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A Scale-Space Information Flux Approach to Natural Irregular Patterns: Methods and Applications

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ABSTRACT. The paper addresses problems related to information management when a multiscale approach is applied to environmental patterns, whether in space or in time. To support the decision-making process concerning the information to be handled on each scale, it introduces the concepts of spatial and temporal informational backbone, and defines the scale space information flux as a quantity that reflects the resolution dependence of the size of the informational backbone. Establishing the scale space information flux helps in the identification of self-affinity properties and the quantitative, scale-range-sensitive pattern evaluation: the information flux is constant on the scale ranges of self-affinity. Pattern change can thereby reliably be recognized and characterized. The introduced procedures support storage-saving data selection and provide a measure of data storage requirements as a function of scale, flexibly adapting the calculation algorithm to the scale ranges of interest to the user. The paper also specifies rules that can be used by geographic information management software for a fast assessment of changes in map connectivity as a function of changes in scale. Practical application examples include synthetic data (Levy flight, Brownian walk, cellular automata output) and real-world patterns (desiccation fracture sets, atmospheric temperature time series). The presented concepts and methods can be applied in environmental science, e.g. to spatial and temporal patterns of pollution, natural resources (spatial distribution of minerals, drainage basins, forests), natural hazards (airborne gamma ray spectrometry maps, earthquake patterns), etc., as well as in other disciplines, such as material science and medical imaging.

Keywords: environmental patterns, geographic information, information flux, informational backbone, information management, multiscale approach, self-affinity

1. Introduction

Information management may be considered among the fastest evolving human activities. The task of handling information is particularly difficult when the objects of study are complex, irregular, and subject to change, which is often the case in environmental science. Methodological progress and technological advances have led to an accelerated information collection rhythm such as has never been experienced before. Challenges related to environmental information management have been shifting from the formerly dominating problems of insufficient data to the need for increasingly effective procedures for the acquisition, transfer, storage and processing techniques able to address large amounts of information at high speeds and to support the understanding of the studied processes.

In this context, scale has changed from what was considered to be a simple, obvious component of scientific data acquisition and processing, to an increasingly important – often subtle – concept in environmental science. It has become obvious that changes in scale may lead to significant, nonlinear effects regarding our understanding of environmental patterns (Turner et al., 1989; Cain et al., 1997; Wilbanks, 2003). In fact, different categories of variability may be expressed on different ranges of scale (Wu et al., 2000; Oldeland et al., 2010), which is especially important for studies concerning pattern change (Gustafson, 2002). Understanding the relation between interrelated processes and scale is recognized as critical (Perveen and James, 2010). Such insights regarding the importance of scale are also reflected in recent changes in environmental research and education (NSF-AC-ERE, 2009). Simply switching to a multiscale approach is not enough either: one must study the implications of the available or selected scale ranges concerning the results and the implied uncertainties (Perveen and James, 2010).

This paper refers to spatial and temporal aspects of environmental systems. It introduces concepts and techniques for the characterization of scale-related aspects of environmental patterns, and highlights implications concerning information management. Section 2 formulates the context and discusses definitions of notions used in this work, section 3 explores informational aspects of spatial and temporal analysis, and the last section is dedicated to conclusions.

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2. Premises

The aim of this section is to provide a brief overview of some concepts and their applications in the study of environmental systems, from the perspective applied in the paper. The selected concepts concern information, scale, affine transformations, and fractals, including self-affinity and self-similarity.

Given the topic of this paper, the first concept requiring definition is "information". Despite the prominent role played by information in many areas of human endeavour, no consensus has been reached regarding such a definition (von Baeyer, 2004). The situation regarding the notion of information is reminiscent of the one pointed out by Augustine in Confessions (Book Eleven, Chapter XIV, Section 17), when he refers to time: "surely we understand it when we speak of it; we understand it also when we hear another speak of it;" "if no one asks me, I know what it is. If I wish to explain it to him who asks me, I do not know" (Augustine, 2002). While undoubtedly important, a discussion on the nature of information is beyond the scope of this paper: for the purpose of the present study we have chosen to refer to information in a concrete way, adopting Shannon's (1948) technology-oriented approach, and addressing quantitative aspects of data handling rather than difficult to quantify and more elusive issues of significations carried by messages.

An essential concept in the framework of this exploration is scale. Although less vague than information, this concept may involve ambiguities that should be addressed from the beginning. Scientists from different fields have distinct meanings for scale; even in the geosciences several different meanings are common (Bian, 1997). Perhaps the most important distinction to be made here is the one that refers to the signification of "small" and "large" scales. In a cartographic sense, scale is defined as the ratio of the distance between two points on the map and the corresponding distance on the ground. Therefore, the scale – specified by a number, the "representative fraction" – is larger for maps that cover smaller areas in more detail. This stands in contrast to the way scale is used in physics, for instance, where by "large scale" and "small scale" processes one refers to large, and small areas, respectively.

In this paper the cartographic sense of scale will be adopted, mainly due to its rigorous definition. While the cartographic sense of the scale has been precise and straightforward when referring to paper maps, the situation has changed with the advent of maps in digital form. In the case of a map that can be displayed on a computer screen, a scale indication based on "map distance" has become less relevant than the resolution that characterizes the digital map. Spatial resolution corresponds to the size of the spatial unit on the ground represented by one pixel (Haining, 2007). Since the significance of scale applied here is consistent with the concept of resolution (e.g. larger resolution and larger scale both mean a more detailed view of a smaller area), both concepts will be applied in this article; their use will depend on the role they play in each situation. While "resolution" may be technically more precise in the digital realm, "scale" is more suggestive, especially in the framework of scientific reasoning about environmental structures and processes.

The entities to be discussed here are parts of the natural environment – possibly affected by anthropic activities. Many patterns of natural environmental systems and of their space-time behaviour exhibit properties that are relevant to studies concerning informational processes. In view of this exploration, several other concepts will be briefly specified.

An affine transformation from $\Re^n$ to $\Re^n$ is based on a linear combination of rotation, dilation, translation and reflection, and has the general form: $S(\mathbf{x}) = T(\mathbf{x}) + \boldsymbol{b}$, where T is a linear transformation and $\boldsymbol{b}$ is a vector in $\Re^n$ (Falconer, 2003). For example, a point in a space of dimension n: $\mathbf{x}(x_1, x_2, ..., x_n)$ can be mapped to the point $\mathbf{x}'$ $(r_1x_1, r_2x_2, \ldots, r_nx_n)$: different magnification ratios r_i are used along the different axes. A feature is called self-affine if its parts and the results of their affine transformations are similar to each other. Beyond the special mathematical appeal of self-affinity (Falconer, 2003), the concept – and the broad field of fractal theory, in which it plays a significant part – enjoy a particular interest in physics, the geosciences, the environmental sciences, etc. In fact, mathematically rigorously self-affine sets do not exist outside the realm of mathematics, but there are numerous real features which emphasize self-affinity properties on a limited range of scales. Typical examples include topographic profiles, fault patterns, streams, etc.: similar shapes are repeatedly found after dilating the images in two directions, x and y, by factors r_x and r_y, respectively. A particular case of self-affinity is self-similarity (Falconer, 2003). In the latter case, parts look similar when they are subject to a scaling operation based on the same magnification for all axes, i.e. $r_1 = r_2 = \ldots = r_n$; also in this case, real features exist for which self-similarity can be identified for limited scale ranges; spatial distributions of minerals, earthquake epicentres, stream patterns of drainage basins, etc. correspond to this latter category (Korvin, 1992; Turcotte, 1997).

Self-affinity and its special case, self-similarity, represent forms of symmetry that are relevant to this investigation on environmental systems. Such properties are ubiquitous in nature: they characterize both the structural aspects of natural systems (shapes, sizes, relative positions of various features), and their temporal behaviour (Korvin, 1992; Takayasu, 1991; Turcotte, 1997). They have direct implications for informational aspects of features in the broad category of "fractals", the objects or patterns characterized by self-affinity. Here again a clear, universally applicable definition is missing. Benoit Mandelbrot (1975), who initiated and developed the growing domain of fractal theory, pointed out the absence of a proper, complete characterization of fractals (Feder, 1988). Actually, in the natural sciences one is confronting features that are not even real fractals, but fractal-like objects or patterns enjoying scaling properties on certain scale intervals. It is no surprise that a mathematician like Mandelbrot makes a statement as general as "a fractal is a shape made of parts similar to the whole in some way" (cited in Feder, 1988). An essential attribute that self-affine features have in common is the lack of a characteristic length scale. The relationships between scale and various parameters evaluated for the self-affine features are typically governed by power laws. Elegant and rigorous expositions of mathematical aspects of fractals and the implied practical cha-

llenges can be found in books such as those of Mandelbrot (1982, 1999, 2002), Barnsley (1988), and Falconer (2003).

3. Informational Properties of Environmental Patterns

Spatial and temporal environmental patterns are approached in this section from a perspective based on the concepts discussed above. Quantities designed to characterize spatial and temporal aspects of patterns, and their relation to scale, will be defined.

Features in the natural environment are characterized by a large diversity of patterns and a remarkable variability in space and in time. Backed by fast evolving technological performance, our need for more information leads to information quantities that become increasingly difficult to manage. An effective selection of the appropriate aspects and levels of detail to be considered for every specific task and for every question is key to successful information management.

Given the variety of features with self-affine properties, it is difficult to develop a rigorous definition suiting all possible situations. Among the definitions referring to fractals, there is at least one that makes an explicit reference to information: "A fractal is a mathematical construct that enjoys information conservation properties with respect to scale change" (Suteanu, 2000). First, the definition makes it clear that real fractals are mathematical concepts; it is the responsibility of the researcher to identify aspects of real-world entities, which may correspond only to a limited extent to the mathematical properties of fractals. Second, although it does not specify the actual conservation properties, this definition may be useful as a starting point for the further exploration of the links between self-affinity properties and information. Such links will be studied here in relation to spatial and temporal environmental patterns. Quantities called the spatial and the temporal informational backbone, respectively, will be defined; based on the relation between scale and the size of the informational backbone, the scale space information flux will be established.

From the point of view of terminology, we should emphasize that the concepts of informational backbone for spatial and temporal patterns as well as scale space information flux are used in this paper as defined below. Even if they may have common elements with similar phrases used in other works, their meaning may be very different from those. Perhaps the most widespread use of "information flux" is the one found in information technology and referring to information transfer. However, many other distinct meanings can be found in other fields. While Di Franco et al. (2007) introduce "information flux" to characterize the influence exerted on an element by other elements in the context of interacting many-body systems, Cotsaftis (2009) applies it to information handling related to system control. The distinctive characteristic of the concept of information flux introduced here is the fact that it refers to the relation between the informational backbone and scale, which is the reason for its full name: scale space information flux.

3.1. Spatial Aspects: the Informational Backbone

Choices regarding spatial information representation in geographic information systems (GIS) are critical for the geographic analysis, because they determine what and how can be represented: the very definition of entities and specification of relationships depend on these choices (Haining, 2007). The two main GIS data models – raster and vector – are characterized by strengths and weaknesses that depend on the geographic reality to be modeled and the type of analysis to be applied. The present paper refers to raster data models, which are especially suitable to the study of natural patterns involving field-based features – either continuous (e.g. temperature) or discrete (e.g. soil types) – and geometric sets including combinations of irregular shapes, branching features, complex point patterns, etc. (Pecknold et al., 1997). Raster data models are particularly useful due to their high flexibility and effectiveness in modeling: they suit especially surface-dominated features and processes, such as diffusion or dispersal events, hydrological flow modeling, etc; the availability of raster datasets from airborne and space-borne sensors also encourages the use of this data model (DeMers, 2002). According to raster data models, the geographic reality is divided up in discrete cells based on a grid of regular shapes (squares being the most common solution), so that the represented property is characterized quantitatively for each cell.

Since the formulation of questions and the applicability of procedures designed for their approach are often scale related, it is desirable to make representations available at multiple scales. Relying on the same dataset, users may pursue different problems and follow distinct paths in their analysis; this requires information to be offered at different scales. The generation of data on many scales starting from the same initial high-resolution dataset using aggregation involves major challenges and sources of error (Bian, 1997; Haining, 2007; Pecknold et al., 1997). The availability of datasets produced directly on multiple scales is becoming a required feature of geographic information databases, and effective solutions such as the quadtree model have been designed for multiple scale information handling (DeMers, 2002).

When the studied features such as streams, pollution patterns, airborne gamma ray spectrometry maps in which the area of interest lies above a certain threshold, etc. are present only in certain areas of the map, it is not necessary to generate and store maps for the whole area on all considered scales. It is possible to decide for which individual area the scale increase would be useful. Such an example is shown in Figure 1. In this case, the point pattern corresponds to a Levy flight (Mandelbrot, 1975).

The approach to this pattern starts from an initial map or "cell", called the *root representation.* This cell is divided in other four equal cells; for simplicity, a square image is used here as a root representation, which does not affect the generality of the outcome. Some of the resulting cells may include a part of the studied feature, while others may not. The cells that include the feature and thus contain relevant information are retained for further zoom-in: for those cells the scale is increased

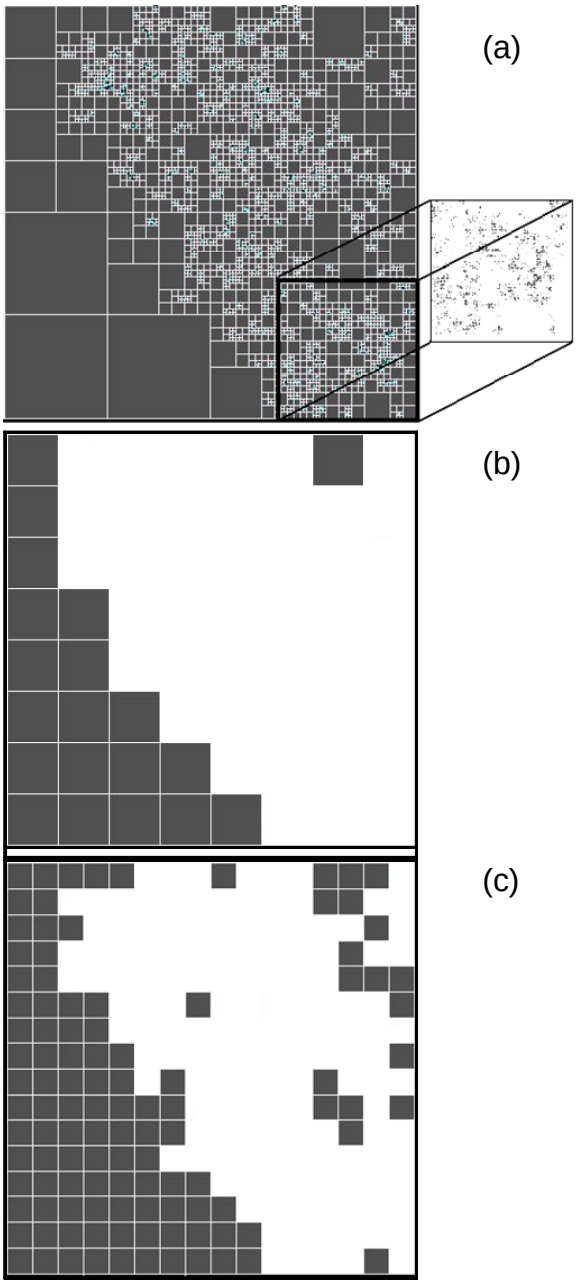

Figure 1. Iterative selection of the relevant cells for a point pattern: a) the resulting cells after six iterations; b) and c): the spatial informational backbone for two different scales.

until they become the size of the initial cell – the root representation. Only these retained cells are stored on this scale; they are then further divided in four cells each, and again the resulting cells containing the feature of interest are retained, the scale for these cells is increased, and the resulting cells are stored. The operation of recursive division continues until the desired maximum resolution is attained. One can recognize the principle of the quadtree model in this process. The approach presented here represents an information selection algorithm, which can be used as a starting point for data compression; in fact, various compression techniques are based on quadtree models (Zhang and Xi, 2007).

Figure 1a shows the resulting cells after six iterations. The square represented to the right of the main image shows a fragment of the initial point pattern.

On each scale the set of retained cells, which constitute the information-bearing parts regarding the studied feature, will be called the *spatial informational backbone* (SIB) of the feature. Figures 1b and 1c present the SIB for two distinct scales. The procedures applied above and their implications can be summarized as follows.

Generation. The spatial informational backbone (SIB) of a feature represented on a map is obtained by recursive application of the division of every area, starting with the initial map (the root representation), in a number of congruent sub-cells. On each scale the SIB consists of the set S of cells that contain the studied feature. Following each division stage, the resulting sub-cells that make up the set S are increased in scale, so that their area becomes equal to the area of the root representation.

Size. The size of the SIB, denoted by $|S|$, is equal to the number of cells in the set S and provides a measure of the amount of information to be stored for the studied feature on each scale. The amount of required storage space is a linear function of the size of the SIB; the exact function depends on the storage format and the information specified for each pixel.

Uniqueness. The SIB for a given feature represented on a map is uniquely defined, as long as the recursive division process is specified. The same feature represented on different maps may correspond to different informational backbones.

Connectivity. This property of the SIB is particularly important for certain spatial analysis operations: it is useful to know whether it is possible to cover the SIB by navigating directly from one cell to another, or if the backbone is composed of isolated groups of cells, since different analysis strategies may be applied in these two cases. For applications involving spatial pattern analysis it is often critical to take in consideration edge effects, which makes the connectivity property particularly relevant. Digital geometry offers a useful theory and tools for the further development of this direction of study; a thorough discussion on this subject is provided in the classic book by Rosenfeld, (1979); a more recent synthesis is presented by Klette and Rosenfeld (2004). According to the theory of digital geometry, considering a 4-adjacency neighbourhood – which consists of the vertical and horizontal neighbours of the cells – a set of cells S is *connected* if one can reach all the cells of S by a path involving only adjacent cells (Acharya and Ray, 2005); if S has no holes, it is considered *simply connected* (Klette and Rosenfeld, 2004). Applying these concepts to the spatial informational backbone, one can notice, for example, that the SIB in Figure 1b is not just connected, but also simply connected; in contrast, the SIB in Figure 1c is not connected. It is useful to note some connectivity properties of SIBs produced from the same root representation; such properties can be applied when designing software for the management of geographic information represented at different scales.

If the SIB is connected – or simply connected – on a certain scale, it is connected – or simply connected, respectively – on any smaller scale. Similarly, if the SIB is disconnected on a certain scale, it is disconnected on any larger scale. Indeed, the SIB on a scale $u' > u$ retains only a fraction of the SIB produced on the scale u, or it covers at most the same area as the latter. Therefore, removing areas from an already disconnected set will keep the set disconnected; on the contrary, integrating in larger cells those that make up a connected set preserves connectivity.

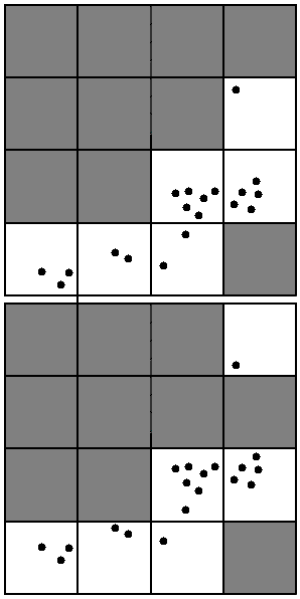

Figure 2. Depending on the actual map, the same pattern may lead to a connected or a disconnected spatial informational backbone: two examples referring to the same pattern.

If the SIB is connected on a scale u, it may be either connected or disconnected on a scale $u' > u$. a) Indeed, any selection of cells on a larger scale u', which corresponds to a reduced total selected area for the SIB compared to scale u, can lead to a disconnected SIB. b) To show that the SIB on a larger scale can still be connected, one can proceed as follows. It has been proven (Rosenfeld, 1979) that any connected set of cells S has at least a cell whose deletion does not disconnect S. One can start with the SIB on the scale u, represented by the set $S(u)$; producing a map on a larger scale $u' > u$ and selecting the SIB on that scale leads to the set $S(u')$. The set $S(u')$ either includes the whole area of $S(u)$, or it leaves one or more cells of the SIB on scale u' unselected. Since removing a whole cell of $S(u)$ may still leave S connected, so does the removal of a sub-cell of $S(u)$, which is a cell of $S(u')$. Similarly, if the SIB is disconnected on a scale u, it may be either connected or disconnected on a scale that is smaller than u.

The connectivity properties of the SIB on any scale produced from the same root representation are uniquely defined, as long as the iterative division algorithm is specified. However, different maps of the same feature may lead to distinct connectivity properties; an example is provided in Figure 2.

3.2. Spatial Aspects: the Scale Space Information Flux

The connectivity properties of the SIB discussed in section 3.1 support a fast identification of implications of scale change from the point of view of the applicability of spatial analysis methods. It would be useful to find a quantity that would not refer only to a certain map and a specific scale, but rather characterize the studied feature. Given the ubiquity of self-affinity properties of environmental systems (Feder, 1988; Korvin, 1992; Mandelbrot, 1975, 1982, 1999, 2002; Sornette, 2006; Takayasu, 1991; Turcotte, 1997), this section will refer to patterns characterized by such properties.

Let us consider a self-affine feature, such as the spatial distribution of rivers in a drainage basin, an environmental distribution pattern like a pollutant in soil, a gamma ray spectrometry map, etc. The scale of the root representation is specified by the representative fraction u_0. Let us suppose that, in order to have access to more details, we divide it – as shown above – in four equal-sized cells, and increase their area by changing the scale to make them equal to the root representation; these views correspond to a representative fraction $u_1 = 2u_0$. If we continue this process for each cell and go iteratively deeper to more and more details, after n iterations we obtain views at the representative fractions:

$$u_n = u_0 2^n \tag{1}$$

The total area A (to be recorded, transferred and stored) corresponding to these views increases according to:

$$A_n = A_0 4^n \tag{2}$$

where A_0 is the area of the root representation. The information to be stored for the whole area would thus increase fast with the iteration number. However, if one collects on a given scale only the SIB, and the feature is self-affine, one may get amounts to be stored that increase at a different rate. The specific rate depends on the self-affinity properties of the studied pattern. Allotting, for simplicity, a value of 1 to the representative fraction of the root representation, we can write the total number of cells N as a function of the scale u as:

$$N(u) = u^2 \tag{3}$$

whereas the number k of cells containing the self-affine feature (Falconer, 2003), and therefore the size of the SIB, written as:

$$k(u) = |S(u)| \tag{4}$$

corresponds to:

$$k(u) = \lfloor u^v \rfloor \tag{5}$$

where $\lfloor x \rfloor$ is the greatest integer less than or equal to x; the exponent v depends on the self-affine properties of the analyzed configuration.

By considering only the SIB of the feature emphasizing self-affinity, we handle a significantly lower amount of infor-

mation. Figure 3 shows an example involving an exponent v equal to 1.7, which is a value consistently found in the case of river networks (Clap and Oliveto, 1996).

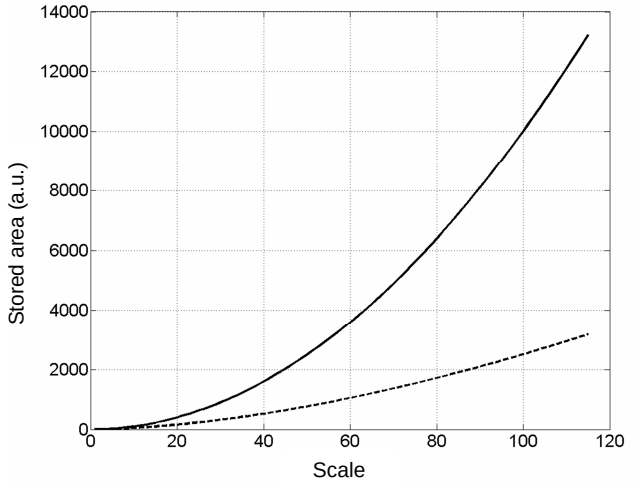

Figure 3. Stored area (in arbitrary units) vs. scale for the process of iterative scale increase: the full line shows the total area and the dotted line represents the area of the spatial informational backbone for a self-affine feature corresponding to an exponent v = 1.7.

We can determine the amount of uncertainty removed in each iteration stage based on a result of Shannon's theory of communication (Shannon, 1949). For example, for a self-affine feature such as the point distribution corresponding to a pollution pattern, we know how many cells will be included, statistically, in the SIB on each scale, but we do not know which ones. We will have *k(u)* cells eventually selected from the total of *N(u)* cells, which results in a number of possible selections given by the combinations:

$$T(u) = \binom{N(u)}{k(u)} \tag{6}$$

Here the number of cells *k(u)* corresponds to the integer given by Equation (5). Since for equal probabilities p_i the entropy corresponding to the removed uncertainty amounts to the maximum value:

$$H_{max}(u) = \log[T(u)] \tag{7}$$

the maximum entropy for the iteration regarding the scale u will be, in the general case:

$$H_{\max}(u) = \log\binom{N(u)}{k(u)} \tag{8}$$

For a self-affine feature, the maximum entropy on the scale u will be:

$$H_{\max}(u) = \log\binom{u^2}{\lfloor u^v \rfloor} \tag{9}$$

Equation (9) indicates the amount of uncertainty removed by the choice of the SIB on a scale u. The value of *H(u)* for any concrete case can be calculated using the Stirling approximation. If the different possible choices of the *k(u)* cells are not equally probable, the entropy *H(u)* has a lower value, which depends on the probabilities corresponding to each selection of the cells.

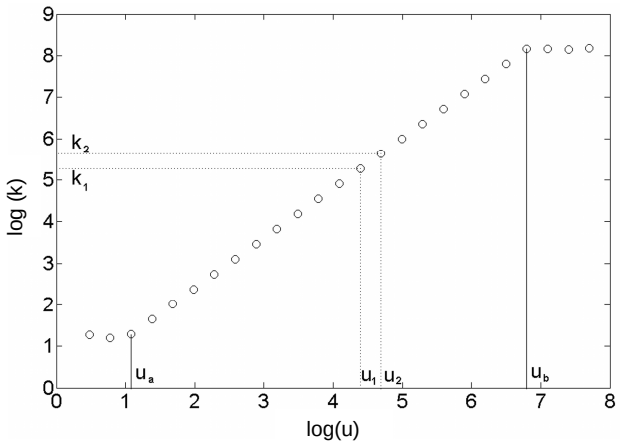

Figure 4. The relation between k (the size of the SIB), and u (the scale) for a self-affine feature identified between the scale limits ua and ub, in logarithmic coordinates.

Generalizing the procedure described above, we may use a succession of scale values u_i so that $u_{i+1}/u_i = m$, with $m > 1$, and follow the multiscale analysis algorithm discussed above. Again, we retain at each stage the k cells that include the SIB. Supposing that the self-affine character of the studied spatial distribution is valid over a scale range delimited by u_a and u_b, we obtain the power law dependence given by Equation (5) for this interval. If we represent log(k) vs. log(u), we obtain a straight line segment defined by: a) the slope v corresponding to the k vs. u relationship; b) the limits between which this relation is found, namely u_a and u_b. The slope v between two points 1 and 2 is:

$$v_{12} = \frac{\log k_2 - \log k_1}{\log u_2 - \log u_1} \tag{10}$$

and thus the dimensionless expression:

$$v_{12} = \frac{\log \frac{k_2}{k_1}}{\log \frac{u_2}{u_1}} \tag{11}$$

characterizes the interval u_1, u_2 (Figure 4).

Since according to its definition the spatial informational backbone represents the relevant information regarding the studied feature, and k is the size of the backbone, given by Equation (4), we will call v_{12} the *scale space information flux* corresponding to the studied feature for the interval u_1, u_2. In fact, the scale space information flux shows the change in the size of the informational backbone with scale. In other words, it reflects the resolution dependence of the amount of information required for the storage of a self-affine feature, on a given resolution interval. In this paper reference will be made to the information flux only in the sense specified above, even if we will avoid repeating the phrase "scale space" every time information flux is mentioned.

Equations (3) and (5) do not depend on a particular map. While the spatial informational backbone is map-specific, the information flux is statistically independent from the map, and characterizes the actual pattern. If scale values are chosen following a geometric progression with the ratio 2, base 2 logarithms can be used and therefore:

$$v_{12} = \log_2 \frac{k_2}{k_1} \tag{12}$$

In other words, the information flux provides the logarithm of the number of times the informational backbone increases when the scale changes by a factor of two. It should be noted that the value v_{12} would be valid for the scale interval u_1 to u_2.

In the case of self-affine patterns, the scale space information flux is constant on the interval of self-affinity. This corresponds to the fact that over this interval, the scale cannot be distinguished based on the represented features: although the details differ from one scale to another, one sees, on each scale, the same kind of pattern. Constant information flux can only be found over limited scale intervals. For example, different physical processes involved in the pattern formation may dominate different scale intervals. Departures from the scaling behaviour that occur on smaller scales can be the result of limitations regarding the access to a statistically relevant number of large features in the studied map, while deviations on larger scales can be a consequence of the fact that small features are not properly recognizable, which leads to biased information acquisition and incomplete data sets. Detailed analyzes regarding sampling effects and scaling regimes related to pattern generation processes have been completed in different domains (examples include Meakin, 1998; Needham et al., 1996; Wojtal, 1996; van der Zee and Urai, 2005).

In summary, the information flux for a self-affine pattern is constant as long as the informational process is not perturbed due to changes either in pattern-forming processes, or in aspects of information handling.

The scale space information flux defined here is not an entirely novel variable. In fact, the procedure starts from a quadtree algorithm, and its numerical result is the same with the box-counting dimension used in fractal analysis (Falconer, 2003). What distinguishes the scale space information flux from other modes of characterizing irregular features consists mainly of its interpretation in terms of its relation to the size of the informational backbone, and the way in which it is used. There are numerous methods for the evaluation of a fractal dimension, characterized by strengths and weaknesses regarding their capability of grasping properties of the analyzed features (Feder, 1988); however, their numerical equivalency – even if established, in theory, for mathematical patterns – is not always true in the case of real-world systems (Ioana et al., 1997; Takayasu, 1991); therefore, the statements made here about the information flux may not always correspond to other "equivalent" fractal dimensions. From a practical point of view, the information flux v provides a way to determine quickly and in a simple way the expected information storage and transfer effort implied by a change in scale. It also specifies the scale limits between which one can reliably apply this evaluation of information storage/transfer effort. We suggest a compact representation of the information flux, presented in the following examples:

$$g = [u_a, v_A, u_b] \tag{13}$$

$$g = [u_a, v_A, u_b, v_B, u_c] \tag{14}$$

$$g = [u_a, v_A, u_b, v_B, u_c] \tag{15}$$

$$g = [u_a, v_A, u_b], [u_c, v_C, u_d] \tag{16}$$

The scale space information flux is thus written as a series of terms between brackets. The odd terms represent limits of the scale intervals, and the even terms represent the information flux values corresponding to these scale intervals. The example in Equation (13) expresses the fact that there is a constant information flux v_A between u_a and u_b, but not outside of this scale interval. Equation (14) shows that, in addition, there is constant information flux v_B between u_b and u_c. The brackets to the left of u_a and to the right of u_c show that the constant information flux intervals have been found to reach only up to those scale limits. In contrast, if one finds from the available data that there is no proof for constant information flux to end on a given scale, one uses round parentheses instead, as shown in Equation (15): in this case, the information flux v_B has been found to reach at least up to the scale u_c, but it is expected to be valid also beyond that scale. Finally, if two scale intervals with constant information flux are not adjacent, one can specify this by using the form shown in Equation (16).

Figure 5 shows an example related to a crack pattern produced experimentally (axes are represented in base 2 logarithms). Starch suspension in water, poured on glass plates, was subject to dessication, and the resulting crack pattern was analyzed; details regarding the experimental conditions can be found in Suteanu et al., 2000. The points connected with a full line lead to $v_F = 1.58 \pm 0.06$. The other group of points, connected with a dotted line, corresponds to an exponent $v_E = 2 \pm 0.2$, which simply shows that on this scale interval the SIB consists of all the available cells. The information flux will be thus written as follows: $g = (u_1, v_E, u_2], [u_3, v_F, u_4)$, and numerically: (4, 2 ± 0.2, 16], [32, 1.58 ± 0.06, 256). All the scale values $u_i > 1$ because the image of the crack pattern has been magnified for the purpose of the analysis.

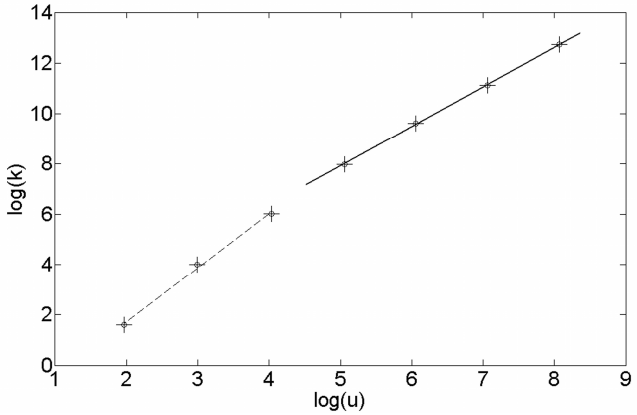

Figure 5. Application example concerning an experimentally obtained fracture pattern. The full line corresponds to v = 1.58 ±0.06. The dotted line connects the points in the interval where the SIB consists of all the cells on each scale (v ≈ 2).

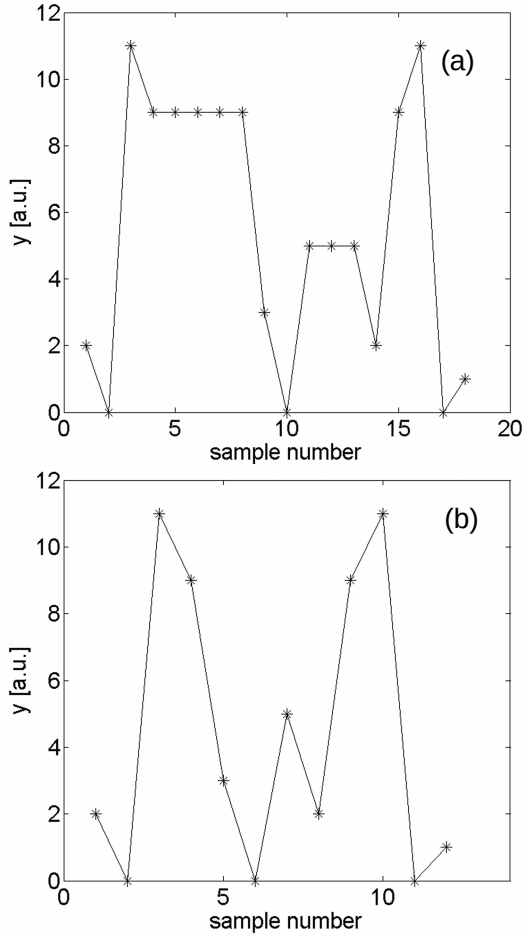

Figure 6. Two representations of the recording of variable *y* (arbitrary units): a) original series, b) series from which multiple occurrences of the same value have been removed.

3.3. Temporal Aspects

Let us suppose that we are recording a certain environmental variable at a given location, at equally spaced moments in time. We denote the variable by *y*, expressed in arbitrary units (one may think of it, for example, as daily maximum temperature in degrees Celsius). We may obtain a series of successive numbers such as: 2, 0, 11, 9, 9, 9, 9, 9, 3, 0, 5, 5, 5, 2, 9, 11, 0, 1. This series shows that a certain resolution (integers in degrees Celsius) was used for the representation of temperature values. It can be noticed that some of the values are repeated on successive days. When the same value is repeated, there is no new information for those days other than the fact that the recorded value has not changed. This, however, can be expressed in a more compact way, for example following an idea of Bigerelle and Iost (2007): 2, 0, 11, 9(4), 3, 0, 5(2), 2, 9, 11, 0, 1. The numbers in parentheses show that four other instances of the number 9 and two other instances of the number 5 are present on successive days. The resulting non-repeating values represent another time series: 2, 0, 11, 9, 3, 0, 5, 2, 9, 11, 0, 1. The new series consists of 12 numbers, while the length of the initial series was 18. The graphs of the two series are shown in Figure 6.

By dividing the available value range of the signal in a number of intervals and by specifying for each sample only the interval to which it belongs, one obtains a "coarser" time series, and the way the signal is transformed by this operation depends on the signal pattern (Munteanu, 1990).

The number of repeated values also depends on the resolution applied in the information acquisition process. The question we are interested in is whether we can characterize the recorded data by the way in which the presence of multiple successive occurrences of values depends on resolution. To approach this question, we may apply more than just one resolution – like in the case of the map study discussed above. To this end, we can divide the range of data of the time series in a number of *q* intervals (for example, 2, 4, 8, 16, 32, 64, etc.), allot to each sample one of the *q* possible values depending on the interval to which it belongs, and then suppress multiple occurrences, as shown in the example provided above. This procedure is exemplified in Figure 7 (details regarding the algorithm used to produce the time series are given below). The resulting series which do not contain multiple occurrences will be called the *temporal informational backbone* (TIB) of the pattern. Similarly to the case of the SIB, the size of the resulting TIB for the resolution *q*, denoted by *T(q)*, can be written as:

$$p(q) = |T(q)| \tag{17}$$

and is equal to the number of values retained in the TIB for that resolution. Like its spatial counterpart, the TIB specifies – for a given resolution – how many elements of the pattern provide relevant (in this case, non-repetitive) values. The amount of stored information is – in this case too – a linear function of the size of the informational backbone. If one specifies the number of occurrences for each of the values in the TIB of the series, the stored quantity increases proportionally to the size of the TIB; however, even the simple example shown above offers a more efficient storage format: in fact, this selection algorithm may represent only a starting point for the development of effective compression techniques.

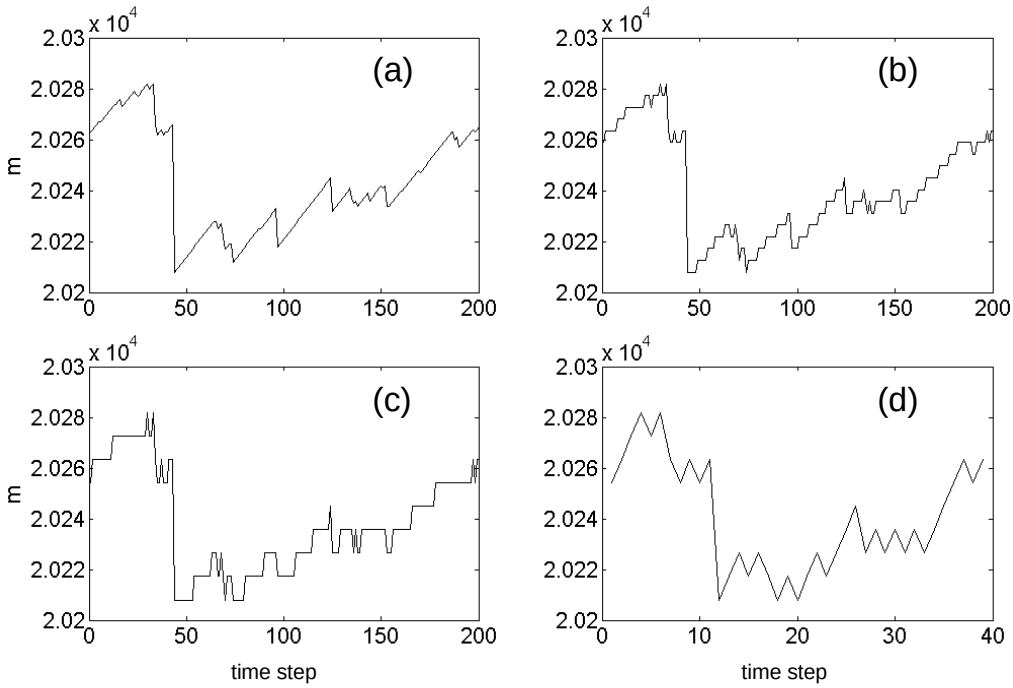

Figure 7. Fragment of an analyzed time series (a), its transformation following a change in resolution to q = 16 (b) and q = 8 (c), and the TIB corresponding to q = 8 (d).

Figure 7d shows the TIB for a resolution $q = 8$ levels: the similarity between the pattern in Figure 7d and those in 7a, 7b, and 7c can be noticed, as well as the contrast between 7d and the other three images from the point of view of the number of samples.

Like in the case of the spatial approach, the informational backbone corresponding to a certain scale is unique as far as we consider one and the same way to "frame" the studied feature: for the spatial approach, it was the map of the feature that was subject to the iterative partition process, which had to be specified; for the temporal approach, it is the value range to which one applies the iterative partition process.

When considering temporal environmental patterns, one is interested also in a frame-independent way to characterize them. Therefore, it is useful to see if and how the size of the TIB depends on the resolution q. Based on the way the TIB was introduced above, and analogously to the spatial approach, we can approach time series with a set of distinct resolutions and identify the TIB.

Similarly with spatial features, time series can be characterized by self-affinity (Kantz and Schreiber, 2004): if one changes the scale for the two coordinates (time and the represented parameter) by different factors, one obtains another curve that is similar to the original one. Numerous processes in the natural environment represented as time series enjoy self-affinity properties. Radiances in the atmosphere, for example, which are strongly coupled with atmospheric dynamics, have been found to emphasize scaling over many orders of magnitude (Lovejoy et al., 2009); scaling aspects have also been found in studies regarding stream flow (Koscielny-Bunde et al., 2006; Sauquet et al., 2008), atmospheric temperature (Kurnaz, 2004; Yu, 2006), sea surface temperature (Monetti et al., 2003), solar wind (Riazantseva and Zastenker, 2008), return intervals between extreme events (Bunde et al., 2005), etc.

This evaluation procedure will be applied to three different features: the output of a computer model with well-studied scaling properties, Brownian walks, and real data from the natural environment. The first example is a time series produced by a cellular automaton working according to the Bak-Tang-Wiesenfeld (BTW) model (Bak et al., 1987). A cellular automaton is a discrete mathematical model based on a uniform set of locally-connected identical cells (Adamatzky et al., 2008). It is defined by the cells composing it, the set of allowed states for each cell, the cell neighbourhood, and the transition rules according to which the state of the automaton is updated simultaneously at discrete time intervals (Schiff, 2007). In its simplest form, the BTW model starts with a plane divided in a set of adjacent square cells capable of hosting a number of "grains": individual grains are added to randomly chosen cells; if the total number of grains in a cell reaches the threshold value of four, a discharge process takes place: the grains are equally distributed to the 4-adjacent neighbours. If the neighbours to which grains are transferred reach in this way their 4-grain threshold, they are subject to discharge in their turn; this process continues until all the cells remain with a number of grains lower than four. At this point, a new random cell is chosen, a grain is added to it, etc. Grains leave the plane of the automaton if they reach its edges. The described processes lead to a "critical" state, when grain avalanches of all sizes form. The model produces patterns with scaling properties which have been extensively studied (Bak, 1996); there are also numerous implications of this model regarding natural and human-made

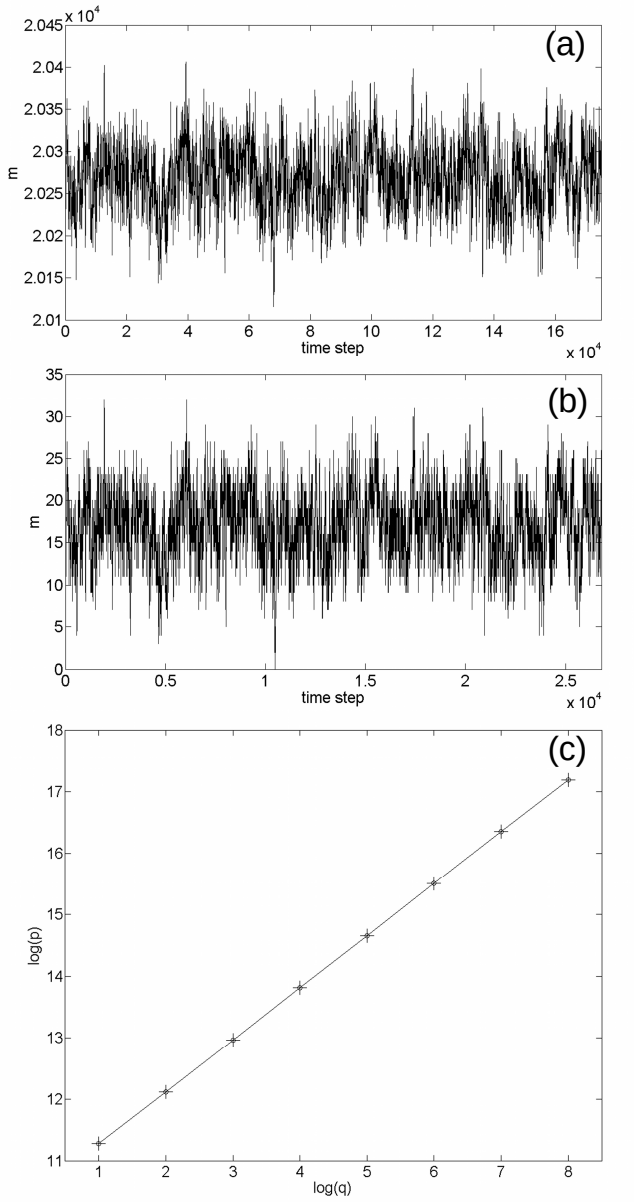

Figure 8. Time series corresponding to the number of "grains" in a cellular automaton functioning according to the Bak-Tang-Wiesenfeld model: (a) original time series, (b) the TIB for a resolution $q = 32$ (note the different number of time steps), (c) the graph showing the resolution dependence of the temporal informational backbone.

systems (Sornette, 2006). The time series analyzed in this study represents the variation of the total number of "grains" on the plane of an automaton of 1600 cells; time series data consist of 180,000 successive samples collected after the model reached its critical state (Figure 8a).

An example TIB is shown in Figure 8b. Figure 8c shows the way in which the size of the TIB varies with resolution (base 2 logarithms were used here too, since resolution values were chosen as powers of 2). The graph shows that a power law governs the relation between the resolution q and the size of the TIB:

$$p(q) = q^w \tag{18}$$

with $w = 0.85 \pm 0.01$. This relation holds for all the resolution values used in the analysis. The consistent relation between $\log(p)$ and $\log(q)$ allows us to find the scale space information flux as:

$$w = \frac{\log \frac{p_{i+1}}{p_i}}{\log \frac{q_{i+1}}{q_i}} \tag{19}$$

Analogously to the spatial context, *the information flux shows the way in which the size of the informational backbone changes with scale.* If the resolution values are chosen according to powers of 2, the information flux is:

$$w = \log_2 \frac{p_{i+1}}{p_i} \tag{20}$$

which means that also in this case the scale space information flux provides the logarithm of the number of times the informational backbone increases when the scale changes by a factor of two. In accordance to the format proposed in section 3.1, the scale space information flux is written as: g = (2, 0.85 ± 0.01, 256); the round parentheses show that neither the lower nor the upper limit of the scaling interval have been reached for these resolution values.

Brownian walks have been studied in detail from the point of view of their mathematical properties (Malamud and Turcotte, 1999), and have been used for the testing and evaluation of time series analysis methods (Ioana et al., 1997). For this study, the data were generated as running sums over series of time series representing white noise; an example of one of the analyzed Brownian walks is provided in Figure 9. A Monte Carlo experiment based on 100 time series produced in this way, with 40,000 samples each, led to g = (2, 0.99 ± 0.09, 64]. In other words, the information stored simply doubles with the doubling of the scale.

As an example of application to real data from the environment, we have used this approach to analyze time series representing temperature patterns: maximum daily values recorded in Yarmouth, Nova Scotia, Canada, between 1942 and 2002 (Environment Canada Climate Archive, 2008). Seasonal trends were removed by subtracting from each day's value the average temperature of that specific day of the year (Monetti et al., 2003). Figure 10 shows the evaluation of the information flux (as base 2 logarithms). The resulting value of the information flux is g = [4, 0.94 ± 0.02, 256).

These studies show that while in the case of Brownian walks the resulting value of the information flux is trivial, different fractional values are obtained for time series from models and real-world data. Further studies on these and other

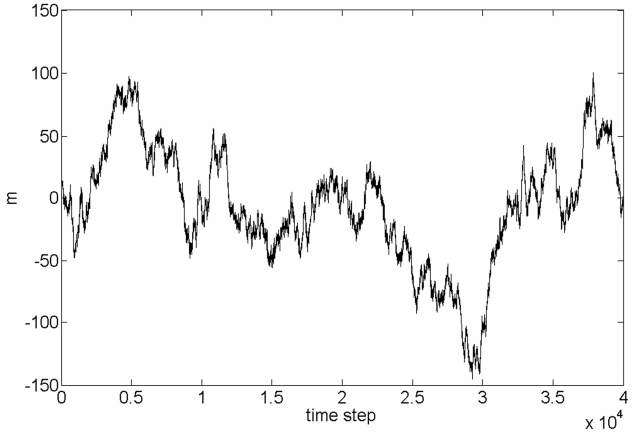

Figure 9. Example of a Brownian walk used for the analysis.

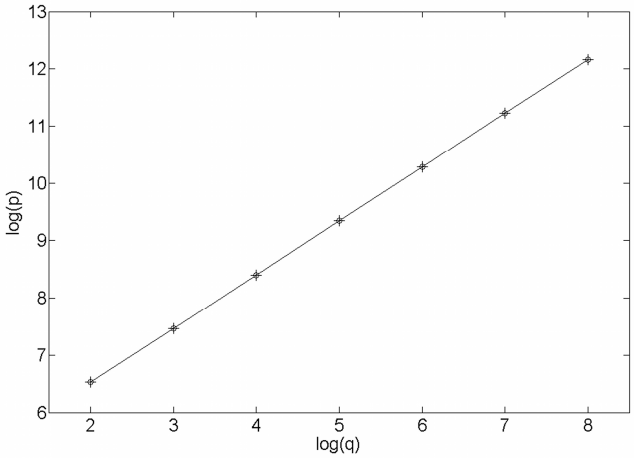

Figure 10. The TIB analysis applied to daily temperature time series collected in Yarmouth, Nova Scotia, between 1942 and 2002.

categories of time series will better reveal the potential of this approach to add to the meaningful description of temporal environmental patterns.

4. Discussion and Conclusions

Spatial and temporal patterns have been studied here from the point of view of the scale dependence of the information to be handled. We have shown that a scale-related informational backbone (in space or in time) can be identified. For patterns with self-affine properties, like those often found in the natural environment, one can establish the scale space information flux that characterizes the way in which the amount of information to be handled depends on scale, usually as a function of the considered scale interval. Furthermore, evaluating the amount of information to be stored, transmitted and analysed when applying a multiscale approach represents an important aspect of information management. The concepts of spatial and temporal informational backbone can be useful in the process of deciding upon the information that should be managed on each scale. The information flux provides a quantitative way of evaluating the scale or resolution dependence of the size of the informational backbone. The process of establishing it supports the identification of self-affinity properties of environmental patterns, which are characterized by constant information flux over certain ranges of scale.

For spatial features, the procedure introduced here supports storage-saving data selection, and also provides a measure of data storage requirements as a function of scale, flexibly adapting the calculation algorithm to the scale ranges of interest to the user. On the other hand, map module connectivity properties for the scale-dependent data selection are important for pattern analysis. Therefore, the paper also specifies rules that can be used by geographic information management software for a fast assessment of changes in connectivity as a function of changes in scale. Furthermore, the scale dependence of the spatial informational backbone is used for the characterization of the pattern for each scale interval of interest. A written form designed to specify the scaling properties corresponding to different scale ranges is introduced and exemplified for different categories of practical situations.

Similarly, for temporal patterns a resolution-dependent data selection algorithm is introduced; as in the case of spatial patterns, a storage-saving procedure is presented. Based on the resolution dependence of the stored temporal informational backbone, the introduced method leads to quantitative, scale-range-sensitive pattern characterization.

We have not discussed here implications of i) varying the information carried by one pixel, and ii) changing the sampling rate of the time series, since these operations lead to changes in information quantity that are, in principle, simpler to foresee; the study of techniques to optimize acquisition methods from this point of view, considering their implications for data compression algorithms, are beyond the scope of this paper. On the other hand, one can naturally develop this approach by including both the spatial and the temporal resolutions in a single integrated space-time informational backbone.

The introduced concepts and methods can be applied to a wide spectrum of situations in environmental science and environmental management; examples include spatial and temporal patterns of pollution, natural resource patterns (spatial distribution of minerals, drainage basins, forests), natural hazards (airborne gamma ray spectrometry maps, earthquake patterns), etc. They are also applicable to features from other disciplines, such as material science and medical imaging. Supported by accumulating achievements in fields such as geographic information science, fractal theory, digital geometry, and time series analysis, the information-focused approach to environmental patterns represents a promising direction of research regarding both theoretical and practical aspects of environmental science.

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