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Scenario-Based Methods for Interval Linear Programming Problems

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ABSTRACT. In this study, a total of twelve system-condition scenarios are considered for the management system. The scenarios correspond on different attitudes of decision makers to the study system. In detail, variations in concerns on objective function values (aggressive, conservative, or neutral), the attitude to the constraints (optimistic or pessimistic), and the preferred types of constricting ratios (consistent or varied) lead to twelve scenarios. Consequently, twelve planning models and solution methods corresponding to different scenarios have been developed. To demonstrate the applicability of the developed methods, a municipal solid waste management problem has been provided in the case study section. The inherent mechanism of the study system could be reflected through a series of considered scenarios. Thus the decision makers could understand the targeted system comprehensively and identify the scenario which best fits the practical condition. Moreover, a number of feasible schemes could be generated under each scenario which allows decision makers to further adjust the obtained solutions and indentify a desired one through incorporation of their experiences, economic situations, social and cultural conditions. In addition, the possibility of infeasible solutions has been greatly reduced with the consideration of twelve scenarios instead of one.

Keywords: interval linear programming, scenario analysis, constricting ratio, three-step method, solid waste management

1. Overview of Interval Linear Programming and Three Step Method

In this study, an interval linear programming (ILP) model will be examined as follows (Huang, 1994, 1998; Sun and Huang, 2010; Yan et al. 2010; Cao et al., 2010a, 2011):

$$\text{Min} \quad f^{\pm} = \sum_{j=1}^{k} c_j^{\pm} x_j^{\pm} + \sum_{j=k+1}^{n} c_j^{\pm} x_j^{\pm} \tag{1a}$$

Subject to:

$$\sum_{j=1}^{k} a_{ij}^{\pm} x_j^{\pm} + \sum_{j=k+1}^{n} a_{ij}^{\pm} x_j^{\pm} \le b_i^{\pm}, \quad i = 1, 2, ..., m . \tag{1b}$$

$$x_j^{\pm} \ge 0, \quad j = 1, 2, ..., n . \tag{1c}$$

where $c_j^{\pm}, a_{ij}^{\pm}, b_i^{\pm}, x_j^{\pm} \in R^{\pm}$, and $R^{\pm}$ denotes a set of interval numbers. For interval coefficients in the objective function ($c_j^{\pm}$), assume that the former k of them are positive, and the latter are negative.

To solve the ILP problem, a ThSM approach has been developed which includes three procedures: two-step-method (TSM), feasibility test, and constricting algorithm (Huang and Cao, 2011). In TSM, interval solutions can be obtained through solving two linear programming submodels (Huang 1998; Huang et al., 2001; Gao et al., 2010; He et al., 2010; Lv et al., 2010). According to the definition of feasible region for ILP, feasibility test can tell whatever the TSM solutions are feasible. Then if they are feasible, the solutions of TSM can be used to generate a number of schemes for decision makers. Otherwise, the non-feasible solutions can be eliminated by means of "constricting" the solutions of TSM. In other words, there are two cases when ThsM is used to solve ILP. Case 1 is that solutions of TSM pass the feasibility test, and thus solutions of ThSM and those of TSM are same. Case 2 is that solutions of TSM fail the feasibility test, and thus solutions obtained through ThSM are "constricting" ones of TSM. Under both cases, solutions of TSM are the foundation of ThSM. Consequently, TSM should be discussed in detail to further explore the solution methods for ILP.

The main idea of TSM is to solve two sub models instead of solving the original ILP model where the submodels are linear programs. Models (2) and (3) show the submodels (assume that $b_i^{\pm} > 0$ and $f^{\pm} > 0$):

$$\text{Min} \quad f^{-} = \sum_{j=1}^{k} c_j^{-} x_j^{-} + \sum_{j=k+1}^{n} c_j^{-} x_j^{+} , \tag{2a}$$

subject to:

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$$\sum_{j=1}^{k}\left|a_{ij}^{\pm}\right|^{+} Sign(a_{ij}^{\pm})x_j^{-} + \sum_{j=k+1}^{n}\left|a_{ij}^{\pm}\right|^{-} Sign(a_{ij}^{\pm})x_j^{+} \le b_i^{+}, i = 1, 2, .., m \quad (2b)$$

$$x_j^{-} \ge 0, \; j = 1, 2, ..., k \;. \quad (2c)$$

$$x_j^{+} \ge 0, \; j = k+1, k+2, ..., n \;. \quad (2d)$$

Solutions of $x_{jopt}^{-} (j = 1, 2, ..., k)$ and $x_{jopt}^{+} (j = k+1, k+2, ..., n)$ can be obtained through solving submodel (2):

$$\text{Min} \;\; f^{+} = \sum_{j=1}^{k} c_j^{+} x_j^{+} + \sum_{j=k+1}^{n} c_j^{+} x_j^{-} \,, \quad (3a)$$

Subject to:

$$\sum_{j=1}^{k}\left|a_{ij}^{\pm}\right|^{-} Sign(a_{ij}^{\pm})x_j^{+} + \sum_{j=k+1}^{n}\left|a_{ij}^{\pm}\right|^{+} Sign(a_{ij}^{\pm})x_j^{-} \le b_i^{-}, i = 1, 2, ..., m \quad (3b)$$

$$x_j^{+} \ge x_{j\,opt}^{-} \,, \; j = 1, \; 2, \; ..., \; k \;. \quad (3c)$$

$$0 \le x_j^{-} \le x_{j\,opt}^{+} \,, \; j = k+1, \; k+2, \; ..., \; n \;. \quad (3d)$$

Hence, solutions of $x_{jopt}^{+} (j = 1, 2, ..., k)$ and $x_{jopt}^{-} (j = k+1, k+2, ..., n)$ can be obtained through solving submodel (3). Thus, the final solution of $f_{opt}^{\pm} = [f_{opt}^{-}, f_{opt}^{+}]$ and $x_{jopt}^{\pm} = [x_{jopt}^{-}, x_{jopt}^{+}]$ can be obtained.

The combinations of coefficient ($a_{ij}^{\pm}$) and decision variable ($x_j^{\pm}$) in models (2) and (3) are indentified through the assumption that the products ($a_{ij}^{\pm} x_j^{\pm}$) obey specific probability distributions (i.e. Normal distributions). Meanwhile, the principles for indentifying the combinations of objective value ($f^{\pm}$) and coefficient ($b_i^{\pm}$) mainly depend on the concerns and attitudes of decision makers. To be specific, in the second submodel, solutions of the first submodel are incorporated as new constraints besides the regular constraints of the original ILP model. As a result, the feasible zone of the second submodel could be reduced. In this sense, the two submodels are not in parallel, and the first submodel holds priority.

If the first model is to solve f^{-}, it indicates the decision makers focus on achieving the lowest value of the objective function. In other words, the lower bound of the objective function is considered more important than its upper one. In real-world applications, for example, the objective is to minimize the total system cost. Decision makers who determine to solve f^{-} first are aggressive and aims to achieve the lowest cost. Otherwise, the upper bound of objective function (f^{+}) should be the first submodel, which means that the decision makers are more interested in for a higher value of objective function. The decision makers are conservative in this case.

The values of $b_i^{\pm}$ in the first submodel reflect the attitudes of decision makers to the constraints. For example, if the first submodel employs the values of b_i^{+}, it suggests that decision makers are optimistic about the constraints. Moreover, the decision makers are considered as pessimistic if the values of b_i^{-} are employed in the first submodel. This is because constraints with b_i^{+} are loose, while those with b_i^{-} are strict. In other words, the feasible zone associated with b_i^{+} is larger than that with b_i^{-}. Thus, when the values of b_i^{+} is used, a more optimal value for the objective function could be obtained.

Therefore, several scenarios indicating different concerns and attitudes of decision makers can be generated. ThSM-I and ThSM-II are two of the scenarios. In detail, if the ILP model is solved through ThSM-I or ThSM-II, it indicates that decision makers are aggressive to achieve the lowest system cost, and hold an optimistic attitude on the constraints. In ThSM-I, decision makers assumed that the constrict ratios for all decision variables are identical, while such constricting ratios varied in ThSM-II. The other possible scenarios, as well as the related applications, will also be presented in the following sections.

2. Alternative Solution Methods

According to the above analysis, TSM presents the combination of pursuing aggressive objective value and employing optimistic constraints. In real-world cases, it is possible that decision makers are aggressive to obtain the lowest system cost. However, they may be pessimistic on the constraints considering the practical situations. For example, in a municipal solid waste (MSW) management system, waste flows delivered to disposal facilities should not exceed their maximum capacities. Although the available capacity of a facility within a range which can be presented as an interval, decision makers may be pessimistic on the actually capacity with their knowledge of overloading operation, outdated maintenance efforts and so on. In order to address such a scenario, solution methods of SOM2-I and SOM2-II will be developed. The specific steps are shown as follows.

SOM2-I and SOM2-II

Step 1. Solve the following submodels:

$$\text{Min} \;\; f^{-} = \sum_{j=1}^{k} c_j^{-} x_j^{-} + \sum_{j=k+1}^{n} c_j^{-} x_j^{+} \quad (4a)$$

subject to:

$$\sum_{j=1}^{k}\left|a_{ij}^{\pm}\right|^{+} Sign(a_{ij}^{\pm})x_j^{-} + \sum_{j=k+1}^{n}\left|a_{ij}^{\pm}\right|^{-} Sign(a_{ij}^{\pm})x_j^{+} \le b_i^{-}, i = 1, 2, ..., m \quad (4b)$$

$$x_j^{-} \ge 0, \; j = 1, 2, ..., k \quad (4c)$$

$$x_j^{+} \ge 0, \; j = k+1, k+2, ..., n \quad (4d)$$

and:

$$\text{Min} \;\; f^{+} = \sum_{j=1}^{k} c_j^{+} x_j^{+} + \sum_{j=k+1}^{n} c_j^{+} x_j^{-} \quad (5a)$$

Subject to:

$$\sum_{j=1}^{k}\left|a_{ij}^{\pm}\right|^{-}Sign(a_{ij}^{\pm})x_j^{+} + \sum_{j=k+1}^{n}\left|a_{ij}^{\pm}\right|^{+}Sign(a_{ij}^{\pm})x_j^{-} \le b_i^{+}, i=1,2,...,m \quad (5b)$$

$$x_j^{+} \ge x_{j\,opt}^{-},\ j=1,\ 2,\ ...,\ k \quad (5c)$$

$$0 \le x_j^{-} \le x_{j\,opt}^{+},\ j=k+1,\ k+2,\ ...,\ n \quad (5d)$$

where $x_{jopt}^{-}(j=1, 2, ..., k)$ and $x_{jopt}^{+}(j=k+1, k+2, ..., n)$ are solutions of model (4). Thus, interval solutions of $f_{opt}^{\pm}=[f_{opt}^{-}, f_{opt}^{+}]$ and $x_{jopt}^{\pm}=[x_{jopt}^{-}, x_{jopt}^{+}]$ can be obtained through solving models (4) and (5).

Step 2. Conduct feasibility test. If pass, then stop. Otherwise, go to step 3.

Step 3. Constricting algorithm. For SOM2-I, assume that all constricting ratios are identical (i.e. q), and the objective is to maximize q. In SOM2-II, the constricting ratios are varied (i.e. q_1, q_2,…), and the objective is to maximize the product of all the nonnegative constricting ratios ($q_1 \times q_2 \times ... \times q_{n_0}$).

Compared with ThSM-I and II, in the first step of SOM2-I and II, the value of b_i^{-} is employed in the first submodel instead of b_i^{+}, while the second submodel would take the value of b_i^{+}. Step 2 in SOM2-I and II is identical to that of ThSM-I and II. Step 3 in SOM2-I is identical with that in ThSM-I, while SOM2-II and ThSM-II also have identical in step 3.

To address scenarios that the decision makers are conservative, as well as the combinations with optimistic and pessimistic attitudes on the constraints, solution methods of SOM3-I, SOM3-II, SOM4-I and SOM4-II are developed as follows:

SOM3-I and SOM3-II

Step 1. Solve submodels (6) and (7):

$$\text{Min}\ \ f^{+}=\sum_{j=1}^{k}c_j^{+}x_j^{+} + \sum_{j=k+1}^{n}c_j^{+}x_j^{-} \quad (6a)$$

Subject to:

$$\sum_{j=1}^{k}\left|a_{ij}^{\pm}\right|^{-}Sign(a_{ij}^{\pm})x_j^{+} + \sum_{j=k+1}^{n}\left|a_{ij}^{\pm}\right|^{+}Sign(a_{ij}^{\pm})x_j^{-} \le b_i^{+}, i=1,2,...,m \quad (6b)$$

$$x_j^{+} \ge 0,\quad j=1,2,...,k \quad (6c)$$

$$x_j^{-} \ge 0,\ \ j=k+1,k+2,...,n \quad (6d)$$

and:

$$\text{Min}\ \ f^{-}=\sum_{j=1}^{k}c_j^{-}x_j^{-} + \sum_{j=k+1}^{n}c_j^{-}x_j^{+} \quad (7a)$$

subject to:

$$\sum_{j=1}^{k}\left|a_{ij}^{\pm}\right|^{+}Sign(a_{ij}^{\pm})x_j^{-} + \sum_{j=k+1}^{n}\left|a_{ij}^{\pm}\right|^{-}Sign(a_{ij}^{\pm})x_j^{+} \le b_i^{-}, i=1,2,...,m \quad (7b)$$

$$0 \le x_j^{-} \le x_{j\,opt}^{+},\ j=1,\ 2,\ ...,\ k \quad (7c)$$

$$x_j^{+} \ge x_{j\,opt}^{-},\ j=k+1,\ k+2,\ ...,\ n \quad (7d)$$

where $x_{jopt}^{-}(j=1,\ 2,\ ...,\ k)$ and $x_{jopt}^{+}(j=k+1,\ k+2,\ ...,\ n)$ are solutions of model (6). Thus, interval solutions of $f_{opt}^{\pm}=[f_{opt}^{-}, f_{opt}^{+}]$ and $x_{jopt}^{\pm}=[x_{jopt}^{-}, x_{jopt}^{+}]$ can be obtained through solving models (6) and (7).

Step 2. Feasibility test which is the same as that in ThSM-I and II.

Step 3. Constricting methods for SOM3-I and II, which are identical to ThSM-I and II, respectively.

SOM4-I and SOM4-II

Step 1. Solve submodels (8) and (9):

$$\text{Min}\ \ f^{+}=\sum_{j=1}^{k}c_j^{+}x_j^{+} + \sum_{j=k+1}^{n}c_j^{+}x_j^{-} \quad (8a)$$

Subject to:

$$\sum_{j=1}^{k}\left|a_{ij}^{\pm}\right|^{-}Sign(a_{ij}^{\pm})x_j^{+} + \sum_{j=k+1}^{n}\left|a_{ij}^{\pm}\right|^{+}Sign(a_{ij}^{\pm})x_j^{-} \le b_i^{-}, i=1,2,...,m \quad (8b)$$

$$x_j^{+} \ge 0,\ j=1,2,...,k \quad (8c)$$

$$x_j^{-} \ge 0,\ \ j=k+1,k+2,...,n \quad (8d)$$

and:

$$\text{Min}\ \ f^{-}=\sum_{j=1}^{k}c_j^{-}x_j^{-} + \sum_{j=k+1}^{n}c_j^{-}x_j^{+} \quad (9a)$$

subject to:

$$\sum_{j=1}^{k}\left|a_{ij}^{\pm}\right|^{+}Sign(a_{ij}^{\pm})x_j^{-} + \sum_{j=k+1}^{n}\left|a_{ij}^{\pm}\right|^{-}Sign(a_{ij}^{\pm})x_j^{+} \le b_i^{+}, i=1,2,...,m \quad (9b)$$

$$0 \le x_j^{-} \le x_{j\,opt}^{+},\ j=1,\ 2,\ ...,\ k \quad (9c)$$

$$x_j^{+} \ge x_{j\,opt}^{-},\ j=k+1,\ k+2,\ ...,\ n \quad (9d)$$

where $x_{jopt}^{-}(j=1,\ 2,\ ...,\ k)$ and $x_{jopt}^{+}(j=k+1,\ k+2,\ ...,\ n)$ are solutions of model (8). Thus, interval solutions of $f_{opt}^{\pm}=[f_{opt}^{-}, f_{opt}^{+}]$ and $x_{jopt}^{\pm}=[x_{jopt}^{-}, x_{jopt}^{+}]$ can be obtained through solving models (8) and (9).

Step 2. Feasibility test which is the same as that in ThSM-I and II.

Step 3. Constricting methods for SOM3-I and II, which are identical to ThSM-I and II, respectively.

In step 1 of SOM3-I and II as well as SOM4-I and II, the upper bound of the objective function (f^{+}) is obtained from the first submodel. SOM3-I and II take the value of b_i^{+} firstly. It indicates the combination of pursuing conservative objecttive and holding optimistic constraints. As for SOM4-I and II, since the value of b_i^- is used in the first submodel, it suggests that decision makers hold a pessimistic attitude on the constraints.

Besides, the decision makers may regard both the lower and upper bounds of the objective function with importance. They maybe concerned about both bounds of the objective instead of one. In this situation, the decision makers show neutral attitude on the objective without preference of lower or upper bound. To address such a scenario, the mid-values of all coefficients ($c_j^\pm$, $a_{ij}^\pm$, and $b_i^\pm$) can be employed to formulate a new linear program:

$$\text{Min} \quad f_m = \sum_{j=1}^{k} c_{jm} x_{jm} + \sum_{j=k+1}^{n} c_{jm} x_{jm} \tag{10a}$$

Subject to:

$$\sum_{j=1}^{k} a_{ijm} x_{jm} + \sum_{j=k+1}^{n} a_{ijm} x_{jm} \le b_{im}, \quad i = 1, 2, ..., m \tag{10b}$$

$$x_{jm} \ge 0, \quad j = 1, 2, ..., n \tag{10c}$$

where c_{jm}, a_{ijm}, b_{im} are the mid-values of $c_j^\pm$, $a_{ij}^\pm$ and $b_i^\pm$, respecttively. Solutions of x_{jm} ($j = 1, 2, \ldots, n$) can be obtained through model (10). Based on the assumption that $x_j^- \le x_{jm} \le x_j^+$, submodels corresponding to the lower and upper bounds of objective function (i.e. f^- and f^+) can be further formulated. Since the values of x_{jm} ($j = 1, 2, ..., n$) could be taken as new constraints, submodels for solving f^- and f^+ would be in parallel. In other words, the order of the submodles has no impact on the solutions. The solution methods of SOM5-I, SOM5-II, SOM6-I and SOM6-II are thus further developed considering different combinations of $f^\pm$ and $b_i^\pm$.

SOM5-I and SOM5-II

Step 1. Solve the model with mid-values of the coefficients [model (10)].

Step 2. Solve the following submodels:

$$\text{Min} \quad f^- = \sum_{j=1}^{k} c_j^- x_j^- + \sum_{j=k+1}^{n} c_j^- x_j^+ \tag{11a}$$

subject to:

$$\sum_{j=1}^{k} |a_{ij}^\pm|^+ Sign(a_{ij}^\pm) x_j^- + \sum_{j=k+1}^{n} |a_{ij}^\pm|^- Sign(a_{ij}^\pm) x_j^+ \le b_i^+, i = 1, 2, ..., m \tag{11b}$$

$$x_j^- \le x_{jm} \quad j = 1, 2, ..., k \tag{11c}$$

$$x_j^+ \ge x_{jm} \quad j = k+1, k+2, ..., n \tag{11d}$$

and:

$$\text{Min} \quad f^+ = \sum_{j=1}^{k} c_j^+ x_j^+ + \sum_{j=k+1}^{n} c_j^+ x_j^- \tag{12a}$$

Subject to:

$$\sum_{j=1}^{k} |a_{ij}^\pm|^- Sign(a_{ij}^\pm) x_j^+ + \sum_{j=k+1}^{n} |a_{ij}^\pm|^+ Sign(a_{ij}^\pm) x_j^- \le b_i^-, i = 1, 2, ..., m \tag{12b}$$

$$x_j^+ \ge x_{jm}, \ j = 1, \ 2, \ ..., \ k. \tag{12c}$$

$$x_j^- \le x_{jm}, \ j = k+1, \ k+2, \ ..., \ n. \tag{12d}$$

Solutions of $x_{jopt}^-(j=1, \ 2, \ ..., \ k)$ and $x_{jopt}^+(j=k+1, \ k+2, \ ..., \ n)$ can be obtained through solving model (11) while solutions of $x_{jopt}^+(j=1, \ 2, \ ..., \ k)$ and $x_{jopt}^-(j=k+1, \ k+2, \ ..., \ n)$ can be obtained through solving model (14). Thus, the final solution of $f_{opt}^\pm = [f_{opt}^-, \ f_{opt}^+]$ and $x_{jopt}^\pm = [x_{jopt}^-, \ x_{jopt}^+]$ can be obtained.

Step 3. Feasibility test which is the same as that in ThSM-I and II.

Step 4. Constricting methods for SOM3-I and II, which are identical to ThSM-I and II, respectively.

SOM6-I and SOM6-II

Step 1. Solve the model with mid-values of the coefficients [model (10)].

Step 2. Solve the following submodels:

$$\text{Min} \quad f^- = \sum_{j=1}^{k} c_j^- x_j^- + \sum_{j=k+1}^{n} c_j^- x_j^+ \tag{13a}$$

subject to:

$$\sum_{j=1}^{k} |a_{ij}^\pm|^+ Sign(a_{ij}^\pm) x_j^- + \sum_{j=k+1}^{n} |a_{ij}^\pm|^- Sign(a_{ij}^\pm) x_j^+ \le b_i^-, i = 1, 2, ..., m \tag{13b}$$

$$x_j^- \le x_{jm} \quad j = 1, 2, ..., k. \tag{13c}$$

$$x_j^+ \ge x_{jm} \quad j = k+1, k+2, ..., n. \tag{13d}$$

and:

$$\text{Min} \quad f^+ = \sum_{j=1}^{k} c_j^+ x_j^+ + \sum_{j=k+1}^{n} c_j^+ x_j^-, \tag{14a}$$

Subject to:

$$\sum_{j=1}^{k} |a_{ij}^\pm|^- Sign(a_{ij}^\pm) x_j^+ + \sum_{j=k+1}^{n} |a_{ij}^\pm|^+ Sign(a_{ij}^\pm) x_j^- \le b_i^+, i = 1, 2, ..., m \tag{14b}$$

$$x_j^+ \ge x_{jm}, \ j = 1, \ 2, \ ..., \ k. \tag{14c}$$

$$x_j^- \le x_{jm}, \ j = k+1, \ k+2, \ ..., \ n. \tag{14d}$$

Solutions of $x_{jopt}^-(j=1, \ 2, \ ..., \ k)$ and $x_{jopt}^+(j=k+1, \ k+2, \ ..., \ n)$ can be obtained through solving model (13) while solutions of $x_{jopt}^+(j=1, \ 2, \ ..., \ k)$ and $x_{jopt}^-(j=k+1, \ k+2, ..., \ n)$ can be obtained through solving model (14). Thus, the final solution of $f_{opt}^\pm = [f_{opt}^-, \ f_{opt}^+]$ and $x_{jopt}^\pm = [x_{jopt}^-, \ x_{jopt}^+]$ can be obtained.

In SOM5-I and II, the value of b_i^+ is taken when solving f^-, while the value of b_i^- is taken in SOM6-I and II. It is con-

Table 1. Twelve Scenario-Based Solution Methods for ILP

Solution method	Objective	Constraints	Constricting ratio
ThSM-I	aggressive	optimistic	consistent
ThSM-II	aggressive	optimistic	varied
SOM2-I	aggressive	pessimistic	consistent
SOM2-II	aggressive	pessimistic	varied
SOM3-I	conservative	optimistic	consistent
SOM3-II	conservative	optimistic	varied
SOM4-I	conservative	pessimistic	consistent
SOM-II	conservative	pessimistic	varied
SOM5-I	neutral	optimistic	consistent
SOM5-II	neutral	optimistic	varied
SOM6-I	neutral	pessimistic	consistent
SOM6-II	neutral	pessimistic	varied

sidered that SOM5-I and II represent an optimistic attitude since the values of f^- obtained through SOM5-I and II are smaller than those through SOM6-I and II. It must be pointed that the criteria of optimistic level used in SOM5-I to SOM6-II are different from those used in ThSM-I to SOM4-II. If the first submodel takes the value of b_i^+ in ThSM-I to SOM-II, the corresponding solution method is considered as optimistic. However, in the solution methods of SOM5-I to SOM6-II, the order of two submodels in step 1 can be changed. Thus, if the submodel corresponding to f^- takes the value of b_i^+ (SOM5-I and II), it is considered optimistic.

In step 1 of SOM6-I and II, the value of b_i^- is used in the submodel of solving f^-; thus this suggests that the decision makers hold a pessimistic attitude. In general, twelve solution methods are developed for the ILP problem as shown in Table 1.

3. Numerical Example

A simplified example of an ILP problem is presented as follows:

$$\text{Max } f^{\pm} = [2, 2.4]x_1^{\pm} - [1, 1.3]x_2^{\pm} + [1.5, 1.8]x_3^{\pm} \quad (15a)$$

$$(\text{Min } f^{\pm} = [-2.4, -2]x_1^{\pm} + [1, 1.3]x_2^{\pm} + [-1.8, -1.5]x_3^{\pm})$$

subject to:

$$[2.6, 3.5]x_1^{\pm} + [2, 2.4]x_2^{\pm} + [3.2, 3.8]x_3^{\pm} \le [18, 22] \quad (15b)$$

$$[4.6, 5.5]x_1^{\pm} + [3, 3.6]x_2^{\pm} - [1.3, 1.6]x_3^{\pm} \le [8, 9] \quad (15c)$$

$$[1, 1.3]x_1^{\pm} - [6, 6.5]x_2^{\pm} + [2, 2.5]x_3^{\pm} \le [2.2, 2.6] \quad (15d)$$

$$x_1^{\pm}, x_2^{\pm}, x_3^{\pm} \ge 0 \quad (15e)$$

The results of step 1 for ThSM-I to SOM4-II, as well as those obtained through steps 1 and 2 for SOM5-I to SOM6-II, are presented in Table 2. The final solutions through the twelve solution methods are shown in Table 3. Compare Tables 2 and 3, it indicates that solutions obtained in step 1 of ThSM-I and II and SOM5-I and II fail in the feasibility test, thus we will move to step 3 of constricting algorithm. The results of step 1 from the other solution methods all pass the feasibility test.

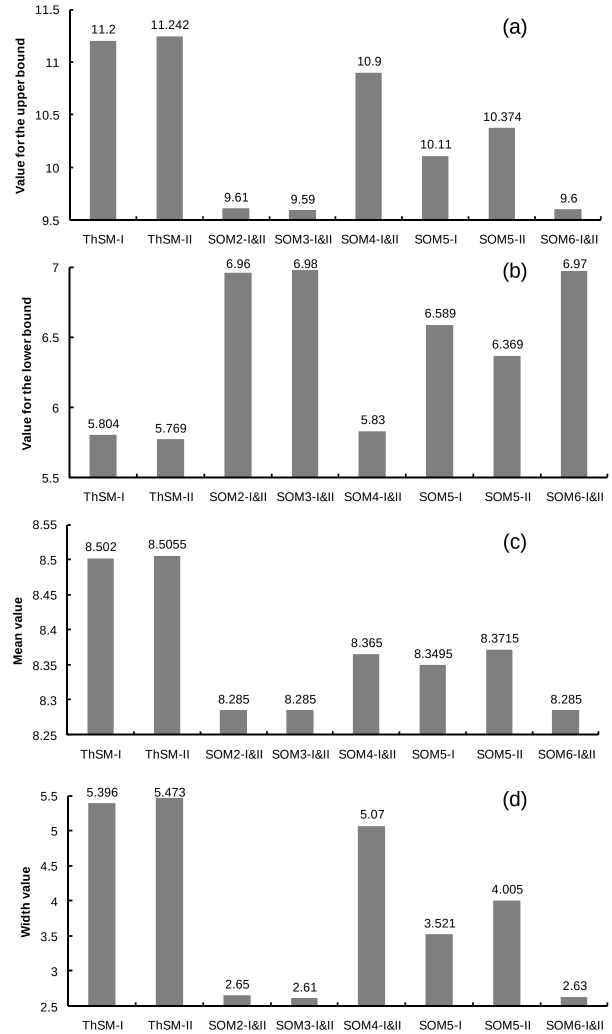

Figure 1. Solutions of upper bound, lower bound, mean and width values of objective function from developed methods.

Since the object is to achieving the maximum value, solution methods with the first submodel containing f^+ are considered aggressive. As for SOM5-I to SOM6-II, the first submodel is to solve the linear program with mid-value coefficients. The submodels of solving f^+ and f^- are independent; thus the order of solving submodels 2 and 3 does not matter. For model (15), solution methods with submodel corresponding to f^+ and b^+ represent an aggressive attitude. Similarly, if combinations of f^- and b^+ are adopted, the related solution methods are considered conservative.

Considering the upper bound of the objective function (f^+), its values obtained through the developed solution methods in a descending order should be ThSM-II > ThSM-I > SOM4-I and II > SOM5-II > SOM5-I > SOM2-I and II > SOM6-I and II > SOM3-I and II, as shown in Figure 1(a). Thus

Table 2. Results of Step 1 or 2

Solution method	$x_1^{\pm}$	$x_2^{\pm}$	$x_3^{\pm}$	$f^{\pm}$
ThSM-I and II	[1.56, 2.18]	1.22	[2.66, 4.18]	[5.51, 11.55]
SOM2-I and II	[1.86, 1.91]	[0.98, 1.36]	3.33	[6.96, 9.61]
SOM3-I and II	[1.87, 1.89]	[0.98, 1.37]	3.35	[6.98, 9.59]
SOM4-I and II	[1.63, 2.17]	1.09	[2.66, 3.77]	[5.83, 10.9]
Mid-value	1.88	1.17	3.34	8.31
SOM5-I and II	[1.59, 2.17]	1.17	[2.66, 4]	[5.65, 11.25]
SOM6-I and II	[1.87, 1.9]	[0.98, 1.36]	3.34	[6.97, 9.6]

Table 3. Solutions for the Numerical Example

Solution method	$x_1^{\pm}$	$x_2^{\pm}$	$x_3^{\pm}$	$f^{\pm}$	(q_1, q_2, q_3)
ThSM-I	[1.61, 2.13]	1.22	[2.78, 4.06]	[5.804, 11.2]	(0.84, 0, 0.84)
ThSM-II	[1.63, 2.11]	1.22	[2.73, 4.11]	[5.769, 11.242]	(0.77, 0, 0.91)
SOM2-I and II	[1.86, 1.91]	[0.98, 1.36]	3.33	[6.96, 9.61]	--
SOM3-I and II	[1.87, 1.89]	[0.98, 1.37]	3.35	[6.98, 9.59]	--
SOM4-I and II	[1.63, 2.17]	1.09	[2.66, 3.77]	[5.83, 10.9]	--
SOM5-I	[1.76, 2]	1.17	[3.06, 3.6]	[6.589, 10.11]	(0.41, 0, 0.41)
SOM5-II	[1.59, 2.17]	1.17	[3.14, 3.52]	[6.369, 10.374]	(1, 0, 0.28)
SOM6-I and II	[1.87, 1.9]	[0.98, 1.36]	3.34	[6.97, 9.6]	--

Thus, ThSM-II could be an appropriate solution method if the decision makers desire to get aggressive results. Mean- while, when the lower bound of the objective function value is considered, the f^- values from these scenario- based solution methods in a descending order should be SOM3-I and II > SOM6-I and II > SOM2-I and II > SOM5-I > SOM5-II > SOM4-I and II > ThSM-I >ThSM-II, as shown in Figure 1(b). In accordance, SOM3-I would be the preferred solution method if the decision makers are conservative and concerned much about the lower bound of the objective function.

It is interesting that the ranking of solution methods with f^+ being considered is totally opposite of that with f^- being concerned. The developed solution methods are based on different scenarios, with each owning both advantages and disadvantages. It is meaningless to say that one solution method is better than another one. According to different criteria or standards, different ranking results for the solution methods can be acquired. For example, if the decision makers consider the means of f^+ and f^- be the most important element, solutions obtained through ThSM-II should be adopted. This is because values for the means of f^+ and f^- obtained through ThSM-I to SOM6-II in a descending order are: ThSM-II > ThSM-I > SOM5-II > SOM4-I and II > SOM5-I > SOM2-I and II = SOM3-I and II = SOM6-I and II. Figure 1(c) shows the results for the means of f^+ and f^-. Besides, the decision makers may consider the range between f^+ and f^- as a critical point. Accordingly, solutions of ThSM-II would be adopted if the decision makers prefer large values for the ranges. If small values for the range are desired, results of SOM3-I and II could be used, as shown in Figure 1(d).

Then the constricting algorithm would be discussed. In ThSM-I and SOM5-I, the constricting ratios are assumed to be identical; in comparison, in ThSM-II and SOM5-II, it is assumed that each decision variable has its own constricting ratio. Results in Table 3 indicates that ThSM-II could get larger value for f^+ and smaller value for f^-, compared with ThSM-I. Similar characteristics exist in the solutions of SOM5-I and II. Thus, if the decision makers prefer large values for f^+ or the range of f^+ and f^-, ThSM-II and SOM5-II should be adopted. Otherwise, the solution methods of ThSM-I and SOM5-I could be used, since they hold an advantage that linearity is guaranteed in the solution process.

In general, different concerns and attitudes of decision makers can lead to different scenarios. The results for the numerical example demonstrate that the twelve solution methods correspond to different scenarios. In real-world cases, desired scenarios and the related solution methods should be identified through analyzing the practical economic, social and political situations. Moreover, to evaluate the developed methods, the criteria or standards must be determined first. Without them, the preferential order of the solution methods means nothing.

4. Case Study

A MSW management case is developed to demonstrate applicability of the above scenario-based solution methods. The study area is assumed to include three cities. An existing landfill and a waste-to-energy (WTE) facility are available to serve the waste disposal needs for the three cities. A planning horizon of 15 years is considered which is further divided into three periods with each having a time interval of 5 years. The cost and technical data used in this study are based on historical literature of solid waste management (Huang et al., 1992, 1995, 1998, 2001; Cheng et al., 2009; Liu et al., 2009; Cao et al., 2010b). Table 4 shows the waste generation rates in the three cities, the operation costs of the two facilities, and the transportation costs for shipping waste flows between the cities and the facilities in the three periods. The capacities of

Table 4. Waste Generation, Transportation, and Facility-Operation Costs

	Time period		
	k = 1	k = 2	k = 3
Waste generation rate, $WG_{jk}^{\pm}$ (t/d):			
City 1	[200,250]	[225,275]	[250,300]
City 2	[350,400]	[375,425]	[400,450]
City 3	[275,325]	[300,350]	[325,375]
Cost of transportation to landfill, $TR_{1jk}^{\pm}$ ($/t):			
City 1	[12.1,16.1]	[13.3,17.7]	[14.6,19.5]
City 2	[10.5,14.0]	[11.6,15.4]	[12.8,16.9]
City 3	[12.7,17.0]	[14.0,18.7]	[15.4,20.6]
WTE	[9,11]	[11,13]	[13,15]
Cost of transportation to WTE facility, $TR_{2jk}^{\pm}$ ($/t):			
City 1	[9.60,12.8]	[10.6,14.1]	[11.7,15.5]
City 2	[10.1,13.4]	[11.1,14.7]	[2.20,16.2]
City 3	[8.80,11.7]	[9.70,12.8]	[10.6,14.0]
Operation costs, $OP_{ik}^{\pm}$ ($/t):			
Landfill	[30,45]	[40,60]	[50,80]
WTE	[55,75]	[60,85]	[65,95]

the landfill and WTE are $[3.5, 4]\times 10^6$ t and [600, 700] t/d, respectively. The WTE facility generates residues of approximately 30% (on a mass basis) of the incoming waste flow. The benefit of WTE is approximately [15, 25] $/t combusted (Huang, 1994, 1998).

The problem under consideration is how to effectively allocate the waste flows under a number of environmental and waste disposal constraints in order to minimize the total system cost. An ILP model can thus be formulated. The decision variables represent waste flows from city *j* to waste disposal facility *i* in period *k*, denoted as x_{ijk} .The objective is to achieve minimum system costs through effectively allocating the waste flows from the three cities to the two waste disposal facilities, and the constraints involve the relationships between the decision variables and the waste generation/treatment conditions. Model (16) presents the formulated ILP model (Huang et al., 1992, 1995, 2001; Liu et al., 2009; Cao et al., 2010b; Xu et al., 2010):

$$\text{Min } f^{\pm} = 1825\sum_{j=1}^{3}\sum_{k=1}^{3}\{\sum_{i=1}^{2} x_{ijk}^{\pm}\left(TR_{ijk}^{\pm}+OP_{ik}^{\pm}\right)+x_{2jk}^{\pm}[FE\left(FT_k^{\pm}+OP_{ik}^{\pm}\right) -RE_k^{\pm}]\} \tag{16a}$$

subject to

$$1825\sum_{j=1}^{3}\sum_{k=1}^{3}\left(x_{ijk}^{\pm}+x_{2jk}^{\pm}FE^{\pm}\right)\le TL^{\pm} \tag{16b}$$

(Landfill capacity constraint)

$$\sum_{j=1}^{3} x_{2jk}^{\pm} \le \mathrm{TE}^{\pm}, \forall k \tag{16c}$$

(WTE facility capacity constraint)

$$\sum_{i=1}^{2} x_{ijk}^{\pm} = WG_{jk}^{\pm}, \forall j,k \tag{16d}$$

(Waste disposal demand constraint)

$$x_{ijk}^{\pm} \ge 0, \forall i,j,k \tag{16e}$$

(Non-negativity constraint)

where

- FE residue flow from the WTE facility to the landfill (% of incoming mass to WTE facility);
- $FT_k^{\pm}$ transportation costs of waste flow from the WTE facility to the landfill in period k ($/t);
- $OP_{ik}^{\pm}$ operating costs of facility *i* in period k ($/t);
- $RE_k^{\pm}$ revenue from the WTE facility in period k ($/t);
- $\mathrm{TE}^{\pm}$ maximum capacity of the WTE facility (t/d);
- $TL^{\pm}$ capacity of the landfill (t);
- $TR_{ijk}^{\pm}$ transportation costs from city j to facility i during period k ($/t);
- $WG_{jk}^{\pm}$ waste generation rate in city j to facility i during period k (t/d);
- $x_{ijk}^{\pm}$ waste flow rate from city j to facility i in period k (t/d), i = 1, 2; j = 1, 2, 3; k = 1, 2, 3;
- I index for facility (i=1 for the landfill, and i = 2 for the WTE facility);
- J index for the three cities (j = 1, 2, 3);
- K index for the time period (k = 1, 2, 3).

The overall system cost includes two parts. One part is the transportation cost of waste delivered to the landfill and WTE facility. The second part is the operation costs of the landfill and WTE facility. As for the WTE facility, its revenue should be subtracted, as shown in formula (16a). Constraint (16b) indicates that the total waste flow delivered to the landfill should be less than its capacity. The capacity constraint for the WTE is shown in (16c). The amount of disposed waste should be equal to that of the generated waste as shown in constraint (16d). The (16e) is non-negativity constraint which means that the waste flow from city *j* to disposal facility *i* in period *k* must be non-negative.

5. Results Analysis

Table 5 presents solutions based on the proposed solution methods. It indicates that no solution can be obtained through ThSM-I, ThSM-II, SOM5-I, or SOM5-II. ThSM-I and II stand for the scenarios that decision makers hold an aggressive attitude on objective and are optimistic in constraints, as shown in Table 1. The submodel corresponding the lower-bound system cost (f^-) and the upper-bound facility-capacity (b^+) is solved first, and the values for the lower-bound waste flow (x_{ijk}^-) can be obtained. Then the submodel corresponding the upper-bound system cost (f^+) could be formulated, where the lower-bound facility-capacity (b^-) is adopted in constraints, values for the lower-bound waste flow (x_{ijk}^-) obtained in the first submodel are incorporated as new constraints. As a result, the feasible region of the second submodel becomes an empty set, and no solution can be obtained. Since the decision makers are agg-

Table 5. Solutions Obtained Through ThSM-I to SOM6-II

Waste flow	SOM2-I and II	SOM3-I and II	SOM4-I and II	SOM6-I and II
$x_{111}^{\pm}$	[200, 250]	[200, 250]	14.73	[200, 250]
$x_{112}^{\pm}$	[0, 23.53]	[225, 275]	25	[225, 274.27]
$x_{113}^{\pm}$	0	0	75	0
$x_{121}^{\pm}$	[350, 400]	[350, 400]	[350, 400]	[350, 400]
$x_{122}^{\pm}$	[375, 425]	[375, 425]	[375, 425]	[375, 425]
$x_{123}^{\pm}$	[400, 425]	[400, 431.12]	[400, 450]	[400, 425]
$x_{131}^{\pm}$	257.58	0	0	6.850
$x_{132}^{\pm}$	0	0	0	0
$x_{133}^{\pm}$	0	0	0	0
$x_{211}^{\pm}$	0	0	[185.27, 235.27]	0
$x_{212}^{\pm}$	[225, 251.47]	0	[200, 250]	[0, 0.73]
$x_{213}^{\pm}$	[250, 300]	[250, 300]	[175, 225]	[250, 300]
$x_{221}^{\pm}$	0	0	0	0
$x_{222}^{\pm}$	0	0	0	0
$x_{223}^{\pm}$	[0, 25]	[0, 18.88]	0	[0, 25]
$x_{231}^{\pm}$	[17.42, 67.42]	[275, 325]	[275, 325]	[268.15, 318.15]
$x_{232}^{\pm}$	[300, 350]	[300, 350]	[300, 350]	[300, 350]
$x_{233}^{\pm}$	[325, 375]	[325, 375]	[325, 375]	[325, 375]
$f^{\pm}$($)	[295754973.2, 495914982.1]	[296895562.5, 495074401.8]	[307621562.5, 508769062.5]	[296673062.5, 495091321.4]

Note: ThSM-I and II as well as SOM5-I and II have no solutions

ressive and optimistic in the constraints when pursuing the lowest system cost, no values for the upper-bound waste flow (x_{ijk}^{+}) can be obtained. In other words, the aggressive and optimistic attitude of the decision makers may result in no solutions for the upper-bound system cost.

As for SOM5-I and II, the submodel formulated with mid-value coefficients should be solved first. Take the landfill as an example. The constraint of landfill capacity is always bound due to its low operation cost. In the solutions of the first submodel, the sum of all the waste flows [i.e. $(x_{1jk})_m$] delivered to the landfill is equal to 3.75×10^6 t (the mid-values of the landfill capacity). When solving for the upper-bound system cost, the upper bounds of all waste flows (x_{1jk}^{+}) delivered to the landfill should be obtained. In this submodel, the sum of x_{1jk}^{+} is required to be less than 3.5×10^6 t (the lower-bound landfill capacity). Obviously, values of x_{1jk}^{+} are no less than those of $(x_{1jk})_m$, which means that the sum of x_{1jk}^{+} would be no less than 3.75×10^6 t. Thus, the sum of x_{1jk}^{+} is required to be no less than 3.75×10^6 t and no more than 3.5×10^6 t, resulting in no solution for submodel with the upper-bound system cost. Basically, the result of no solution is due to the optimistic attitude of the decision makers. The upper-bound landfill capacity is used when solving the submodel of the lower-bound system cost since decision makers desire the lowest system cost. If they are not so optimistic, then with the 3.5×10^6 t landfill capacity when solving for f^- and 4×10^6 t landfill capacity when solving for f^+, the case of infeasibility may be avoided. In fact, SOM6-I and II represent these scenarios, and interval solutions are obtained as shown in Table 5.

According to all of the obtained solutions, the majority of the generated waste in city 2 would be transported to landfill during the planning horizon, because city 2 is close to the landfill and the operation cost of landfill is relatively low. As for city 3, most of the generated waste in the planning periods would be shipped to the WTE facility according to results obtained through SOM3-I and II, SOM4-I and II, and SOM6-I and II. However, solutions of SOM2-I and II indicate that most of the generated waste should be delivered to the landfill in period 1 and to the WTE facility in periods 2 and 3. The allocation of waste generated in city 1 varied according to the solutions obtained through different methods. Results of SOM2-I and II show that the waste flows should be delivered to the landfill in period 1 and to the WTE facility in periods 2 and 3. According to solutions of SOM3-I and II and SOM6-I and II, the waste flows would be shipped to the landfill in periods 1 and 2, and to the WTE facility in period 3. In addition, solutions of SOM4-I and II suggest that most of the waste flows would be transported to the WTE facility. Since the operation cost of WTE is higher than that of landfill, the total system costs from SOM4-I and II are the highest among the solutions from the developed solution methods.

Values obtained for f^- obey the following order: SOM2- I and II < SOM6-I and II < SOM3-I and II < SOM4-I and II, as presented in Figure 2. If achieving the lowest system cost is the most important consideration for the decision makers, SOM2-I and II should be adopted. However, Figure 2 also shows that the upper-bound system costs from SOM2-I and II are higher than those from SOM6-I and II and SOM3-I and II. Since the overall system cost could fluctuate within its lower and upper bounds, it is possible that the total costs take values of the upper bounds. In other words, when results of SOM2-I and II are used in the MSW management problem, it may achieve the lowest system cost. However, this also implies risks of attaining the value of f^+ (obtained from SOM2-I and

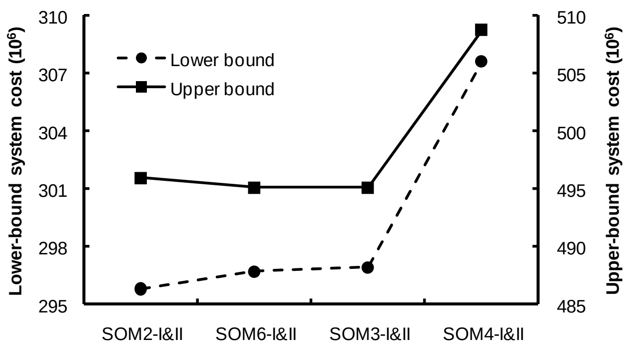

Figure 2. System cost obtained through the developed methods.

II) which is higher than that from SOM6-I and II, SOM3-I and II.

SOM4-I and II represent relatively conservative and pessimistic scenarios. Figure 2 shows that the lower and upper bounds of system costs obtained through SOM4-I and II are of the highest values. This is because the submodel corresponding to f^+ is solved firstly, and in this submodel the lower-bound facility capacities are adopted. Take the capacity constraint for WTE as an example. The sum of x_{2jk}^+ is required to be no more than 600 t/d in the first submodel. The sum of x_{2jk}^- must be no more than that of x_{2jk}^+, and thus the sum of x_{2jk}^- would be absolutely lower than 600 t/d. Meanwhile, in the second submodel which is corresponding to f^- and x_{ijk}^-, the sum of x_{2jk}^- is required to be no more than 700 t/d (upper-bound WTE capacity). The WTE-capacity constraint in the second submodel could be not bound. It indicates that the WTE facility may be keep idle when the waste generation rates are relatively low. In real-world cases, when SOM4-I and II are applied, it is possible that the waste disposal facilities are not operated with full loads. However, it could assure that all generated waste be disposed even when the waste generation rates of the 3 cities reach their upper bounds. In this sense, system reliability under this scenario is high. Also the total system cost is rather higher than those from the other solution methods.

Similarly, SOM6-I and II represent that the decision makers hold pessimistic attitude toward the constraints, being similar to SOM2-I and II and SOM4-I and II. The decision makers may show aggressive, neutral and conservative attitudes to the objecttive in SOM6-I and II, SOM2-I and II, and SOM4-I and II, respectively. As a result, the values of f^- and z f^+ in SOM6-I and II are between those of SOM2-I and II and those of SOM4-I and II. As for scenarios with optimistic attitudes of the constraints (i.e. SOM1-I and II, SOM3-I and II, and SOM5-I and II), only SOM3-I and II have feasible solutions, since the attitudes toward the objective represented in SOM1- I and II, SOM3-I and II, and SOM5-I and II are aggressive, conservative, and neutral, respectively.

The solutions under different scenarios can be used for helping decision makers to indentify desired management schemes. When the planning aims towards a lower system cost, results of SOM2-I and II can be used to generate desired management schemes. When the waste disposal requirement is considered as the most important target, SOM4-I and II may be expedient although the corresponding system cost would be high. In other words, when the decision makers choose aggressive and optimistic solution methods, low system cost may be obtained but with high system-failure risk (e.g. the waste disposal requirement may not be adequately met). Planning with a high system cost, on the other hand, could guarantee high system reliability. Therefore, these solutions could reflect tradeoffs between total cost and system reliability. The economic, social and political preferences could be incorporated when decision makers finalize a desired management strategy.

6. Discussions

Through the twelve solution methods, feasible ranges for decision variables have been obtained. The decision makers could choose the scenario that best fits the practical situations. The feasible schemes generated could be potentially adjusted through incorporation of implicit knowledge of decision makers, stakeholders, and particular economic, social and culture conditions. Moreover, post optimimality techniques like multi-criteria decision analysis (MCDA) could be used to help the decision makers to indentify the final scheme. MCDA is a technique for making preference decisions (e.g., evaluation, prioritisation and selection) on available alternatives in terms of multiple, usually conflicting, criteria (Zhang et al., 2004; Zhang et al., 2008, 2010). A combination of the developed scenario-based solution methods with MCDA could integrate unquantifiable knowledge (e.g. special social customs, and past experiences) into the decision process.

The assumption that the product ($a_{ij}^{\pm} x_j^{\pm}$) obeys specific probability distributions as mentioned in the first section should be met when the developed solution methods are adopted to solve ILP models. Different solution methods correspond to different scenarios which reflect different concerns and attitudes of the decision makers; however all the developed approaches are based on the aforementioned assumptions. If the assumed probability distributions cannot be satisfied, it is then risky to apply the developed methods. Investigation of the above issues in detail is important for making the modeling result applicable. Further studies can be focused on these subjects.

7. Conclusions

In this study, twelve methods corresponding to different system-condition scenarios have been developed for solving interval linear programming (ILP) problems. The scenarios correspond on different attitudes of decision makers to the study system. In detail, variations in concerns on objective function values (aggressive, conservative, or neutral), the attitude to the constraints (optimistic or pessimistic), and the preferred types of constricting ratios (consistent or varied) lead to a total of twelve scenarios. Consequently, twelve solution methods representing these scenarios have been developed. A numerical example has been presented to demonstrate applicability of the developed methods. Interval solutions have been obtained under each scenario. Compared with the previous studies, more information and options could be provided to the decision makers.

The developed methods have also been applied to a municipal solid waste management problem. The inherent mechanism (i.e. tradeoffs between system cost and system-failure risk) of the study system could be reflected through the obtained solutions under multiple scenarios. As a result, decision makers could understand the study system comprehensively and identify a desired scenario which is suitable to the practical situations. Moreover, a number of feasible schemes could be generated under each scenario which allows decision makers to further adjust the obtained solutions and indentify a desired one through incorporation of their experiences, economic situations, social and cultural conditions. In addition, the possibility of infeasible solutions has been greatly reduced with the consideration of twelve scenarios instead of one.

The accuracy and flexibility of the decision-making process could be enhanced since more information and schemes have been provided. The developed methods are also useful for dealing with other planning problems that can be formulated as ILP models. Integration of post optimality techniques into the scenario-based solution methods would be a subject for further studies.

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