ISEIS

Journal of Environmental Informatics 16(2) 80-93 (2010)

Journal of Environmental Informatics

www.iseis.org/jei

Inexact Piecewise Quadratic Programming for Waste Flow Allocation under Uncertainty and Nonlinearity

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Received 4 March 2010; revised 8 August 2010; accepted 25 August 2010; published online 10 October 2010

ABSTRACT. In practical waste management systems, most relationships among different system components are nonlinear in nature. Effects of economies-of-scale can often bring about such nonlinearity in objective functions within an inexact optimization framework. To handle both nonlinearity and uncertainty, an inexact piecewise quadratic programming (IPQP) model was developed through coupling piecewise linear regression with interval linear programming. In IPQP, uncertainties expressed as intervals for transportation/operation costs, treatment capacities, waste generation rates, waste flows/amounts were reflected; a more accurate approximation for nonlinearities reflecting effects of economies-of-scale between unit transportation costs and waste flows as well as between unit operation costs and waste treatment amounts were provided. An interactive algorithm was designed for solving IPQP. IPQP was applied to a hypothesis case of waste allocation planning and compared with a conventional inexact quadratic programming model (IQP). The results indicated that, in the investigated waste allocation system, the optimized waste flows from the districts to the waste treatment facilities (WTFs) and the optimized waste treatment amounts in WTFs had no significant differences between both models. However, most of unit transportation costs or unit operation costs in IPQP were less than those in IQP, which finally contributed to a lower net system costs in IPQP than IQP. This implied that the often ignored effects of economies-of-scale should be considered accurately in the real-world waste management system to obtain lower costs. Strategies to balance the tradeoff between approximation accuracy and computational complexity for IPQP were also discussed.

Keywords: quadratic programming, piecewise linear regression, waste management, operation costs, transportation costs, economies of scale

1. Introduction

Waste flow allocation (WFA) is critical in municipal solid waste management (Everett and Modak, 1996; Li and Huang, 2007). To address the WFA problem under uncertainties, three major types of inexact mathematical programming methods have been proposed, including stochastic programming, fuzzy programming, and interval parameter programming (Chang et al., 1997; Chang and Wang, 1996; Huang et al., 2005; Li and Huang, 2007; Maqsood and Huang, 2003; Sahinidis, 2004; Xu et al., 2010).These programming methods can tackle various uncertainties expressed as random variables, fuzzy sets and discrete intervals; meanwhile; combinations of these methods can address multiple uncertainties (Huang et al., 1992; Huang et al., 1996; Huang and Moore, 1993; Li et al., 2006; Liu et al., 2009; Xu et al., 2009a; Xu et al., 2009b). However, most of these methods are based on an assumption of linear objective function. In practical waste management systems, most relationships among different system components are nonlinear in nature (Huang et al., 2010), which can be described accurately only if a nonlinear model is expressed (Wu et al., 2006). Thus, incorporation of nonlinearities and uncertainties within a general optimization framework is desired to comprehend tradeoffs among various waste management practice and policies.

Effects of economies-of-scale can often bring about nonlinearities in objective functions in a WFA planning under uncertainties. Previously, two types of approaches were employed to deal with the scale effects within an inexact optimization framework. One approach was to find efficient algorithms to directly solve the resulting nonlinear objective functions with inexact information. However, this approach needs much effort of strict mathematical proofs (Sun et al., 2009b). For instance, a derivative algorithm was proved for solving the inexact quadratic programming model (IQP) with much lower computational efforts (Chen and Huang, 2001). An interval nonlinear programming model with a satisfactory algorithm was proposed and applied to the planning of waste management activities in the Hamilton-Wentworth Region of Ontario, Canada (Wu et al., 2006). The other type of approach was to approximate nonlinear expressions so that existing algorithms could be applied. A simple linear approximation was often employed to substitute nonlinear expressions previously. For instance, with the consideration of data availability and computational effi-

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ISSN: 1726-2135 print/1684-8799 online
 doi:10.3808/jei.201000180

ciency, two IQP methods were proposed through introduction of intervals and fuzzy numbers within quadratic programming frameworks (Huang et al., 1994; Huang and Baetz, 1995). A fuzzy two-stage quadratic programming method was proposed to better reflect uncertainties expressed as both probability-density and fuzzy membership functions as well as nonlinearity in the form of quadratic terms as a whole (Li and Huang, 2007).

In fact, nonlinear systems usually can be approximated more accurately by piecewise linear functions through splitting the state space into piecewise regions and assuming sub-system is linear within each region (Croxton et al., 2003; Keha et al., 2004). The concept of piecewise has many applications in the field of automatic control (Bemporad et al., 2000; Bemporad and Morari, 1999; Rantzer and Johansson, 2000). There have been a few applications of piecewise-linearization-based linear programming to dealing with filter management in fluid power systems (Nie et al., 2009, 2010a, 2010b), coastal subsurface water management problems (Karterakis et al., 2007), hydroelectric generation resources management (Moraga et al., 2007), reservoir system scheduling (Pereira and Pinto, 1991), and optimal synthesis of an integrated water system (Karuppiah and Grossmann, 2006). Notably, piecewise linearization was incorporated for dealing with interval-fuzzy nonlinear programming models in water quality management (Qin et al., 2007; Zhu et al., 2009). However, few application of inexact piecewise quadratic programming to waste management problems was reported.

Therefore, the objective of this study is to develop an inexact piecewise quadratic programming method (IPQP) and to apply it to a WFA problem. The performance of IPQP will be campared with that of conventional inexact quadratic programming model (IQP). A representative waste management planning case will then be employed to test the models in dealing with both uncertainties in waste management conditions and nonlinearities reflecting effects of economies-of-scale simultaneously. The effects of different nonlinearity approximation methods on both transportation and operations costs in IPQP and IQP will be analyzed and compared.

2. An Interval Piecewise Quadratic Programming

2.1. Interval Programming with Nonlinear Objective

Consider an interval nonlinear programming where parameters ($c_j^{\pm}$) in the objective function are expressed as nonlinear functions ($g_j(x_j^{\pm})$) of the corresponding decision variable ($x_j^{\pm}$):

$$\text{Min } f^{\pm} = \sum_{j=1}^{n} c_j^{\pm} x_j^{\pm} \tag{1a}$$

subject to:

$$c_j^{\pm} = g_j(x_j^{\pm}), \ j = 1, 2, \ldots, n \tag{1b}$$

$$\sum_{j=1}^{n} a_{ij}^{\pm} x_j^{\pm} \le b_i^{\pm}, \ i = 1, 2, \ldots, m \tag{1c}$$

$$x_j^{\pm} \ge 0, \ \forall j \tag{1d}$$

where $a_{ij}^{\pm}$, $b_i^{\pm}$, $c_j^{\pm}$ and $x_j^{\pm}$ are interval parameters or variables. Since it is difficult to find general arithmetic algorithms to solve the interval nonlinear problem directly, a straightforward solution is to transfer Model (1) into an approximated linear programming.

2.2. Formulation of Interval Piecewise Quadratic Programming

Because $g_j(x_j^{\pm})$ is only associated with one independent variable ($x_j^{\pm}$), a piecewise linear regression (PLR) model could be fitted to approximate equation (1b) as follows:

$$g_j(x_j^{\pm}) \approx \gamma_j^{\pm} x_j^{\pm} + \delta_j^{\pm} = \begin{cases} \gamma_{j1}^{\pm} x_j^{\pm} + \delta_{j1}^{\pm} & (P_{j1}^{L} \le x_j^{\pm} < P_{j1}^{U}) \\ \ldots & \\ \gamma_{jh}^{\pm} x_j^{\pm} + \delta_{jh}^{\pm} & (P_{jh}^{L} \le x_j^{\pm} < P_{jh}^{U}) \\ \ldots & \\ \gamma_{jl_P}^{\pm} x_j^{\pm} + \delta_{jl_P}^{\pm} & (P_{jl_P}^{L} \le x_j^{\pm} < P_{jl_P}^{U}) \end{cases} \tag{2}$$

where $\gamma_j^{\pm}$ and $\delta_j^{\pm}$ are the interval slope and the interval intersection in the PLR model to approximate $g_j(x_j^{\pm})$, respectively;

Substitute Equations (2) into Model (1) and consider the features of PLR, then we have the Interval Piecewise Quadratic Programming (IPQP) model:

$$\text{Min } f^{\pm} = \sum_{j=1}^{n} (\gamma_j^{\pm} x_j^{\pm 2} + \delta_j^{\pm} x_j^{\pm}) \tag{3a}$$

subject to:

$$\sum_{h=1}^{l_P} (\mu_{jh} P_{jh}^{L}) \le x_j^{\pm} \le \sum_{h=1}^{l_P} (\mu_{jh} P_{jh}^{U}), \forall j \tag{3b}$$

$$\gamma_j^{\pm} = \left[\sum_{h=1}^{l_P} (\mu_{jh} \gamma_{jh}^{-}), \sum_{h=1}^{l_P} (\mu_{jh} \gamma_{jh}^{+}) \right], \forall j \tag{3c}$$

$$\delta_j^{\pm} = \left[\sum_{h=1}^{l_P} (\mu_{jh} \delta_{jh}^{-}), \sum_{h=1}^{l_P} (\mu_{jh} \delta_{jh}^{+}) \right], \forall j \tag{3d}$$

$$\sum_{h=1}^{l_P} \mu_{jh} = 1, \mu_{jh} \in \{0,1\}, \ j = 1, 2, \ldots, n \tag{3e}$$

$$\sum_{j=1}^{n} a_{ij}^{\pm} x_j^{\pm} \le b_i^{\pm}, \ i = 1, 2, \ldots, m \tag{3f}$$

$$x_j^{\pm} \ge 0, \ \forall j \tag{3g}$$

where h is piece number in the PLR model for $g_j(x_j^{\pm})$; l_P is the max piece number in the PLR model for $g_j(x_j^{\pm})$; P_{jh}^{L} and P_{jh}^{U} are lower and upper bounds of the range for $x_j^{\pm}$ in piece h, respectively; γ_{jh}^- and γ_{jh}^+ are lower and upper bounds of $\gamma_j^\pm$ in piece h, respectively; δ_{jh}^- and δ_{jh}^+ are lower and upper bounds of $\delta_j^\pm$ in piece h, respectively; μ_{jh} is 0 or 1 to indicate if Piece h is selected in the PLR model for $g_j(x_j^\pm)$.

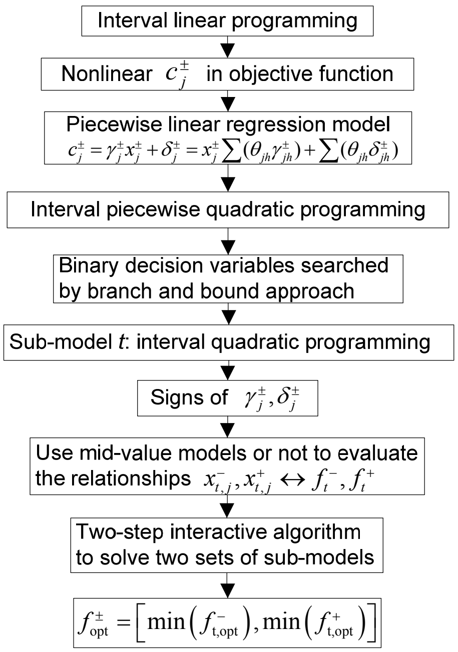

Figure 1. Framework of inexact piecewise quadratic programming.

2.3. Solution Algorithm

Figure 1 shows the general framework of the IPQP model. In nature, the IPQP Model is an interval-parameter mixed-integer quadratic programming model. Without loss of generality, the binary decision variable μ_{jh} could be firstly determined by the branch and bound approach. Due to the constraint from Equation (3e), there will be at most $(l_P)^n$ combinations of μ_{jh} (In real applications, the number of combinations could be further reduced due to the individual constraints of each $x_j^\pm$). When μ_{jh} is searched and determined in sequence among all combinations, the IPQP model then can be transformed into two sets of sub-models under two situations by the interactive algorithm in the previous work (Chen and Huang, 2001; Huang and Baetz, 1995; Huang and Loucks, 2000). The pairwise submodels can be solved since they are deterministic quadratic programming models.

(i) when the signs of $\gamma_j^\pm$ and $\delta_j^\pm$ are the same (if $\gamma_j^\pm \geq 0$, i.e. $\gamma_{jh}^- \geq 0, \gamma_{jh}^+ \geq 0, \forall h$, then $\delta_j^\pm$ should be $\geq$ 0, i.e. $\delta_{jh}^- \geq 0$, $\delta_{jh}^+ \geq 0, \forall h$; vice versa), the bounds for $\gamma_{jh}^\pm$, $\delta_{jh}^\pm$ and $x_{t,j}^\pm$ corresponding to f_t^- and f_t^+ can be easily defined under each $\mu_{t,jh}$ (t = 1, 2,…, $(l_P)^n$). Then model (3) can be formulated as follows:

$$\text{Min} f_t^- = \sum_{j=1}^{k}[\sum_{h=1}^{l_P}(\mu_{t,jh}\gamma_{jh}^-)\, x_{t,j}^- + \sum_{h=1}^{l_P}(\mu_{t,jh}\delta_{jh}^-)(x_{t,j}^-)^2] + \sum_{j=k+1}^{n}[\sum_{h=1}^{l_P}(\mu_{t,jh}\gamma_{jh}^-)x_{t,j}^+ + \sum_{h=1}^{l_P}(\mu_{t,jh}\delta_{jh}^-)(x_{t,j}^+)^2] \quad (4a)$$

$$\gamma_{jh}^- \geq 0, \delta_{jh}^- \geq 0, j = 1, 2, \cdots, k \quad (4b)$$

$$\gamma_{jh}^- < 0, \delta_{jh}^- < 0, j = k+1, k+2, \cdots, n \quad (4c)$$

subject to:

$$\sum_{h=1}^{l_P}(\mu_{t,jh}P_{jh}^L) \leq x_{t,j}^- \leq \sum_{h=1}^{l_P}(\mu_{t,jh}P_{jh}^U), \ j = 1, 2, \cdots, k \quad (4d)$$

$$\sum_{h=1}^{l_P}(\mu_{t,jh}P_{jh}^L) \leq x_{t,j}^+ \leq \sum_{h=1}^{l_P}(\mu_{t,jh}P_{jh}^U), \ j = k+1, k+2, \cdots, n \quad (4e)$$

$$\sum_{h=1}^{l_P}\mu_{t,jh} = 1, \mu_{t,jh} \in \{0,1\}, j = 1,\ 2,\ \ldots,\ n \quad (4f)$$

$$\sum_{j=1}^{k_1}|\, a_{ij}\,|^+ \ \text{Sign}(a_{ij}^+)x_{t,j}^- + \sum_{j=k_1+1}^{n}|\, a_{ij}\,|^- \ \text{Sign}(a_{ij}^-)x_{t,j}^+ \leq b_i^+, \ \ \forall i \quad (4g)$$

$$x_{t,j}^- \geq 0, \ j = 1, 2, \cdots, k \quad (4h)$$

$$x_{t,j}^+ \geq 0, \ j = k+1, k+2, \cdots, n \quad (4i)$$

The solutions of sub-model (4) can be obtained as follows:

$$f_{\text{opt}}^- = \overset{t=\phi}{\min}\left\{f_{\text{t,opt}}^-\right\} \quad (5a)$$

$$x_{j,\text{opt}}^- = \left\{x_{t,j,\text{opt}}^-\right\}_{t=\phi}, \ j = 1, 2, \cdots, k \quad (5b)$$

$$x_{j,\text{opt}}^+ = \left\{x_{t,j,\text{opt}}^+\right\}_{t=\phi}, \ j = k+1, k+2, \cdots, n \quad (5c)$$

The solutions of sub-model (4) then can be substituted to sub-model (6) as constraints. Thus, we have:

$$\text{Min} f_t^+ = \sum_{j=1}^{k}[\sum_{h=1}^{l_P}(\mu_{t,jh}\gamma_{jh}^+)x_{t,j}^+ + \sum_{h=1}^{l_P}(\mu_{t,jh}\delta_{jh}^+)(x_{t,j}^+)^2] + \sum_{j=k+1}^{n}[\sum_{h=1}^{l_P}(\mu_{t,jh}\gamma_{jh}^+)x_{t,j}^- + \sum_{h=1}^{l_P}(\mu_{t,jh}\delta_{jh}^+)(x_{t,j}^-)^2] \quad (6a)$$

$$\gamma_{jh}^+ \geq 0, \delta_{jh}^+ \geq 0, j = 1, 2, \cdots, k \quad (6b)$$

$$\gamma_{jh}^+ < 0, \delta_{jh}^+ < 0, j = k+1, k+2, \cdots, n \quad (6c)$$

subject to:

$$\sum_{h=1}^{l_P}(\mu_{t,jh}P_{jh}^L) \leq x_{t,j}^+ \leq \sum_{h=1}^{l_P}(\mu_{t,jh}P_{jh}^U), \ j = 1, 2, \cdots, k \quad (6d)$$

$$\sum_{h=1}^{l_P}(\mu_{t,jh}P_{jh}^L) \leq x_{t,j}^- \leq \sum_{h=1}^{l_P}(\mu_{t,jh}P_{jh}^U), \ j = k+1, k+2, \cdots, n \quad (6e)$$

$$\sum_{h=1}^{l_P} \mu_{t,jh} = 1,\ \mu_{t,jh} \in \{0,1\},\ j = 1,\ 2,\ \ldots,\ n \tag{6f}$$

$$\sum_{j=1}^{k_1} |a_{ij}|^+ \operatorname{Sign}(a_{ij}^+) x_{t,j}^- + \sum_{j=k_1+1}^{n} |a_{ij}|^- \operatorname{Sign}(a_{ij}^-) x_{t,j}^+ \le b_i^-,\ \forall i \tag{6g}$$

$$x_j^+ \ge x_{j,opt}^-,\ j = 1,2,\cdots,k \tag{6h}$$

$$0 \le x_j^- \le x_{j,opt}^+,\ j = k+1, k+2, \cdots, n \tag{6i}$$

Solutions of sub-model (6) can be obtained as:

$$f_{\text{opt}}^+ = \min^{t=\psi} \{ f_{\text{t,opt}}^+ \} \tag{7a}$$

$$x_{j,\text{opt}}^+ = \{ x_{t,j,\text{opt}}^+ \}_{t=\psi},\ j = 1,2,\cdots,k \tag{7b}$$

$$x_{j,\text{opt}}^- = \{ x_{t,j,\text{opt}}^- \}_{t=\psi},\ j = k+1, k+2, \cdots, n \tag{7c}$$

Combining solutions of sub-models (4) and (6), the solutions of model (3) can be obtained when the signs of $\gamma_j^\pm$ and $\delta_j^\pm$ are the same as follows:

$$x_{j,\text{opt}}^\pm = [x_{j,\text{opt}}^-,\ x_{j,\text{opt}}^+],\ \forall j \tag{8a}$$

$$f_{\text{opt}}^\pm = [f_{\text{opt}}^-,\ f_{\text{opt}}^+] \tag{8b}$$

(ii) when the signs of $\gamma_j^\pm$ and $\delta_j^\pm$ are different, a two-step process is needed to determine the corresponding relationships among $\gamma_{jh}^\pm$, $\delta_{jh}^\pm$, $x_{t,j}^\pm$ and $f_t^\pm$. When $\mu_{t,jh}$ is searched and selected by the branch and bound approach ($t = 1,2,\cdots,(l_P)^n$), through letting all left- and/or right-hand-side coefficients of Model (3) be equal to their mid-values, Model (3) then becomes a deterministic quadratic programming (QP) model as follows:

$$\text{Min } f_{t,m} = \sum_{j=1}^{n} \left\{ \sum_{h=1}^{l_P} \left[\mu_{t,jh} (\gamma_{jh})_m \right] (x_{t,j})_m^2 + \sum_{h=1}^{l_P} \left[\mu_{t,jh} (\delta_{jh})_m \right] (x_{t,j})_m \right\} \tag{9a}$$

subject to:

$$\sum_{h=1}^{l_P} (\mu_{t,jh} P_{jh}^L) \le (x_{t,j})_m \le \sum_{h=1}^{l_P} (\mu_{t,jh} P_{jh}^U),\ \forall j \tag{9b}$$

$$\sum_{h=1}^{l_P} \mu_{t,jh} = 1,\ \mu_{t,jh} \in \{0,1\},\ j = 1,\ 2,\ \ldots,\ n \tag{9c}$$

$$\sum_{j=1}^{n} \left[(a_{ij})_m (x_{t,j})_m \right] \le (b_i)_m,\ i = 1, 2,\ \ldots, m \tag{9d}$$

$$(x_{t,j})_m \ge 0,\ \forall j \tag{9e}$$

where $(\gamma_{jh})_m$, $(\delta_{jh})_m$, $(a_{ij})_m$ and $(b_i)_m$ are mid-values of $\gamma_{jh}^\pm$, $\delta_{jh}^\pm$, $a_{ij}^\pm$ and $b_i^\pm$ [e.g., $(\gamma_{jh})_\text{m} = (\gamma_{jh}^- + \gamma_{jh}^+)/2$]. Through solving model (9), we can obtain the optimal solutions of $(x_{t,j})_{\text{m,opt}} \in [x_{t,j,\text{opt}}^-,\ x_{t,j,\text{opt}}^+]$, $\forall j$. Thus, the relationships between $x_{t,j}^\pm$ and $f_t^\pm$ can be identified according to the following criteria:

$$f_{t,j}^-(x_{t,j\,\text{opt}}^-) \le f_{t,j}^-(x_{t,j\,\text{opt}}^+),\ \text{when } 2\left[\sum_{h=1}^{l_P} (\mu_{t,jh} \gamma_{jh}^+) \right] (x_{t,j})_{\text{m,opt}} + \sum_{h=1}^{l_P} (\mu_{t,jh} \delta_{jh}^+) > 0 \tag{10a}$$

$$f_{t,j}^-(x_{t,j\,\text{opt}}^-) \ge f_{t,j}^-(x_{t,j\,\text{opt}}^+),\ \text{when } 2\left[\sum_{h=1}^{l_P} (\mu_{t,jh} \gamma_{jh}^+) \right] (x_{t,j})_{\text{m,opt}} + \sum_{h=1}^{l_P} (\mu_{t,jh} \delta_{jh}^+) < 0 \tag{10b}$$

When criterion (10a) is satisfied, $x_{t,j}^-$ corresponds to f_t^-; when criterion (10b) is satisfied, $x_{t,j}^+$ corresponds to f_t^-. Thus, two sets of sub-models similar to sub-models (4) and (6) can be further developed to obtain the interval solutions ($x_{t,j\,\text{opt}}^\pm$) and the corresponding interval values of objective functions ($f_{\text{t,opt}}^\pm$). Finally, the corresponding upper or lower bound solutions ($x_{j\,\text{opt}}^-$, $x_{j\,\text{opt}}^+$) will be obtained through searching $\min\left(f_{\text{t,opt}}^-\right)$ or $\min\left(f_{\text{t,opt}}^+\right)$; the bound solutions will then be combined to form the general solutions ($x_{j\,\text{opt}}^\pm$) for Model (3) under this situation (when $\gamma_j^\pm$ and $\delta_j^\pm$ have different signs).

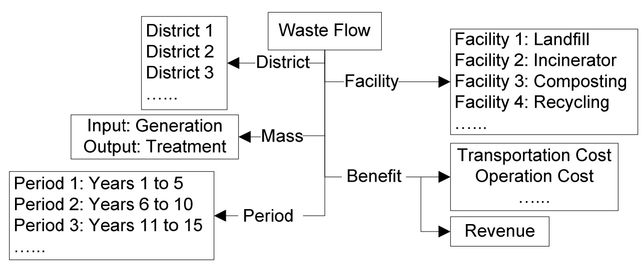

Figure 2. Multiple dimensions of waste flow allocation system.

3. Overview of Waste Flow Allocation Problem

3.1. General Complexity

Scientifically solving a WFA problem by a regional manager is to allocate reasonable amounts of waste flows from multiple districts to different waste treatment facilities (WTFs) over several periods across the region. The WFA problem is multiple-dimensional in nature (Figure 2). The planning time consists of several equally-long periods. The allocation region is divided into several districts. The WTFs usually include landfill, incineration, composting, and recycle. Waste flows generated and transferred in this multiple-dimension system result in interconnections among districts and facilities. A landfill is the disposal of waste materials by burial, which receives wastes from different districts and residues from other facilities. A composting facility turns the organic wastes into environment-friendly humic matters (Sun et al., 2009a). A recycling facility converts the wastes into useful products. An incineration facility serves as burning the allocated wastes. Those interconnections lead to various complexities involving uncertainties in wastes flow, transportation/operation costs and WTF capacities, constraints in treatment capacities of different WTFs, balances among waste amounts of generation, transportation and treatment, as well as nonlinearity existing between the waste flows and their transportation/operation costs caused by effects of economies-of-scale.

3.2. Case Study

Based on representative data from governmental reports and related references, a hypothetical case is considered wherein a regional manager is responsible for reasonably allocating waste flows over a three-period planning horizon (each period is 5-year) in order to minimize the net system cost. The waste generation rates in district 1 are [125, 185], [165, 215], and [185, 245] t/d in the three periods and those in district 2 are [155, 205], [175, 220], and [195, 245] t/d, individually. Two available WTEs (one landfill and one incinerator) serve the MSW treatment/disposal needs from the districts. At least 40% (a diversion rate) of waste flows is forced to be treated by the incinerator due to the growing opposition from the public with regard to landfill disposal. Approximately 30% (on a mass basis) residues of the incoming waste flows to the incinerator are further disposed at the landfill. The incinerator has a capacity of [200, 240] t/day while the landfill has a capacity of [1.7, 2.1] million tones. Revenue from incinerator is [15, 20], [20, 25], and [25, 30] $/t in the three periods, individually. The costs of waste transportation and operation vary from districts to WTFs in the three periods. When waste flows are high or transportation distances are long, the effects of economies-of-scale in terms of waste transportation to or operations in WTFs could be expressed as a sizing model with a power law (Thuesen et al., 1977; Huang et al., 1995):

$$TR = \frac{TR_{re}(X / X_{re,tr})^m}{X} \tag{11a}$$

$$OP = \frac{OP_{re}(Y / Y_{re,op})^n}{Y} \tag{11b}$$

where X is a waste flow variable (t/d); $X_{re,tr}$ is a reference waste flow (t/d); TR is the unit transportation cost ($/t); TR_{re} is a known transportation cost for reference waste flow $X_{re,tr}$ ($/t); Y is a waste treatment amount variable (equals to a sum of waste flow which enters the WTF, t/d); $Y_{re,op}$ is a reference waste treatment amount (t/d); OP is the unit operation cost in WTFs ($/t); OP_{re} is a known operation cost for reference waste flow $Y_{re,\text{op}}$ in WTFs ($/t); and m and n are the corresponding economies-of-scale exponents ($0 < m,\ n < 1$). For waste transportation and operation cost, m or n value is approximately 0.8 to 0.9 (Huang and Baetz, 1995).

Assume that there are l pieces within the feasible range of waste flows or waste treatment amounts. Accordingly, Equation (11) can be approximated as PLR models as follows:

$$TR = \begin{cases} \alpha_1 X + \beta_1 & (P_1 \le X < P_2) \\ \cdots & \\ \alpha_i X + \beta_i & (P_i \le X < P_{i+1}) \\ \cdots & \\ \alpha_l X + \beta_l & (P_{l-1} \le X \le P_l) \end{cases} \tag{12a}$$

$$OP = \begin{cases} \gamma_1 Y + \delta_1 & (Q_1 \le Y < Q_2) \\ \cdots & \\ \gamma_j Y + \delta_j & (Q_j \le Y < Q_{j+1}) \\ \cdots & \\ \gamma_l Y + \delta_l & (Q_{l-1} \le Y \le Q_l) \end{cases} \tag{12b}$$

where α_i, and β_i are the slopes and the intercepts for ith PLR model for waste transportation cost (i = 1, 2, ... , l); the lower and upper bounds for the range of waste flows are P_i and P_{i+1} individually; γ_j and δ_j are the slopes and the intercepts for jth PLR model for waste treatment cost (j = 1, 2, ... , l); The lower and upper bounds for the range of waste treatment amounts are Q_j and Q_{j+1} individually.

Since effects of economies-of-scale may change due to variations of factors affecting transportation and operation costs, different levels of scale exponents (m and n) are assumed. In the meantime, an increasing piecewise level will not only bring about decreasing approximation errors between the sizing model and the PLR models but also results in increased numbers of decision variables. Thus, a proper piecewise level of 4 is chosen for all PLR models in this study.

4. Application

The proposed IPQP model is applied to tackle the above planning problem for WFA. The objective is to minimize the net system cost (i.e. subtracting revenues from transportation cost and operation costs in WTFs). The decision variables represent waste flows from district j to WTE i in period k. The constraints involve all relationships among the decision variables under representative waste management conditions. Thus, we have:

$$Min\ f^{\pm} = L_k \left\{ \sum_{i=1}^{u}\sum_{j=1}^{v}\sum_{k=1}^{q}\left(X_{ijk}^{\pm} TR_{ijk}^{\pm}\right) + \sum_{i=2}^{u}\sum_{k=1}^{q}\left(Y_k^{\pm} FT_k^{\pm}\right) + \sum_{k=1}^{q}\left(Z_{1k}^{\pm} OP_{1k}^{\pm}\right) + \sum_{k=1}^{q}\left[Z_{2k}^{\pm}\left(OP_{2k}^{\pm} - RE_k^{\pm}\right)\right] \right\} \tag{13a}$$

$$TR_{ijk}^{\pm} \approx \alpha_{ijk}^{\pm} X_{ijk}^{\pm} + \beta_{ijk}^{\pm} = \begin{cases} \alpha_{ijk1}^{\pm} X_{ijk}^{\pm} + \beta_{ijk1}^{\pm} & (P_{ijk1}^{L} \le X_{ijk}^{\pm} < P_{ijk1}^{U}) \\ \ldots & \\ \alpha_{ijkh}^{\pm} X_{ijk}^{\pm} + \beta_{ijkh}^{\pm} & (P_{ijkh}^{L} \le X_{ijk}^{\pm} < P_{ijkh}^{U}) \\ \ldots & \\ \alpha_{ijkl}^{\pm} X_{ijk}^{\pm} + \beta_{ijkl}^{\pm} & (P_{ijkl}^{L} \le X_{ijk}^{\pm} < P_{ijkl}^{U}) \end{cases} \tag{13b}$$

$$FT_k^{\pm} \approx \varepsilon_k^{\pm} Y_k^{\pm} + \zeta_k^{\pm} = \begin{cases} \varepsilon_{k1}^{\pm} Y_k^{\pm} + \zeta_{k1}^{\pm} & (Q_{k1}^L \le Y_k^{\pm} < Q_{k1}^U) \\ \cdots \\ \varepsilon_{kh}^{\pm} Y_k^{\pm} + \zeta_{kh}^{\pm} & (Q_{kh}^L \le Y_k^{\pm} < Q_{kh}^U) \\ \cdots \\ \varepsilon_{kl}^{\pm} Y_k^{\pm} + \zeta_{kl}^{\pm} & (Q_{kl}^L \le Y_k^{\pm} < Q_{kl}^U) \end{cases} \quad (13c)$$

$$OP_{ik}^{\pm} \approx \gamma_{ik}^{\pm} Z_{ik}^{\pm} + \delta_{ik}^{\pm} = \begin{cases} \gamma_{ik1}^{\pm} Z_{ik}^{\pm} + \delta_{ik1}^{\pm} & (R_{ik1}^L \le Z_{ik}^{\pm} < R_{ik1}^U) \\ \cdots \\ \gamma_{ikh}^{\pm} Z_{ik}^{\pm} + \delta_{ikh}^{\pm} & (R_{ikh}^L \le Z_{ik}^{\pm} < R_{ikh}^U) \\ \cdots \\ \gamma_{ikl}^{\pm} Z_{ik}^{\pm} + \delta_{ikl}^{\pm} & (R_{ikl}^L \le Z_{ik}^{\pm} < R_{ikl}^U) \end{cases} \quad (13d)$$

where: $f^{\pm}$ is net system cost ($); $X_{ijk}^{\pm}$ $Y_k^{\pm}$ and $Z_{ik}^{\pm}$ are decision variables; $X_{ijk}^{\pm}$ is waste flow from district j to facility i during period k (t/d); $Y_k^{\pm}$ is waste residue flow from incinerator to landfill during period k (t/d); $Z_{ik}^{\pm}$ is waste treatment amount in facility i during period k (t/d); i is type of waste management facility (i = 1 for landfill and 2 for incinerator); j is name of district (j = 1 and 2); k is planning period (k = 1, 2 and 3); L_k is length of period k (day); h is piece number in the PLR models for $TR_{ijk}^{\pm}$, $FT_k^{\pm}$ and $OP_{ik}^{\pm}$ (h =1, 2, …, l); $TR_{ijk}^{\pm}$ is transportation cost for waste flow from district j to facility i during period k ($/t); $\alpha_{ijk}^{\pm}$ and $\beta_{ijk}^{\pm}$ are slope and y-intercept in the PLR model to approximate $TR_{ijk}^{\pm}$; α_{ijkh}^{-} and α_{ijkh}^{+} are lower and upper bounds of $\alpha_{ijk}^{\pm}$ in piece h, respectively; β_{ijkh}^{-} and β_{ijkh}^{+} are lower and upper bounds of $\beta_{ijk}^{\pm}$ in piece h, respectively; P_{ijkh}^{L} and P_{ijkh}^{U} are lower and upper bounds of the range for $X_{ijk}^{\pm}$ in piece h, respectively; $FT_k^{\pm}$ is transportation cost for residue flow from the incinerator to the landfill during period k ($/t); $\varepsilon_k^{\pm}$ and $\zeta_k^{\pm}$ are slope and y-intercept in the PLR model to approximate $FT_k^{\pm}$; ε_{kh}^{-} and ε_{kh}^{+} are lower and upper bounds of $\varepsilon_k^{\pm}$ in piece h, respectively; ζ_{kh}^{-} and ζ_{kh}^{+} are lower and upper bounds of $\zeta_k^{\pm}$ in piece h, respectively; Q_{kh}^{L} and Q_{kh}^{U} are lower and upper bounds of the range for $Y_k^{\pm}$ in piece h; $OP_{ik}^{\pm}$ is operating cost in facility i for waste treatment amount during period k ($/t); $\gamma_{ik}^{\pm}$ and $\delta_{ik}^{\pm}$ are slope and y-intercept in the PLR model to approximate $OP_{ik}^{\pm}$; γ_{ikh}^{-} and γ_{ikh}^{+} are lower and upper bounds of $\gamma_{ik}^{\pm}$ in piece h, respectively; δ_{ikh}^{-} and δ_{ikh}^{+} are lower and upper bounds of $\delta_{ik}^{\pm}$ in piece h, respectively; R_{ikh}^{L} and R_{ikh}^{U} are lower and upper bounds of the range for $Z_{ik}^{\pm}$ in piece h; $RE_k^{\pm}$ is revenue from the incinerator during period k ($/t).

Substitute Equations 13b - 13d into 13a, then we have the IPQP model as follows:

$$\text{Min } f^{\pm} = L_k \left\{ \sum_{i=1}^{u}\sum_{j=1}^{v}\sum_{k=1}^{q}\left[\alpha_{ijk}^{\pm}\left(X_{ijk}^{\pm}\right)^2 + \beta_{ijk}^{\pm}X_{ijk}^{\pm}\right] + \sum_{i=2}^{u}\sum_{k=1}^{q}[\varepsilon_k^{\pm}\left(Y_k^{\pm}\right)^2 + \zeta_k^{\pm}Y_k^{\pm}] + \sum_{k=1}^{q}\left[\gamma_{ik}^{\pm}\left(Z_{1k}^{\pm}\right)^2 + \delta_{ik}^{\pm}Z_{1k}^{\pm}\right] + \sum_{k=1}^{q}\left[\gamma_{2k}^{\pm}\left(Z_{2k}^{\pm}\right)^2 + \delta_{2k}^{\pm}Z_{2k}^{\pm} - RE_k^{\pm}Z_{2k}^{\pm}\right]\right\} \quad (14a)$$

subject to:

[Piecewise constraints related to transportation cost from districts to WTFs]:

$$\sum_{h=1}^{l}(\theta_{ijkh}P_{ijkh}^{L}) \le X_{ijk}^{\pm} \le \sum_{h=1}^{l}(\theta_{ijkh}P_{ijkh}^{U}), \forall i, j, k \quad (14b)$$

$$\alpha_{ijk}^{\pm} = \left[\sum_{h=1}^{l}(\theta_{ijkh}\alpha_{ijkh}^{-}), \sum_{h=1}^{l}(\theta_{ijkh}\alpha_{ijkh}^{+})\right], \beta_{ijk}^{\pm} = \left[\sum_{h=1}^{l}(\theta_{ijkh}\beta_{ijkh}^{-}), \sum_{h=1}^{l}(\theta_{ijkh}\beta_{ijkh}^{+})\right] \quad \forall i, j, k \quad (14c)$$

$$\sum_{h=1}^{l}\theta_{ijkh} = 1, \quad \theta_{ijkh} \in \{0,1\}, \quad \forall i, j, k \quad (14d)$$

[Residue mass balance in WTFs]:

$$Y_k^{\pm} \ge \sum_{j=1}^{v}\left(X_{2jk}^{\pm}FE^{\pm}\right), \quad \forall k \quad (14e)$$

[Piecewise constraints related to transportation cost from WTFs to landfill]:

$$\sum_{h=1}^{l}(\eta_{kh}Q_{kh}^{L}) \le Y_k^{\pm} \le \sum_{h=1}^{l}(\eta_{kh}Q_{kh}^{U}), \quad \forall k \quad (14f)$$

$$\varepsilon_k^{\pm} = \left[\sum_{h=1}^{l}(\eta_{kh}\varepsilon_{kh}^{-}), \sum_{h=1}^{l}(\eta_{kh}\varepsilon_{kh}^{+})\right], \zeta_k^{\pm} = \left[\sum_{h=1}^{l}(\eta_{kh}\zeta_{kh}^{-}), \sum_{h=1}^{l}(\eta_{kh}\zeta_{kh}^{+})\right] \quad \forall k \quad (14g)$$

$$\sum_{h=1}^{l}\eta_{kh} = 1, \quad \eta_{kh} \in \{0,1\} \quad \forall k \quad (14h)$$

[Mass balance for waste treatment]:

$$Z_{1k}^{\pm} \ge \sum_{j=1}^{v}\left(X_{1jk}^{\pm}\right) + Y_k^{\pm}, \quad \forall k \quad (14i)$$

$$Z_{2k}^{\pm} \ge \sum_{j=1}^{v}X_{2jk}^{\pm}, \quad \forall k \quad (14j)$$

[Piecewise constraints related to operation costs in WTFs]:

$$\sum_{h=1}^{l}(\lambda_{ikh}R_{ikh}^{L}) \le Z_{ik}^{\pm} \le \sum_{h=1}^{l}(\lambda_{ikh}R_{ikh}^{U}), \quad \forall i, k \quad (14k)$$

$$\gamma_{ik}^{\pm} = \left[\sum_{h=1}^{l}(\lambda_{ikh}\gamma_{ikh}^{-}), \sum_{h=1}^{l}(\lambda_{ikh}\gamma_{ikh}^{+})\right], \delta_{ik}^{\pm} = \left[\sum_{h=1}^{l}(\lambda_{ikh}\delta_{ikh}^{-}), \sum_{h=1}^{l}(\lambda_{ikh}\delta_{ikh}^{+})\right] \quad \forall i, k \quad (14l)$$

$$\sum_{h=1}^{l}\lambda_{ikh} = 1, \quad \lambda_{ikh} \in \{0,1\} \quad \forall i, k \quad (14m)$$

Table 1. Optimized Waste Flows ($X_{ijk}^{\pm}$) from Districts to WTFs and the Corresponding Transportation Costs ($TR_{ijk}^{\pm}$)

Facility	District	Period	IQP		IPQP		
i	*j*	*k*	Waste Flow (t/d)	Transportation Cost ($/t)	Piece *h*	Waste Flow (t/d)	Transportation Cost ($/t)
Landfill	1	1	[75.0, 111.0]	[14.1, 16.6]	1, 2	[75.0, 106.5]	[14.1, 16.4]
Landfill	1	2	[99.0, 129.0]	[15.0, 18.0]	2, 2	[99.0, 114.0]	[14.8, 18.2]
Landfill	1	3	[111.0, 147.0]	[16.3, 19.2]	2, 3	[102.5, 143.0]	[16.2, 19.1]
Landfill	2	1	[93.0, 123.0]	[11.5, 15.5]	2, 2	[93.0, 123.0]	[11.2, 15.1]
Landfill	2	2	[105.0, 135.0]	[12.4, 15.9]	2, 3	[105.0, 131.5]	[12.0, 15.7]
Landfill	2	3	[117.0, 147.0]	[13.4, 16.8]	2, 3	[117.0, 147.0]	[13.0, 16.6]
Incinerator	1	1	[50.0, 74.0]	[11.8, 15.1]	1, 1	[50.0, 78.5]	[12.2, 14.9]
Incinerator	1	2	[66.0, 86.0]	[12.1, 15.7]	1, 2	[66.0, 101.0]	[12.3, 15.1]
Incinerator	1	3	[74.0, 98.0]	[12.8, 16.3]	2, 2	[82.5, 102.0]	[12.4, 15.9]
Incinerator	2	1	[62.0, 82.0]	[12.3, 15.8]	1, 2	[62.0, 82.0]	[12.4, 15.5]
Incinerator	2	2	[70.0, 90.0]	[12.9, 16.5]	1, 2	[70.0, 93.5]	[12.9, 16.1]
Incinerator	2	3	[78.0, 98.0]	[13.5, 17.6]	2, 2	[78.0, 98.0]	[13.2, 17.1]

[Capacity constraints for Landfill]:

$$L_k \sum_{k=1}^{q} Z_{1k}^{\pm} \le LC^{\pm} \tag{14n}$$

[Capacity constraints for Incinerator]:

$$Z_{2k}^{\pm} \le TC_{ik}^{\pm}, \quad \forall k \tag{14o}$$

[Waste disposal demand constraints]:

$$\sum_{i=1}^{u} X_{ijk}^{\pm} \ge WG_{jk}^{\pm}, \quad \forall j, k \tag{14p}$$

$$X_{ijk}^{\pm} \ge DG_{ik}^{\pm}\, WG_{jk}, \quad \forall j,k; \; i = 2, 3, \cdots, u \tag{14q}$$

[Non-negativity constraints]:

$$X_{ijk}^{\pm} \ge 0, \quad \forall i, j, k \tag{14r}$$

where θ_{ijkh} is 0 or 1 to indicate if piece h is selected in the PLR model for $TR_{ijk}^{\pm}$; $FE^{\pm}$ is residue flow rate from incinerator to landfill (% of incoming mass to incinerator); η_{kh} is 0 or 1 to indicate if Piece h is selected in the PLR model for $FT_k^{\pm}$; λ_{ikh} is 0 or 1 to indicate if Piece h is selected in the PLR model for $OP_{ik}^{\pm}$; $LC^{\pm}$ is capacity of landfill (tonne or t); $TC_{ik}^{\pm}$ is capacity of WTF i (t/day) in period k; $WG_{jk}^{\pm}$ is amount of waste generated in district j in period k; $DG_{ik}^{\pm}$ is diversion rate of waste flow to facility i regulated by the authority in period k.

To verify the performance of IPQP, a conventional inexact quadratic programming model (IQP) is introduced for comparison. The IQP model can be formulated based on Model (14) with removal of piecewise constraints (Equations (14b) to (14d), (14f) to (14h), and (14k) to (14m)). Accordingly, in IQP model, $\alpha_{ijk}^{\pm}$ and $\beta_{ijk}^{\pm}$ become slope and y-intercept in a linear regression (LR) model to approximate $TR_{ijk}^{\pm}$; $\varepsilon_k^{\pm}$ and $\zeta_k^{\pm}$ change to slope and y-intercept in a LR model to approximate $OP_{ik}^{\pm}$; and $\gamma_{ik}^{\pm}$ and $\delta_{ik}^{\pm}$ are slope and y-intercept in a LR model to approximate $OP_{ik}^{\pm}$. Obviously, IQP could be considered as the simplest type of IPQP where the max piece number is 1 and the unit cost functions in the entire feasible region is linear.

5. Result Analysis

Table 1 presents the optimized waste flows and the corresponding unit transportation costs from the two districts to two WTFs, which are obtained through both IPQP and IQP models. Since the inexact waste-management conditions vary temporally and spatially, the patterns of optimized waste flows change accordingly in the multidimensional system. Figures 3 and 4 shows the lower and upper bound curves of waste transportation cost from districts to the landfill and to the incinerator, which are based on raw data (generated by exponential functions), linear regression models and PLR models for all of the pieces. The bold pieces in Figures indicated those which were optimally selected in the final solution for IPQP model. Better agreement between raw data and the PLR models than the LR models indicated that better fitting ability of PLR than LR when approximating nonlinear scale effects between waste flows and transportation costs. This implied more accurate transportation costs would be calculated in the PLR-based IPQP model compared with the LR-based IQP model.

Based on the IPQP model (Table 1), for the landfill, the optimized waste flows from district 1 would be [75.0, 106.5] t/d in period 1, [99.0, 114.0] t/d in period 2, and [102.5, 143.0] t/d in period 3. The corresponding transportation costs related to these waste flows would be [14.1, 16.4], [14.8, 18.2], and [16.2, 19.1] $/t in periods 1, 2 and 3. The optimally selected pieces in the IPQP model for calculated lower- and upper-bound of these costs would be number 1 and 2, 2 and 2, and 2 and 3 in each PLR model (Figure 3), respectively; similar to district 1, the optimized flows from district 2 to the landfill would be [93.0, 123.0], [105.0, 131.5], and [117.0, 147.0] t/d in periods 1 to 3, individually. The associated transportation costs would be [11.2, 15.1], [12.0, 15.7] and [13.0, 16.6] $/t. The obtained

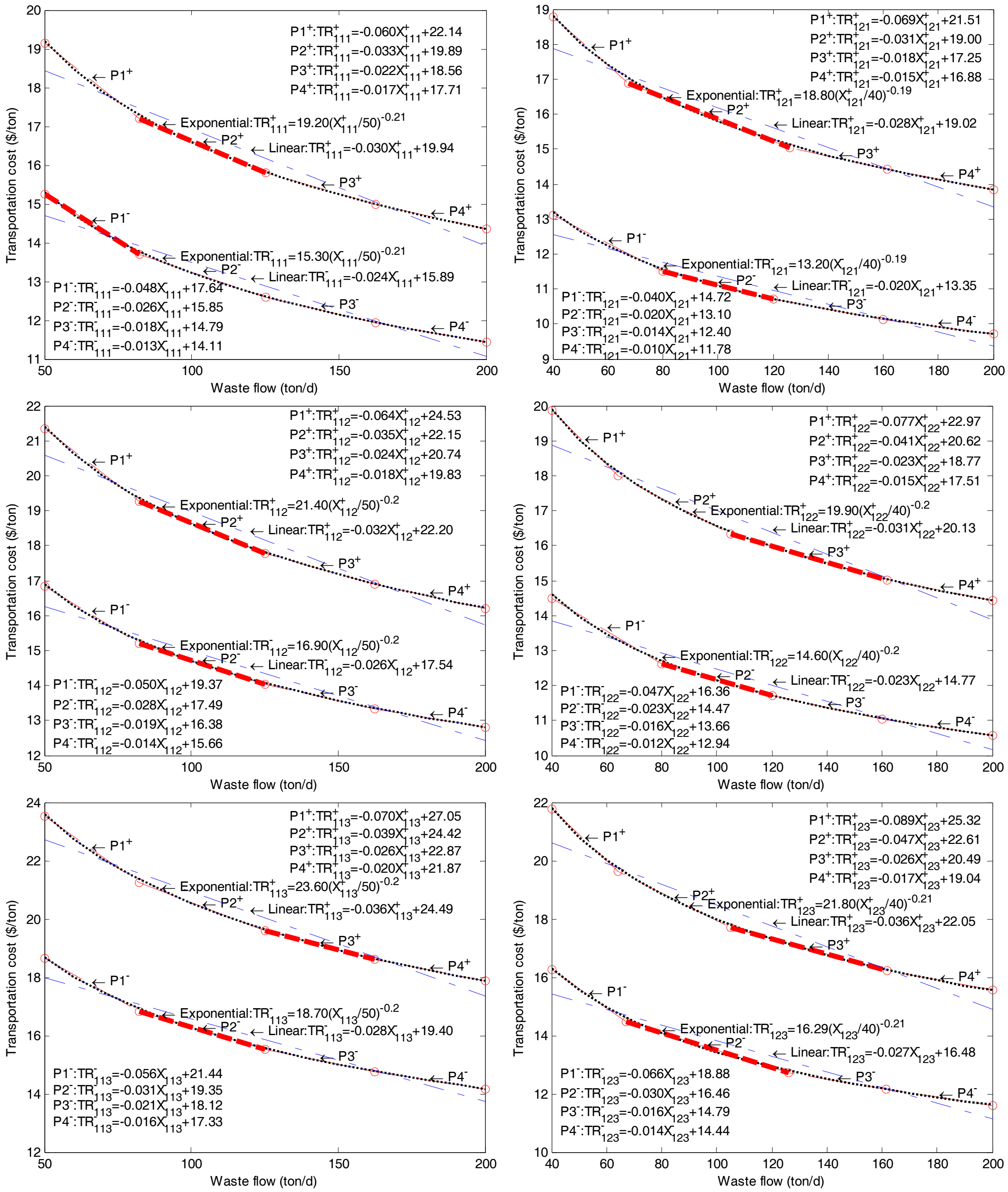

Figure 3. Waste transportation costs to the landfill ($TR_{11k}^{\pm}$ and $TR_{12k}^{\pm}$).

pieces for calculating lower-bound and upper-bound of these costs would be number 2 and 2, 2 and 3, and 2 and 3 in each PLR model (Figure 3), respectively. For the incinerator, the optimized flows from district 1 would be [50.0, 78.5], [66.0, 101.0], and [82.5, 102.0] t/d, the related transportation costs would be [12.2, 14.9], [12.3, 15.1] and [12.4, 15.9] $/t in periods 1 to 3, and the matching piece numbers for lower- and upper-bounds of these costs would be 1 and 1, 1 and 2, and 2 and 2 (Figure 4), separately; those from district 2 would be [62.0, 82.0], [70.0, 93.5], and [78.0, 98.0] t/d which are associated with transportation costs of [12.4,15.5], [12.9,16.1] and [13.2, 17.1] $/t and piece numbers of 1 and 2, 1 and 2, and 2 and 2, in periods 1 to 3 (Figure 4), respectively.

In comparison, the interval solutions of the IQP are close to those of the IPQP (Table 1). Most lower-bounds of optimized waste flows in IQP are almost the same as those in IPQP. Only two differences between lower-bounds can be found when waste flows in Period 3 are allocated from district 1 to the landfill (102.5 vs. 111.0) and from district 1 to the incinerator (82.5 vs. 74.0). Most upper-bounds of optimized waste flows in IQP are slightly different from those in IPQP while four upper-bounds of waste flows from district 2 to both landfill and incinerator

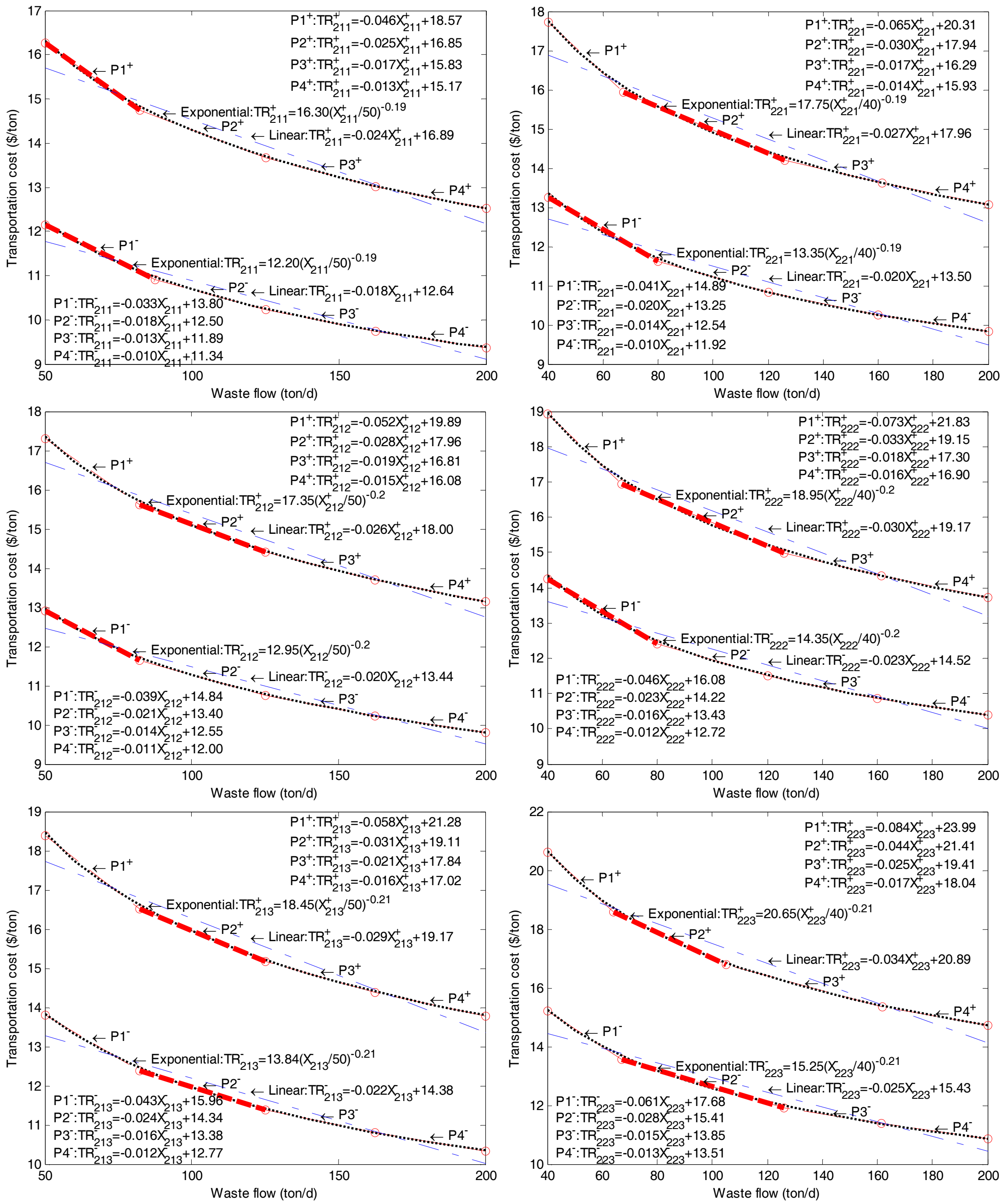

Figure 4. Waste transportation costs to the incinerator ($TR_{21k}^{\pm}$ and $TR_{22k}^{\pm}$).

in Periods 1 and 3 have no differences in both models (123.0 vs. 123.0 and 147.0 vs. 147.0, 82.0 vs. 82.0 and 98.0 vs. 98.0). Meanwhile, most of unit transportation cost intervals in IQP are larger than those in IPQP (i.e. both lower-bound and upper-bound are larger or equal) except for four exceptions. The exceptions are unit transportation cost intervals from district 1 to landfill in period 2 ([14.8, 18.2] vs. [15.0, 18.0]), from district 1 to incinerator in periods 1 and 2 ([12.2, 14.9] vs. [11.8, 15.1], and [12.3, 15.1] vs. [12.1, 15.7]), and from district 2 to incinerator in period 1 ([12.4, 15.5] vs. [12.3, 15.8]). The differences for unit transportation cost can be due to the fitting degrees between raw data and these regression models, which can be obviously observed in Figures 3 and 4. Since most of selected piece numbers are 2 or 3, the curves of PLR models are closer to the raw data and thus lower than the curves of LR models. Therefore when lower and upper bounds of optimized waste flows in both models are near to each other, the corresponding unit transportation cost calculated by PLR models would be less than by LR models. On the contrary, in the four exceptions, when the piece number is optimally selected as 1 and

Table 2. Optimized Residues ($Y_k^{\pm}$) from Incinerator to Landfill and the Corresponding Transportation cost ($FT_k^{\pm}$)

Period	IQP		IPQP		
k	Waste Flow (t/d)	Transportation Cost ($/t)	Piece *h*	Waste Flow (t/d)	Transportation Cost ($/t)
1	[33.6, 46.8]	[6.6, 8.1]	1, 2	[33.6, 48.1]	[6.5, 7.8]
2	[40.8, 52.8]	[7.0, 8.8]	2, 3	[40.8, 58.4]	[6.8, 8.3]
3	[45.6, 58.8]	[7.4, 9.3]	2, 3	[48.1, 60.0]	[7.2, 9.1]

Table 3. Optimized Waste Treatment Amounts in WTFs ($Z_{ik}^{\pm}$) and the Corresponding Operation Costs ($OP_{ik}^{\pm}$)

Facility	Period	IQP			IPQP	
i	*k*	Waste Generation (t/d)	Operation Cost ($/t)	Piece *h*	Waste Generation (t/d)	Operation Cost ($/t)
Landfill	1	[201.6, 280.8]	[22.5, 30.7]	2, 3	[201.6, 277.7]	[22.2, 30.6]
Landfill	2	[244.8, 316.8]	[25.9, 37.4]	3, 3	[244.8, 303.8]	[25.6, 37.5]
Landfill	3	[273.6, 352.8]	[33.7, 48.1]	3, 4	[267.7, 350.0]	[33.5, 48.7]
Incinerator	1	[112.0, 156.0]	[51.5, 68.7]	2, 3	[112.0, 160.5]	[51.2, 67.4]
Incinerator	2	[136.0, 176.0]	[54.6, 75.9]	2, 3	[136.0, 194.5]	[53.8, 73.9]
Incinerator	3	[152.0, 196.0]	[58.1, 82.9]	3, 3	[160.5, 200.0]	[56.7, 81.2]

the corresponding lines for PLR models (related to the optimized waste) are higher than that for LR models, unit transportation costs based on PLR models would be larger than LR models.

Similarly, Table 2 presents the optimized residue and the corresponding unit transportation costs from the incinerator to landfill based on IPQP and IQP models. Figure 5 shows the lower and upper bound curves of unit transportation costs, which further confirms that prediction of the PLR models are much closer to raw data than that of the LR models. For this reason, even intervals (or only lower-bounds or upper-bounds) of optimized residue waste flows in IPQP are larger than those in IQP models ([33.6, 48.1] > [33.6, 46.8] t/d in period 1, [40.8, 58.4] > [40.8, 52.8] t/d in period 2, and [48.1, 60.0] > [45.6, 58.8] t/d in period 3), the unit transportation costs for these waste flows in IPQP model are still less than those in IQP models ([6.5, 7.8] < [6.6, 8.1] $/t in period 1, [6.8, 8.3] < [7.0, 8.8] $/t in period 2, and [7.2, 9.1] < [7.4, 9.3] $/t). It thus would be concluded that PLR-based IPQP model can calculate unit transportation costs more accurately compared with the LR-based IQP model.

Besides waste flows and unit transportation costs among districts and WTFs, Table 3 further lists optimized waste treatment amounts in WTFs and the corresponding unit operation costs. The patterns of optimized waste treatment amounts vary in periods in two WTFs while the differences of waste treatment amounts between two models are not significant. Figure 6 illustrates the lower-bound and upper-bound curves of unit operation costs in WTFs in three different forms. Although the slopes of unit operation cost curves are gentler than those of unit transportation costs, curves of the PLR models are still nearer to raw operation cost data than that of the LR models. For example, the results of IPQP indicate that waste treatment amounts in landfill would be [201.6, 277.7], [244.8, 303.8], and [267.7, 350.0] t/d in periods 1 to 3; in comparison, slightly higher waste treatment amounts obtained by IQP would be [201.6, 280.8], [244.8, 316.8], and [273.6, 352.8] t/d in periods 1 to 3. Meanwhile, the corresponding unit operation costs of IPQP are lower than those of IQP in Periods 1 and 2 ([22.2, 30.6] < [22.5, 30.7] $/t and [25.6, 37.5] < [25.9, 37.4] $/t) while the upper-bound of unit operation costs of IPQP are slightly larger than that of IQP in Period 3 (48.7 > 48.1 $/t). This exception can be explained by the fact that Piece 4 is selected in the IPQP model, the curves of IPQP are higher than that of IQP in the specific ranges, and the two corresponding waste treatment amounts are almost the same (350.0 vs. 352.8 t/d). Different from landfill, the waste treatment amounts in incinerator of IPQP would be larger than those of IQP in periods 1 to 3 ([112.0, 160.5] > [112.0, 156.0] t/d, [136.0, 194.5] > [136.0, 176.0] t/d, and [160.5, 200.0] > [152.0, 196.0] t/d). Despite this, the unit operation costs of IPQP would be still lower than those of IQP in periods 1 to 3 ([51.2, 67.4] < [51.5, 68.7] $/t, [53.8, 73.9] < [54.6, 75.9] $/t, [56.7, 81.2] < [58.1, 82.9] $/t), which would be due to the curves of IPQP are much lower than those of IQP in the range where Piece Number is 2 or 3 (Figure 6).

Table 4. Cost Comparison for Waste Flow Allocation between Two Models

Expense	Specification	IQP ($$10^6$)	IPQP ($$10^6$)
Transportation Cost	Districts to WTFs	[24.32, 40.32]	[24.00, 39.59]
	Incinerator to Landfill	[1.544, 2.533]	[1.538, 2.569]
Operation Cost	Landfill	[40.20, 73.59]	[40.41, 75.63]
	Incinerator	[36.67, 68.32]	[35.92, 67.42]
Revenue	Incinerator	[-19.64, -18.62]	[-20.62,-19.08]
Net Cost	System	[84.12, 165.13]	[82.79,164.59]

In order to testify how the unit cost differences (transportation and operation costs) in IPQP and IQP models would affect the net system costs (objective functions), Table 4 presents the sum of costs for waste flow allocation. The sum of transportation cost in IPQP from districts to WTFs in the waste

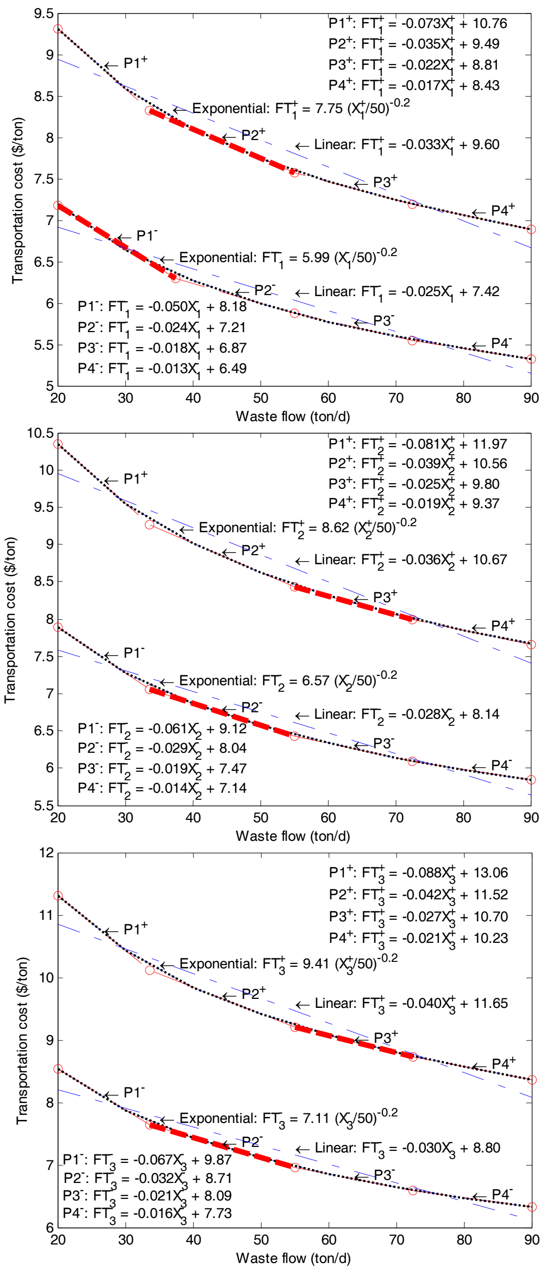

Figure 5. Residue transportation costs from the incinerator to the landfill (FT_k^-).

management system is $[24.00, 39.59] ×10^6, being lower than that in IPQP ($ [24.32, 40.32] × 10^6). The lower-bound of the sum of transportation cost from the incinerator to the landfill in IPQP is lower than that in IQP ($1.538 × 10^6 < 1.544 × 10^6) whereas upper-bound of residue transportation cost sum in IPQP is slightly higher than that in IQP ($2.569 × 10^6 > 2.533 × 10^6). However, the sum of residue transportation cost from the incinerator to the landfill is relatively low, which would have no significant effects on the difference between two models' objective function values. Meanwhile, the sum of operations cost in landfill in IPQP is higher than that in IQP ($[40.41, 75.63] ×10^6 > [40.20, 73.59] ×10^6) while the sum of operations costs in incinerator in IPQP is lower than that in IQP ($[35.92, 67.42] ×10^6 < [36.67, 68.32] ×10^6). The incinerator's revenue in IPQP is also less than that in IQP ($[-20.62, -19.08] × 10^6 < [-19.64, -18.62] × 10^6). Thus, the overall net cost (objective function) for the entire waste allocation system by IPQP is lower than by IQP ($[82.79, 164.59] ×10^6 < [84.12, 165.13] ×10^6). The bound difference between two objective function values would be $1.32 × 10^6 and $0.54 × 10^6, respectively. This supports the fact that the differences of unit transportation and operation costs between two waste flow cost models (PLR vs. LR) would result in lower net costs in PLR-based IPQP than LR-based IQP. This further implies that the often negligible effects of scale should be considered accurately in waste flow allocation system.

6. Discussion

In general, the IPQP model can not only reflect uncertainties expressed as intervals among costs, capacities, waste generation rates, waste flows, and waste treatment amounts and so on, but also provide a more accurate approximation for nonlinearity existing in the objective function between unit transportation costs and waste flows as well as between unit operation costs and waste treatment amounts due to the effects of economic scale. This integration indicates a capability enhancement of the IPQP in dealing with multiple complexities (i.e. both uncertainties and nonlinearity) so as to further decrease the net costs for the overall waste management system to some degrees. In comparison, the conventional IQP model usually handles only uncertainties and inaccurate approximation for nonlinearity in effects of economies-of-scale.

It is interesting to note that, in the investigated waste allocation system, the optimized waste flows from the districts to the WTFs and the optimized waste treatment amounts in WTFs obtained through both the IPQP and IQP models have no significant differences. This maybe mainly because the added piecewise constraints (Equations 14b to 14d, 14f to 14h, and 14k to 14m) could not change the feasible region for both models in this study. Equations 14b, 14f and 14k can not restrict the ranges of waste flows or waste treatment amounts at all because each piece has the chance to be selected and determined by restrictions from other constraints. The solution differences between the IPQP and IQP models would be due to the degrees that objective functions to be approximated as well as the concomitant change of interactive locations between approximated objective functions and the same feasible region. Correspondingly, once these optimized waste flows and waste treatment amounts lie in the ranges where the PLR curves are much closer to the raw data (usually lower than the LR curves; Piece number equals 2 or 3 in this study), most of the unit transporttation costs or unit operation costs in IPQP model would be less than those in IQP models. These unit cost differences would finally contribute to the fewer sums of net costs (objecttive function values) in IPQP than those in IQP.

In principle, any smooth nonlinear system can be piecewise approximated to an arbitrary accuracy. In this sense, the nonlinear curves (waste flow vs. unit transportation cost and

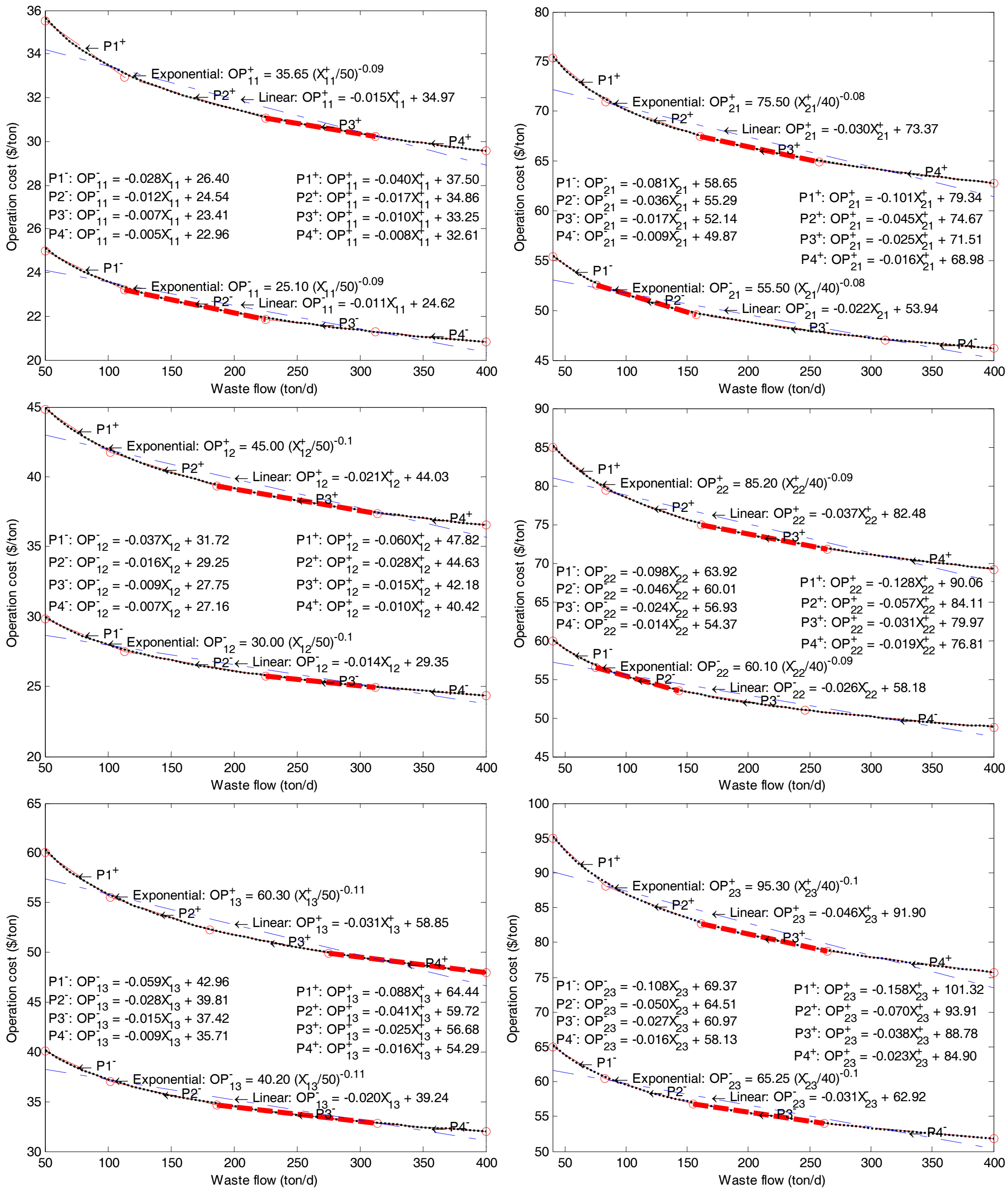

Figure 6. Operation costs for waste treatment facilities (landfill and incinerator; $OP_{1k}^{\pm}$ and $OP_{2k}^{\pm}$).

waste treatment amount vs. unit operation cost) in the waste management system can be further approximated by increasing the max piecewise number in IPQP model to obtain more accurate and fewer net costs. However, the number of binary variables (l^n) would exponentially increase accordingly in the practical application. Therefore, the max piecewise number and the number of decision variables with considered scale effects need appropriate selections through the trial and error method. When the increasing piece number could not change the feasible region so as the decision variables would not change significantly, a reasonable strategy would be: 1) ensure max piece numbers large enough in a wider feasible region and solve the model; 2) gradually decrease the max piece numbers but narrow down the feasible region of decision variables to the neighborhood of solutions obtained in the first step; 3) repeat steps 1 and 2 until the tradeoff between approximation accuracy and computational complexity is carefully balanced.

7. Conclusions

An inexact piecewise quadratic programming (IPQP) model has been developed for municipal solid waste allocation under uncertainty and nonlinearity. In IPQP, uncertainties expressed as intervals among transportation/operation costs, treatment capacities, waste generation rates, and waste flows/amounts can be reflected; a more accurate approximation for nonlinearities reflecting effects of economies-of-scale in the objective function between unit transportation costs and waste flows as well as between unit operation costs and waste treatment amounts can be provided. The developed method has been applied to a hypothesis case of waste allocation planning. An interactive algorithm is designed for solving the developed IPQP model. A conventional inexact quadratic programming model (IQP) is chosen to compare its performances with IPQP. The results indicate that, in the investigated waste allocation system, the optimized waste flows from the districts to the WTFs and the optimized waste treatment amounts in WTFs through IPQP and IQP models have no significant differences. However, most of unit transportation costs or unit operation costs in IPQP would be less than those in IQP. These unit cost differences would finally contribute to a lower net system costs (objective function values) in IPQP than those in IQP. This further implies that the often ignored effects of economies-of-scale should be considered accurately in the real-world waste management system to obtain lower costs. Although this study is the first attempt for waste management application through developing the IPQP approach, the IPQP is applicable to other environmental problems under uncertainty and nonlinearity.

Acknowledgments. This research was supported by the Major State Basic Research Development Program of MOST (2005CB724207) and the Natural Science and Engineering Research Council of Canada.

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