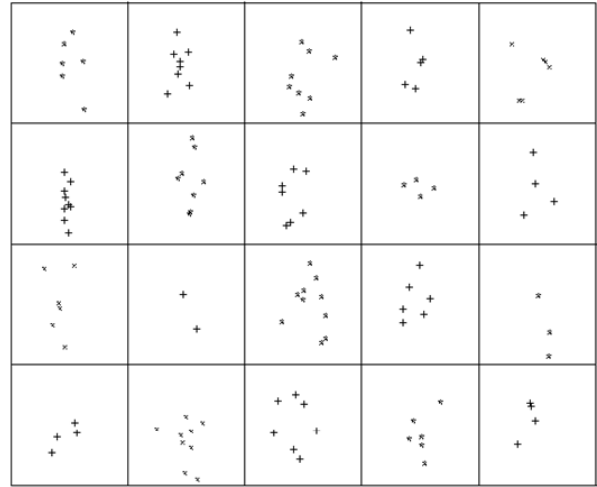
### 3.1 Estimation Procedure for the Variance of $\hat{\tau}_{est}$

We use the variance formula of the HT method and obtain the estimated variance  $\hat{v}(\hat{\tau}_{est})$  of our proposed estimator  $\hat{\tau}_{est}$ :

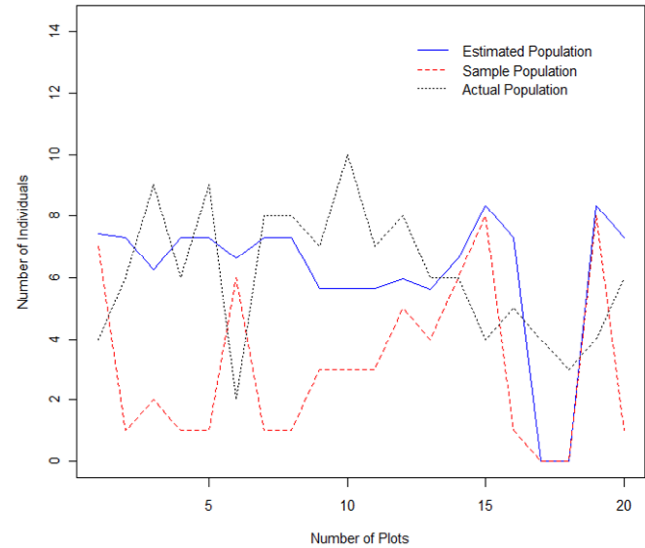
$$\hat{v}(\hat{\tau}_{est}) = \sum (1 - \delta_i \pi_i) \frac{\hat{\tau}_i^2}{\pi_i^2} + \sum_{i=1}^v \sum_{i \neq j} \left( \frac{\pi_{ik} - \pi_i \pi_k}{\pi_{ik} \pi_i \pi_k} \right) \hat{\tau}_i \hat{\tau}_k, (i \neq j) \quad (12)$$

in which  $\pi_{ik}$  is the probability that sites  $i$  and  $k$  are both included in the sample. However, without a replacement sampling scheme, it is complicated to derive and sometimes gives negative results (Thompson, 1992). Instead, a simple approximation of the estimated variance can be used (Brewer and Hanif, 1983):

$$\hat{v}(\hat{\tau}_{est}) = \left( \frac{N-v}{N} \right) \frac{\sum_i^2}{v} + \sum_{i=1}^v \left( \frac{1 - \delta_i}{\pi_i} \right) \frac{\hat{\tau}_i^2}{\pi_i} \quad (13)$$



**Figure 1.** Randomly generated objects from Poisson distribution with parameter 5 for 20 plots in the area.



**Figure 2.** Actual, sampled, and estimated populations by plots.

where  $\sum_i^2 = 1 / (v-1) \sum_{i=1}^v (t_i - \hat{\tau}_{est})^2$ , and  $t_i = v \hat{\tau}_i / \pi_i$  for  $i = 1, 2, \dots, v$ .

Equation (13) has the advantages of reducing the computational burden and not yielding negative estimates. However, it is biased, as we replace the value of  $\delta_i$  with  $\hat{\delta}_i$ , and the result tends to be larger than the actual variance. Finally, we calculate the mean squared error (MSE) of our proposed estimator  $\hat{\tau}_{est}$  with:

$$MSE(\hat{\tau}_{est}) = \hat{v}(\hat{\tau}_{est}) + bias^2 \quad (14)$$

where  $bias^2 = (\hat{\tau}_{est} - \tau)^2$ . Previous studies (e.g., Cochran, 1977; Thompson, 1992) provide the SRS estimator:

$$\hat{\tau}_{SRS} = \sum_{i=1}^v \frac{y_i}{\pi_i} = \sum_{i=1}^v N \frac{y_i}{v} = N\bar{y} \tag{15}$$

In this paper we use simple random sampling (SRS) estimation procedure to compare our proposed method, because literally it does not have any difference with the sampling scheme of SRS. Whereas, other estimation procedures for environmental populations consider different sampling approaches (e.g. capture–recapture, line–transect, line–intercept, distance–sampling, etc.), which increases the expenses of sampling cost and time.

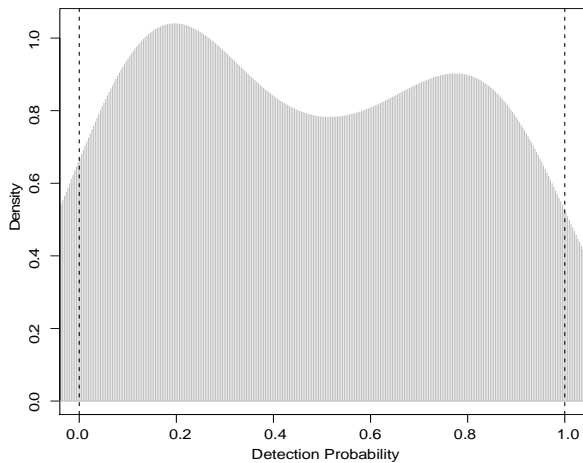


Figure 3. Density plot of the estimated detectability.

### 4. Simulated Results

In this section, we will provide simulated results obtained from both the proposed and the SRS method of estimation. Figure 1 shows a set of randomly generated observations  $M_i$  from Poisson distribution with parameter 5 (say) for 20 plots. We randomly sample  $m_i$  observations from  $M_i$  and estimate the total  $\hat{\tau}_i$  in each site  $i = 1, \dots, v$ . Figure 2 shows the total number of observations, sample observations, and estimated results for each site.

The population total is about 91 according to the randomly generated observations, and our method yields a very good approximation of the population total: 89.02. But the SRS method produces an estimate of 50, far below the population total. The standard deviation (SD) of the estimated population total is 6.25 for the proposed method, lower than the SD of SRS. Our method clearly produces a better result than the SRS estimation procedure. The density function of the detectability is plotted in Figure 3, which shows that it exhibits a bimodal distribution.

More results obtained from the proposed and SRS methods are given in Table 1 for 5,000 simulations. Columns 3 ~ 5 of Table 1 list the median of the simulated observations of the actual population total ( $\tau$ ), estimated population total for the proposed

method ( $\hat{\tau}_{est}$ ), and estimated population total for the SRS method ( $\hat{\tau}_{SRS}$ ). To reduce the calculation burden, we use 20 sites ( $N$ ). We also apply the likelihood ratio (LR) test to determine whether any significant relationship exists between the estimated and actual population total for both the proposed and SRS methods. For both methods, percentage of rejection of null hypothesis ( $H_0$ : Actual and estimated totals are same) at the 5% level of significance are given in columns 6 and 7 of Table 1.

Table 1. Results (5000 simulations) of the Proposed and SRS Methods for  $N = 20$

$\lambda$	$v$	Median			Reject $H_0$	
		$\tau$	(%) $\hat{\tau}_{est}$	(%) $\hat{\tau}_{SRS}$	Proposed	SRS
5	20	100	90.08	50	33.8	100
	15	100	89.98	49	36.1	99.7
	10	100	90.76	50	37.2	96.2
4	20	80	70.31	40	31.8	99.9
	15	80	69.61	40	32.4	99.5
	10	80	69.59	40	32.2	94.1
3	20	60	51.70	30	25.92	99.9
	15	60	51.99	29	25.42	97.9
	10	60	52.61	30	24.22	88.8
2	20	40	37.86	20	15.4	98.7
	15	40	36.90	20	16.2	91.5
	10	40	35.51	20	19.4	77.6
1	20	20	18.58	10	9.4	80.5
	15	20	18.03	10	10.9	67.4
	10	20	18.35	9	13.3	52.1

From Table 1 we can observe that the estimates obtained from our proposed method ( $\hat{\tau}_{est}$ ) is close to the actual values ( $\tau$ ) compare to the estimates obtained from SRS. The percentage rejection is also higher for the proposed method compare to the SRS method of estimation for different sample sizes. Moreover, from Figure 4 we can also observe that for small sample sizes, our method gives better result for estimating the population total compare to SRS, because for different values of  $\lambda$  the percentage rejection of  $H_0$  is lower for our method.

Tables 2 and 3 provide the simulated results of absolute percentage relative bias (APBR) and mean squared error (MSE) for the proposed and SRS methods for different values of  $\lambda$ . Columns 3 and 4 in Tables 2 and 3 show the percentage of the relative bias. Column 5 gives the median of the MSE ratio (i.e., median of the simulated observations of the term  $MSE(\hat{\tau}_{SRS}) / MSE(\hat{\tau}_{est})$ ), and the last column in both tables represents the percentage of the MSE ratio greater than one. We choose different values of  $v$  and  $N$  to gauge their effects on the simulated result.

From both Tables 2 and 3 we can observe small absolute percentage relative bias for the proposed method. The ratio of MSE is also greater than one, which explains the MSE of the proposed method is lower than the MSE of the SRS method of estimation. For small number of sample sizes the APBR increase for the proposed method, however the amount of bias is very low compare to the SRS method.

**Table 2.** Results (5000 simulations) of the Proposed and SRS Methods for  $\lambda = 5$

N	v	Median APRB (%)		MSE Ratio (SRS/Proposed)	
		Proposed	SRS	Average	Ratio > 1 (%)
10	N = 10	18.19	50.00	3.49	93.58
	N/2 = 5	27.31	51.02	1.49	84.60
	N/3 = 3	35.54	51.39	1.07	84.18
	N/4 = 2	42.21	52.83	1.01	84.08
20	N = 20	13.99	50.00	6.20	98.86
	N/2 = 10	20.58	50.40	3.00	91.34
	N/3 = 6	27.77	50.79	1.87	89.44
	N/4 = 5	28.29	50.82	1.47	83.82
30	N = 30	12.13	50.00	8.21	99.64
	N/2 = 15	16.97	50.30	4.51	95.70
	N/3 = 10	20.21	50.68	3.04	91.10
	N/4 = 7	26.35	50.77	2.09	89.62
40	N = 40	10.89	50.00	10.23	99.92
	N/2 = 20	15.09	50.00	5.66	96.98
	N/3 = 13	19.05	50.02	3.76	94.58
	N/4 = 10	20.98	50.54	2.92	90.70

**Table 3.** Results (5000 simulations) of the Proposed and SRS Methods for  $\lambda = 3$

N	v	Median APRB (%)		MSE Ratio (SRS/Proposed)	
		Proposed	SRS	Average	Ratio > 1 (%)
10	N = 10	17.79	50.00	3.47	95.20
	N/2 = 5	26.42	51.35	1.92	84.92
	N/3 = 3	34.11	52.38	1.42	82.40
	N/4 = 2	38.56	54.54	1.41	81.45
20	N = 20	14.58	50.00	5.62	99.38
	N/2 = 10	20.04	50.00	3.24	92.82
	N/3 = 6	27.72	51.51	2.06	91.22
	N/4 = 5	27.11	50.87	1.86	84.74
30	N = 30	13.76	50.00	6.88	99.84
	N/2 = 15	17.55	50.62	4.31	96.72
	N/3 = 10	20.62	50.63	3.15	92.22
	N/4 = 7	26.46	51.29	2.27	90.88
40	N = 40	13.29	50.00	7.87	99.96
	N/2 = 20	15.91	50.00	5.28	98.36
	N/3 = 13	20.15	50.76	3.62	95.48
	N/4 = 10	21.57	50.82	3.10	91.88

### 5. Application to Tiger Census Data

The Bangladesh–India Joint Tiger Census-2004 was carried out from February to March in 2004. It took 9 hours/day for 7 days to cover the whole reserve forest of the Sundarbans. Thirty-two groups were formed to collect data from the 55 compartments. Each group had 10 members: one group leader Deputy Ranger/Forester, three Forest Guards/Boatmen, one Bangladesh National Cadet Core (BNCC) Cadet/Student, one Non-Government Organization representative, two labourers and two accompanying trawlers/country boat crews (see details

in Tiger Census, 2004). Figure 5 shows the map of the Sundarbans river forest with the cluster and sub-cluster boundaries of Tiger Census-2004.

The proposed estimation method is applied to estimate the population total of male tigers in the Sundarban Reserved Forest (SRF) to see whether it provides the anticipated result. The total area of the Sundarbans (6,017 km<sup>2</sup> including land and water bodies) is divided into 19 sub-clusters (N), and 121 male tigers are observed in that area. We randomly sample sub-clusters four times with different number of sizes. We take the first sample size as  $v = N$ , the second as  $v = N/2$ , the third as  $v = N/3$ , and the fourth as  $v = N/4$ .

**Table 4.** Results (5000 simulations) of the Proposed and SRS Methods for Tiger Census Data for Different Sample Sizes

Median	Sample Sizes ( v )			
	N = 19	N/2 = 9	N/3 = 6	N/4 = 4
$\hat{\tau}_{est}$	137.50	117.7	104.4	81.02
$\hat{\tau}_{SRS}$	60.00	59.11	57.00	52.25
$\frac{MSE(\hat{\tau}_{SRS})}{MSE(\hat{\tau}_{est})}$	3.86	2.12	1.86	1.49
APRB( $\hat{\tau}_{est}$ )	13.63	2.72	13.72	33.86
APRB( $\hat{\tau}_{SRS}$ )	50.41	51.15	52.89	56.82
Percentage (%) of $\frac{MSE(\hat{\tau}_{SRS})}{MSE(\hat{\tau}_{est})} > 1$	73.08	69.48	68.98	66.68

Table 4 shows the results obtained from the male tiger census data for both our proposed and the SRS method of estimation. We simulate the results 5,000 times to validate our results and provide absolute percentage relative bias (APRB) and the percentage of mean squared error (MSE) ratio greater than one for both methods.

We can observe the estimated population total produced by the proposed method yields a better result than the SRS method for the male tiger census data. The median of the MSE ratio (i.e.,  $MSE(\hat{\tau}_{SRS}) / MSE(\hat{\tau}_{est})$ ) is greater than one for all samples. Even with the sample size  $v = 4$  (i.e.,  $19/4 \approx 4$ ), the MSE ratio is greater than 1, which demonstrates that the MSE of the proposed method is smaller than that of the SRS method.

The APRB for the proposed estimator is also smaller than the SRS estimator for all sample sizes. We can observe our method shows best result for sample size half of the total number of clusters. The APRB increase when the sample size decreases, however the value is always smaller than the bias obtained from SRS estimates. Thus our proposed method of estimation gives better result and hence is very efficient to estimate the male tiger population.

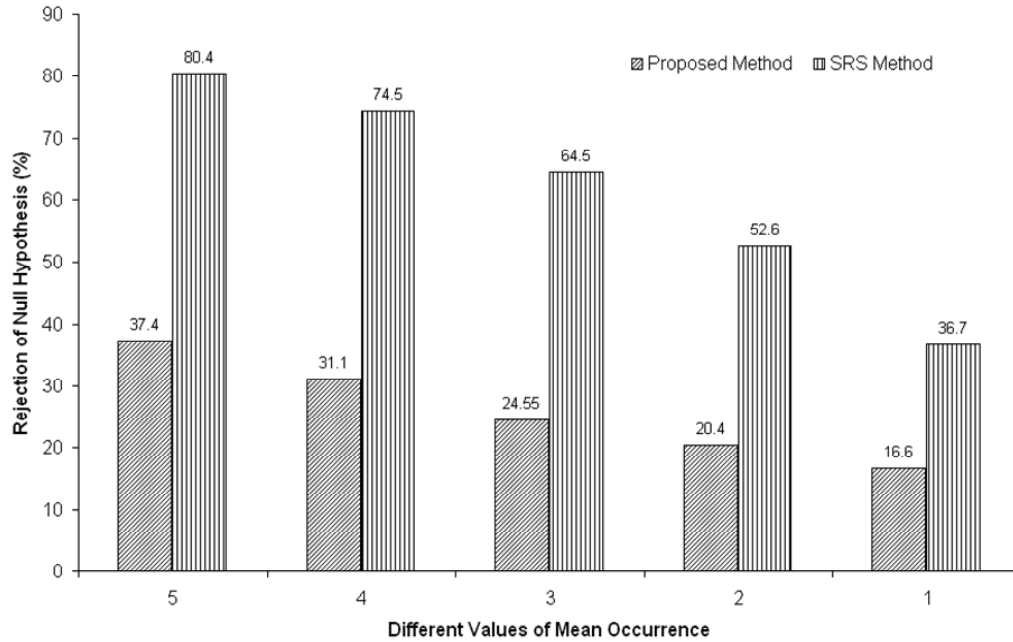


Figure 4. Percentage rejection of null hypothesis for proposed and SRS method for small sample sizes.

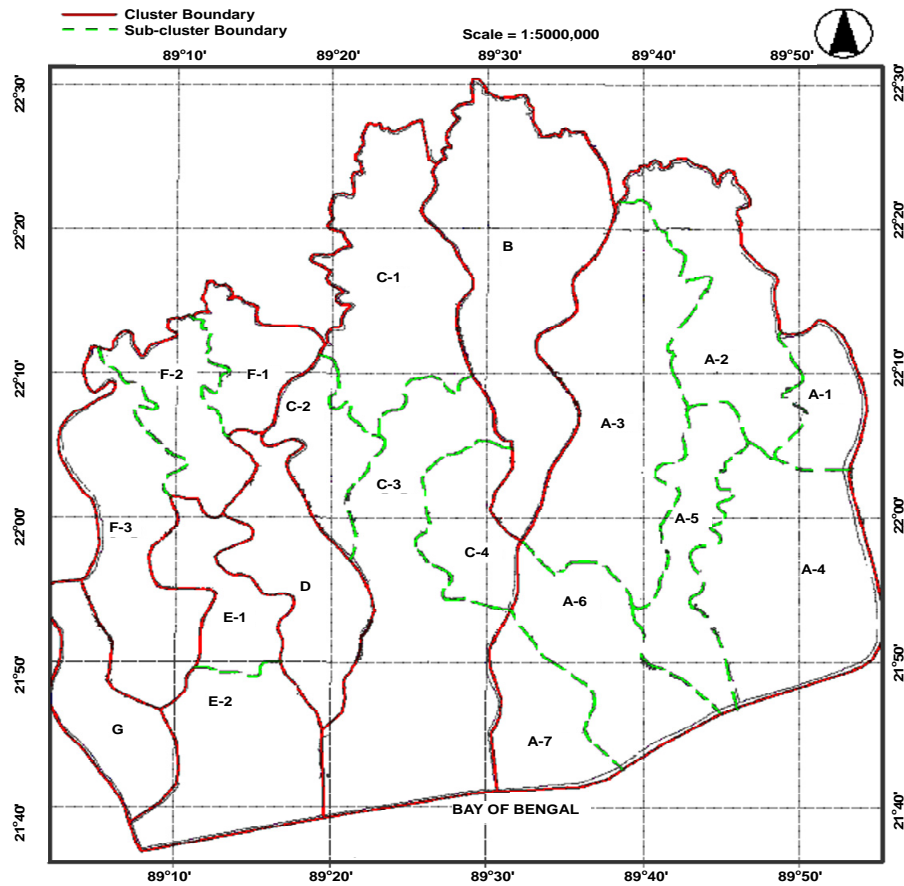


Figure 5. Division of Sundarban river forest into clusters and sub-clusters for tiger census – 2004 (source: tiger census report – 2004).

## 6. Conclusions

In this paper we consider the occurrence pattern of the environmental population and propose a new procedure to estimate the population total. We also compare the results obtained from our method with results of the simple random sampling (SRS) method, the mostly used estimation procedures for population total. In this paper we use the SRS estimates to compare our method because our sampling scheme is related to the simple random sampling approach. This simple principle benefits our proposed method with less expense in both time and cost compare to all other existing methods of estimation of environmental populations.

From the results we can observe the percentage rejection of the null hypothesis is lower for our proposed method compared to the SRS method for different values of mean occurrence and sample sizes. We can also observe the median absolute percentage relative bias (APRB) for the proposed method shows a better result. Tables 2 and 3 also show the ratio of mean squared error (MSE) is greater than one for all cases, demonstrating that the MSE of the proposed method is lower than the MSE of the SRS method. In our analysis, the statistical location parameter median is used as a robust indicator to eliminate the effect of outliers generated from the samples.

Both the simulation and tiger census example reveal that our proposed method for estimating the total population is superior to the SRS method. Even when the sample size is small our method gives better result for estimating the population total. However, the proposed method has its bias, and for that reason, we use a robust statistic to lower the bias. In this paper, we assume a homogeneous Poisson process, which implies the assumption that the mean occurrence is constant throughout the area. Further study with a non-homogeneous Poisson process could be conduct with  $\lambda$  values different for each site. Moreover, the Bayesian approach can be applied if prior information is available.

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