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Management of Uncertain Information for Environmental Systems Using a Multistage Fuzzy-Stochastic Programming Model with Soft Constraints

Y. P. Li*, G. H. Huang, and W. Sun

MOE Key Laboratory of Regional Energy Systems Optimization, S-C Energy and Environmental Research Academy, North China Electric Power University, Beijing 102206, China

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ABSTRACT. In this study, a multistage fuzzy-stochastic programming model with soft constraints (FSPM-SC) is developed for managing environmental systems associated with uncertain information. The developed model can deal with uncertainties expressed as probability distributions and fuzzy sets; it can also reflect the dynamics in terms of decisions for waste-flow allocation and capacity expansion, through transactions at discrete points of a complete scenario set over a multistage context. The results indicate that solutions have been generated for binary and continuous decision variables under fuzzy and random information. They can be used for generating waste-flow-allocation pattern and facility-capacity-expansion scheme with a cost-effective manner. Sensitivity analyses are also conducted to demonstrate that the violation of waste-disposal-demand constraint has significant effect on reducing system cost.

Keywords: decision making, fuzzy programming, management, stochastic programming, uncertain information, environmental systems

1. Introduction

Municipal solid waste (MSW) management is crucial for environmental protection and public health in urban regions. Due to the waste management hierarchy, one of the greatest challenges that decision makers face is to figure out how to diversify the treatment options, increase the reliability of infrastructure systems, and leverage the redistribution of waste streams among landfilling, incineration, compost, recycling and other facilities (Chang and Davila, 2007). Systems analysis technique plays an important role for managing MSW in cost-effective and environmentally benign ways, and modeling results can provide decision makers with break-through insights and risk-informed strategies. Since the 1960s, a number of mathematical programming models have been developed for supporting decisions of MSW management and evaluating relevant operational and investment policies (Anderson and Nigam, 1968; Kirca and Erkip, 1988; Baetz, 1990; Frey et al., 2003; Lv et al., 2010; Sun and Huang, 2010).

In the MSW management, however, uncertainties often exist in the related costs, impact factors and objectives, which can affect the optimization processes and the decision schemes generated (Huang et al., 2001). These uncertainties may be further amplified by the complex features of the system components, as well as by their associations with economic implications and environmental concerns (Li et al., 2006). Fuzzy programming is capable of dealing with decision problems under fuzzy goal and constraints and handling ambiguous coefficients of objective function and constraints caused by imprecision and vagueness. Stochastic programming is an extension of mathematical programming to decision problems whose coefficients (input data) are not certainly known but could be represented as chances or probabilities. Previously, applications of fuzzy and stochastic programming methods to MSW management and planning were found in many literatures (Davila et al., 2005; Li et al., 2008; Xu et al., 2009). For example, Jaung et al. (1995) used fuzzy set theory to tackle decisions for siting landfills, where a procedure for systematic evaluation and ranking of prospective sites was provided. Wilson and Baetz (2001a, b) developed a derived probability model for curbside waste collection activities that allowed for analyzing stochastic information in the MSW management. Nie et al. (2007) proposed an interval-parameter fuzzy robust programming method for the planning of the solid waste management, where uncertain parameters represented as interval numbers and/or fuzzy membership functions could be effectively reflected. Chang et al. (2008) proposed a fuzzy multicriteria decision analysis alongside with a geospatial analysis for the selection of landfill sites. Li and Huang (2010) proposed an inexact scenario-based probabilistic programming method for supporting MSW management under uncertainties expressed as probability distributions and interval numbers over a multistage context. El Hanandeh and El-Zein (2010) proposed a stochastic integrated waste management model based on a life-cycle inventory approach, which allowed a systematic consideration of uncertainty.

* Corresponding author. Tel.: +86 10 61772977; fax: +86 10 61772982.
E-mail address: yongping.li33@gmail.com (Y. P. Li).

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In general, fuzzy programming considers uncertainties as fuzzy sets, and is effective in reflecting ambiguity and vagueness in resource availabilities; however, it has difficulties in dealing with uncertainties expressed as random variables. Stochastic programming can deal with various probabilistic uncertainties; however, the increased data requirements for specifying the parameters' probability distributions can affect their practical applicability (Li et al., 2009; Ping et al., 2010; Desharnais et al., 2011). In fact, in real-world decision problems, some parameters may present as fuzzy sets, while the others may be associated with probabilistic information; moreover, some system components contain not only randomness with probability distributions but also fuzziness in individual events with varied probability levels (i.e. randomness and fuzziness). If merely individual fuzzy or stochastic methods are employed under such complexities, robustness of the optimization results may be significantly influenced due to the problems of over-simplification or over-specification for uncertainties.

Therefore, one potential approach in response to tackling such uncertainties is to couple fuzzy programming with stochastic programming, leading to a multistage fuzzy-stochastic programming model with soft constraints (FSPM-SC). Then, the developed model will be applied to a case study of long-term MSW management and planning under varied violation levels for objective and constraints. This paper will be organized as follows: section 2 describes the development process of a multistage fuzzy-stochastic programming model with soft constraints (FSPM-SC); section 3 provides a case study of managing uncertain information in environmental systems through the developed model; section 4 presents result analysis and discussion, where a number of cases based on different violation levels are analyzed; section 5 draws some conclusions.

2. Methodology

Multistage stochastic programming with recourse (MSP) is developed as an extension of dynamic stochastic optimization methods to reflect the dynamic variations of system conditions, particularly for large-scale problems with a sequential structure. The uncertain information in the MSP is often modeled through a multi-layer scenario tree. When the planning horizon (τ) is finite, a MSP problem whose random elements have a discrete distribution with finite support can be formulated as (Rosa and Takriti, 1999):

$$\min_{x_1} \left\{ c_1^T x_1 - E_{\xi_2}\left[\min_{x_2}(c_2^T x_2 + \cdots + E_{\xi_\tau|\xi_2,\cdots,\xi_{\tau-1}} \min_{x_\tau} c_\tau^T x_T) \right] \right\} \quad (1a)$$

subject to:

$$\begin{array}{lllllll} A_1 x_1 & & & & = & b_1, & \\ T_2 x_1 & + & W_2 x_2 & & = & b_2, & \\ & & \cdots & & = & \vdots & (1b) \\ & & T_\tau x_{\tau-1} & + \; W_\tau x_\tau & = & b_\tau, & \\ x_1, & & x_2, \; \cdots, & x_\tau & \geq & 0. & \end{array}$$

where $\xi_t = (b_t,\ c_t,\ T_t,\ W_t),\ t = 2,\ 3, \cdots,\ \tau$ are random vectors of appropriate dimension. In model (1), decision variables are divided into two subsets: those (i.e. the first-stage decision variables) that have to be selected before the random information is revealed, whereas those (i.e. the recourse variables) that can be allowed to adapt to this information (i.e. after the realized random-variable values are available) (Ahmed et al., 2003; Li and Huang, 2010). The stages do not necessarily refer to time periods; they correspond to steps in the decision process (Dupačová et al., 2000). Assume that the probability associated with each realization of the random vector is known, the problem can be equivalently formulated as a linear program as follows:

$$\text{Min} \quad f = \sum_{t=1}^{T} C_t X_t + \sum_{t=1}^{T} \sum_{k=1}^{K_t} p_{tk} D_{tk} Y_{tk} \quad (2a)$$

subject to:

$$A_{rt} X_t \leq B_{rt}, \quad r = 1, 2, \cdots, m_1;\ t = 1, 2, \cdots, T \quad (2b)$$

$$A_{it} X_t + A'_{itk} Y_{tk} \leq \widehat{w_{itk}}\ ,\ i = 1,\ 2,\ \ldots,\ m_2;\ t = 1,\ 2,\ \ldots,\ T;\ k = 1,\ 2,\ \ldots,\ K_t \quad (2c)$$

$$x_{jt} \geq 0, \quad x_{jt} \in X_t,\ j = 1, 2, \cdots, n_1;\ t = 1,\ 2,\ \cdots,\ T \quad (2d)$$

$$y_{jtk} \geq 0,\ y_{jtk} \in Y_{tk}\ ,\ j = 1,\ 2, \ldots,\ n_2;\ t = 1,\ 2, \ldots,\ T;\ k = 1,2, \ldots,\ K_t \quad (2e)$$

where p_{tk} is probability of occurrence for scenario k in period t, with $p_{tk} > 0$; C_t are coefficients of first-stage variables (X_t) in the objective function; D_{tk} are coefficients of recourse variables (Y_{tk}) in the objective function; A_{rt} and A_{it} are coefficients of X_t in constraints r and i; A'_{itk} are coefficients of Y_{tk} in constraint i; $\widehat{w_{itk}}$ is random variable of constraint i, which is associated with probability level p_{tk}; K_t is the number of scenarios in period t.

Although the MSP model can deal with uncertainty presented as random variable with known probability distribution, vague information may exist in the objective function and the constraints. Fuzzy flexible programming is effective for dealing with decision problems under fuzzy goal and constraints, in which the flexibility in the constraints and fuzziness in the objective (i.e. presented by fuzzy sets and denoted as "fuzzy constraints" and "fuzzy goal" respectively) are introduced into conventional mathematical programming models (Zimmermann, 1996). In fact, a decision in a fuzzy environment can be defined as the intersection of membership functions corresponding to fuzzy objective and constraints (Chang et al., 1997; Li and Huang, 2007). Given a fuzzy goal (G) and a fuzzy constraint (C) in a space of alternatives (X), a fuzzy decision set (D) can then be formed in the intersection of G and C. In a symbolic form, $D = G \cap C$, and correspondingly:

$$\mu_D = \text{Min}\{\mu_G,\ \mu_C\} \quad (3)$$

where μ_D, μ_G and μ_C denote membership functions of fuzzy decision D, fuzzy goal G, and fuzzy constraint C, respectively. Letting $\mu_C(X)$ be membership functions of constraints C_i (i = 1, 2, …, m) and $\mu_G(X)$ be those of goals G_j (j = 1, 2, …, n), a decision can then be defined by the following membership functionn (Huang et al., 2001):

$$\mu_D(X) = \mu_{C_i}(X) * \mu_{G_j}(X) \tag{4}$$

where X represents a set of fuzzy decision variables; "*" denotes an appropriate and possibly context-dependent "aggregator". Then, consider a fuzzy linear programming (FLP) problem:

$$\text{Min } f \ \tilde{=} \ CX \tag{5a}$$

subject to:

$$AX \lesssim B \tag{5b}$$

$$X \ \geq \ 0 \tag{5c}$$

where A={a_{ij}} and $A \in R^{m\times n}$; B = {b_i} and $B \in R^{m\times 1}$; C = {c_i} and $C \in R^{1\times n}$; X = {x_i}; $X \in R^{n\times 1}$; R denote a set of real numbers; symbols $\tilde{=}$ and $\lesssim$ represent fuzzy equality and inequality. According to Zimmermann (1996), decision makers can establish an aspiration level (f') for the objective function value they desire to achieve, and each of the constraints can be modeled as a fuzzy set. Thus model (5) can be converted into:

$$CX \lesssim f' \tag{6a}$$

$$AX \lesssim B \tag{6b}$$

$$X \ \geq \ 0 \tag{6c}$$

Model (6) can be re-formulated as:

$$EX \lesssim B' \tag{7a}$$

$$X \ \geq \ 0 \tag{7b}$$

where $E = \begin{bmatrix} C \\ A \end{bmatrix}$ and $B' = \begin{bmatrix} f_1 \\ B \end{bmatrix}$.

Each of the $m+1$ rows in E and B' is represented by a fuzzy set with a membership function $\mu_i(X)$. Thus, the membership function of the fuzzy decision can be expressed as follows:

$$\mu_D(X) = \text{Min}\{\mu_i(X)\} \ \big| \ i = 1,\ 2,\ ...,\ m+1\} \tag{8}$$

where $\mu_i(X)$ can be interpreted as the degree to which X satisfies fuzzy inequality $E_i X \lesssim b_i'$ (where E_i denotes the ith row of E; b_i' denotes the ith row of B'). A desired decision is thus the one with the highest $\mu_D(X)$ value:

$$\text{Max } \mu_D(X) = \text{Max Min}[\mu_i(X)],\ X \geq 0 \tag{9}$$

where $\mu_i(X)$ would be 0 if the constraints (including the aspired objective) are violated, and 1 if they are totally satisfied. Assume that membership grades $\mu_i(X)$ are linearly increasing over the tolerance intervals (b_i' , $b_i' + p_i$), where p_i denote admissible violations of system objective and constraint i (i = 1, 2, …, m + 1). The $\mu_i(X)$ values can thus be calculated as follows:

$$\mu_i(X) = \begin{cases} 1, & \text{if } E_i X \leq b_i', \\ 1 - \dfrac{E_i X - b_i'}{p_i}, & \text{if } b_i' < E_i X \leq b_i' + p_i,\ i = 1,\ 2,\ \cdots,\ m+1 \\ 0, & \text{if } E_i X > b_i', \end{cases} \tag{10}$$

Then, equation (9) can be converted into:

$$\text{Max } \mu_D(X) = \text{Max Min}[1 - (E_i X - b_i') / p_i],\ X \geq 0 \tag{11}$$

Through introducing a new variable of $\lambda = \mu_D(X)$ (which corresponds to the membership function of the fuzzy decision), problem (11) can be equivalent to the following model (Negoita and Minoiu, 1976; Zimmermann, 1996):

$$\text{Max } \lambda \tag{12a}$$

subject to:

$$E_i X \leq \ b_i' + (1-\lambda) p_i,\ i = 1,\ 2,\ \cdots,\ m+1 \tag{12b}$$

$$X \ \geq \ 0 \tag{12c}$$

$$0 \ \leq \ \lambda \ \leq \ 1 \tag{12d}$$

where λ is the control variable corresponding to the degree (membership grade) of satisfaction for the fuzzy decision. A λ level close to 1 would correspond to a high possibility of satisfying the constraints/objective; conversely, a λ value near 0 would be related to a solution that has a low possibility of satisfying the constraints/objective. Model (12) can handle vague information corresponding to fuzzy objective and constraints. However, it has difficulties in tackling uncertainties expressed as random variables in a non-fuzzy decision space and in providing a linkage between the pre-regulated policies (i.e. policies which are first formulated before values of random variables are known) and the associated implications (i.e. recourse actions which are made after the random events have occurred). Therefore, one potential approach for handling such complexities is to integrate fuzzy programming and stochastic programming, leading to a multistage fuzzy-stochastic programming model with soft constraints (FSPM-SC) as follows:

$$\text{Max } \lambda \tag{13a}$$

subject to:

$$\sum_{t=1}^{T} C_t X_t + \sum_{t=1}^{T}\sum_{k=1}^{K_t} p_{tk} D_{tk} Y_{tk} \le f_s - \lambda \Delta V_f \tag{13b}$$

$$A_{rt} X_t \le B_{rt} + (1-\lambda)\Delta V_{B_{rt}}, \quad r = 1, 2, \cdots, m_1;\ t = 1, 2, \cdots, T \tag{13c}$$

$$A_{it} X_t + A'_{itk} Y_{tk} \le \widehat{w_{itk}} + (1-\lambda)\Delta V_{\widehat{w_{itk}}},\quad i = 1, 2, \ldots, m_2;\ t = 1, 2, \ldots, T;\ k = 1, 2, \ldots, K_t \tag{13d}$$

$$x_{jt} \ge 0, \quad x_{jt} \in X_t,\ j = 1, 2, \cdots, n_1;\ t = 1, 2, \cdots, T \tag{13e}$$

$$y_{jtk} \ge 0,\ y_{jtk} \in Y_{tk},\ j = 1, \ldots, n_2;\ t = 1, \ldots, T;\ k = 1, \ldots, K_t \tag{13f}$$

where f_s is the solution of the objective-function value from model (12); ΔV_f is violation variable for the objective function; $\Delta V_{B_{rt}}$ and $\Delta V_{\widehat{w_{itk}}}$ are violation variables for constraints (12c) and (12d), respectively. Obviously, with varied violation levels, solutions associated with different λ values will be generated, corresponding to different system costs and constraint-violation risks. They can help investigate the risk levels of violating the objective and constraints and thus generate desired decision alternatives.

3. Case Study

Consider a waste-management system wherein a manager is responsible for allocating waste flows from multiple districts of one city to multiple facilities within multiple periods; the waste treatment options include landfilling, recycling, incinerating and composting. When the waste generation has reached the limits of what the MSW management system can handle, more excess waste will become a major obstacle to economic development and environmental protection for one region. However, a variety of uncertainties exist in the system components such as waste generation amounts, cost and revenue data, waste-management-facility capacities, and waste diversion goals, which will bring significant difficulties to formulate waste management models and generation of effective solutions (Huang et al., 2005a, b). Uncertain waste-generation rates can be mainly attributed to population growth and migration; besides, underlying economic development, household size, employment variation, consumption-behave change, and waste-recycling impact would influence the solid waste generation interactively. Estimation of solid waste generation frequently counts on the demographic factors on a percapita basis, while the percapita coefficients may be taken as fixed over time or they may be projected to change with time (Dyson and Chang, 2005). Since the waste-generation rates are uncertain, it will be difficult for the manager to pre-regulate a deterministic waste-allocation level to each district from a long-term planning time perspective. If the pre-regulated waste level is not exceeded, it will result in a regular cost to the system. However, if it is exceeded, the surplus waste flow will have to be disposed with a higher cost, resulting in an excess cost (i.e. economic penalty expressed in terms of raised transportation and operation costs) to the system.

Generally, waste must be collected, transported and disposed in an environmentally and economically efficient manner, such that the effective planning of the associated waste-management activities is desired. Mathematical programming model is useful for analyzing the interactions and the related tradeoffs and thus providing bases for generating desired decision alternatives under the above complexities and uncertainties. Therefore, the study problem can be formulated as a multistage stochastic programming model (MSPM) as follows:

$$\begin{aligned}\text{Min } f = {} & \sum_{j=1}^{J}\sum_{t=1}^{T} L_t X_{1jt}(TR_{1jt} + OP_{1t}) + \sum_{i=2}^{I}\sum_{j=1}^{J}\sum_{t=1}^{T} L_t X_{ijt}[TR_{ijt} + OP_{it} + \\ & FE_i\,(FT_{it} + OP_{1t}) - RE_{it}] + \sum_{j=1}^{J}\sum_{t=1}^{T}\sum_{k=1}^{K_t^{(j)}} L_t p_{jtk} Y_{1jtk}(DR_{1jt} + DP_{1t}) + \\ & \sum_{i=2}^{I}\sum_{j=1}^{J}\sum_{t=1}^{T}\sum_{k=1}^{K_t^{(j)}} L_t p_{jtk} Y_{ijtk}[DR_{ijt} + DP_{it} + FE_i\,(DT_{it} + DP_{1t}) - RM_{it}] + \\ & \prod_{j=1}^{J} p_{jtk} \sum_{t=1}^{T}\sum_{k=1}^{K_t^{(j)}} (FLC_{1t} BV_{1tk} + VLC_{it} Z_{1tk}) + \sum_{i=2}^{I}\{\prod_{j=1}^{J} p_{jtk}\sum_{t=1}^{T}\sum_{k=1}^{K_t^{(j)}}(FTC_{it} BV_{itk} \\ & + VTC_{it} Z_{itk})\}\end{aligned} \tag{14a}$$

subject to:

$$\sum_{j=1}^{J}\sum_{t=1}^{t'} L_t\{(X_{1jt} + Y_{1jtk}) + \sum_{i=2}^{I} FE_i\,(X_{ijt} + Y_{ijtk})\} \le LC + \sum_{t=1}^{t'} Z_{1tk},\ t' = 1, \ldots, T;\ k = 1, 2, \cdots, K_t^{(j)} \tag{14b}$$

$$\sum_{j=1}^{J}(X_{ijt} + Y_{ijtk}) \le TC_i + \sum_{t=1}^{t'} Z_{itk},\ t' = 1, 2, \cdots, T;\ i = 2, 3, \cdots, I;\ k = 1, 2, \cdots, K_t^{(j)} \tag{14c}$$

$$\sum_{i=1}^{I}(X_{ijt} + Y_{ijtk}) \ge WG_{jtk}, \quad \forall j, t;\ k = 1, 2, \cdots, K_t^{(j)} \tag{14d}$$

$$\sum_{j=1}^{J}(X_{1jt} + Y_{1jtk}) \le DG_{1t}\sum_{j=1}^{J} WG_{jtk}, \quad \forall t; k = 1, 2, \cdots, K_t^{(j)} \tag{14e}$$

$$BV_{itk}\begin{cases} = 1, \text{ if capacity expansion is undertaken} \\ = 0, \text{ if otherwise}\end{cases},\ \forall i, t;\ k = 1, 2, \ldots, K_t^{(j)} \tag{14f}$$

$$\sum_{t=1}^{t'} BV_{1tk} \le 1,\ t' = 1, 2, \cdots, T;\ k = 1, 2, \cdots, K_t^{(j)} \tag{14g}$$

$$0 \le Z_{itk} \le N_{it} BV_{itk},\ \forall i, t;\ k = 1, 2, \cdots, K_t^{(j)} \tag{14h}$$

$$0 \le Y_{ijtk} \le X_{ijt}, \quad \forall i, j, t;\ k = 1, 2, \cdots, K_t^{(j)} \tag{14i}$$

The detailed nomenclatures for the variables and parameters are provided in the Appendix. In model (14), decision variables can be sorted into two categories: continuous and binary. The continuous variables represent pre-regulated waste flows, probabilistic surplus flows and expanded capacities, while the binary ones indicate whether individual facility-expansion actions need be undertaken. The objective is to minimize the expected net system cost through allocating waste flows to facilities over a multistage context. The objective value involves the cost for regular waste flows, the penalty for violating the pre-regulated targets, and the capital for facility-capacity expansion. The constraints can help define the interrelationships among the decision variables and the waste management conditions.

Model (14) can reflect uncertain information (in waste-generation amount) expressed as random variables with known probability distributions. However, in the study problem, uncertainties in waste amounts contain not only randomness with probability distributions but also fuzziness in individual events (of the realized inflows) with varied probability levels. For example, the capacity of WTE facility could hardly be identified as deterministic value due to its intrinsic fluctuations such as (a) variations in working hours, (b) requirements for system maintenance, (c) inconsistent manners among workers in operating the facility, and (d) occurrence of unplanned or accidental events and natural disasters; the capacity of WTE may have statements expressed as "possibly 130 to 150 tonne per day" when decision makers express different subjective judgments. Model (14) has difficulties in tackling uncertainties presented in terms of fuzzy sets. Therefore, to reflect uncertainties presented as fuzzy sets, the above problem can be reformulated as a multistage fuzzy-stochastic model with soft constraints (FSPM-SC) as follows:

$$\text{Max } \lambda \tag{15a}$$

subject to:

$$\sum_{j=1}^{J}\sum_{t=1}^{T} L_t X_{1jt}(TR_{1jt} + OP_{1t}) + \sum_{i=2}^{I}\sum_{j=1}^{J}\sum_{t=1}^{T} L_t X_{ijt}[TR_{ijt} + OP_{it} + FE_i\,(FT_{it} + OP_{1t}) - RE_{it}] + \sum_{j=1}^{J}\sum_{t=1}^{T}\sum_{k=1}^{K_t^{(j)}} L_t p_{jtk} Y_{1jtk}(DR_{1jt} + DP_{1t}) + \sum_{i=2}^{I}\sum_{j=1}^{J}\sum_{t=1}^{T}\sum_{k=1}^{K_t^{(j)}} L_t p_{jtk} Y_{ijtk}[DR_{ijt} + DP_{it} + FE_i\,(DT_{it} + DP_{1t}) - RM_{it}] + \prod_{j=1}^{J} p_{jtk} \sum_{t=1}^{T}\sum_{k=1}^{K_t^{(j)}} (FLC_{1t}BV_{1tk} + VLC_{it}Z_{1tk}) + \sum_{i=2}^{I}\{\prod_{j=1}^{J} p_{jtk} \sum_{t=1}^{T}\sum_{k=1}^{K_t^{(j)}} (FTC_{it}BV_{itk} + VTC_{it}Z_{itk})\} \le f_s^{*} - \lambda \Delta V_f \tag{15b}$$

$$\sum_{j=1}^{J}\sum_{t=1}^{t'} L_t\{(X_{1jt} + Y_{1jtk}) + \sum_{i=2}^{I} FE_i\,(X_{ijt} + Y_{ijtk})\} - \sum_{t=1}^{t'} Z_{1tk} \le LC^{(s)} + (1 - \lambda)\Delta V_{LC}, t' = 1,\ 2,\ \cdots,\ T;\ k = 1,\ 2,\ \cdots, K_t^{(j)} \tag{15c}$$

$$\sum_{j=1}^{J}(X_{ijt} + Y_{ijtk}) - \sum_{t=1}^{t'} Z_{itk} \le TC_i^{(s)} + (1-\lambda)\Delta V_{TC_i}, t' = 1,\ 2,\ \cdots,\ T;\ i = 2, 3,\ \cdots,\ I;\ k = 1,\ 2,\ \cdots, K_t^{(j)} \tag{15d}$$

$$\sum_{i=1}^{I}(X_{ijt} + Y_{ijtk}) \ge WG_{jtk}^{(s)} - \lambda \Delta V_{WG_{jtk}},\ \forall j, t;\ k = 1,\ \cdots, K_t^{(j)} \tag{15e}$$

$$\sum_{j=1}^{J}(X_{1jt} + Y_{1jtk}) \le DG_{1t}\sum_{j=1}^{J}[WG_{jtk}^{(s)} + (1\text{ - }\lambda)\Delta V_{WG_{jtk}}],\ \forall t; k = 1,\ 2,\ ..., K_t^{(j)} \tag{15f}$$

$$BV_{itk}\begin{cases} = 1,\ \text{if capacity expansion is undertaken} \\ = 0,\ \text{if otherwise} \end{cases},\ \forall i,t;\ k = 1, 2,\ ..., K_t^{(j)} \tag{15g}$$

$$\sum_{t=1}^{t'} BV_{1tk} \le 1,\ t' = 1,\ 2,\ \cdots,\ T;\ k = 1,\ 2,\ \cdots, K_t^{(j)} \tag{15h}$$

$$0 \le Z_{itk} \le N_{itk}BV_{itk},\ \forall i, t;\ \ k = 1,\ 2,\ \cdots, K_t^{(j)} \tag{15i}$$

$$0 \le Y_{ijtk} \le X_{ijt},\quad \forall i, j, t;\ k = 1,\ 2,\ \cdots, K_t^{(j)} \tag{15j}$$

where f_s is the solution of the system cost from model (14); ΔV_f is a violation variable for the objective function; ΔV_{LC} is violation variable for landfill's disposal capacity; ΔV_{TC_i} is violation variable for the capacity of waste-diversion facility i; $\Delta V_{WG_{jtk}}$ is violation variable for waste-generation amount in city j.

Table 1 presents the waste generation rates and the associated probabilities of occurrence during the planning periods. The study time horizon is 15 years, consisting of three 5-year periods. Table 2 presents the discounted fixed and variable costs for capacity expansion/development of the three facilities (i.e. landfill, incinerator and composting facility). Table 3 contains the normal cost for treating regular wastes as well as penalties for excess flows. It is indicated that the penalty is much higher than the regular cost. The surplus waste flow (i.e. when the projected waste level is exceeded) will be disposed of at a premium, resulting in a raised cost (penalty) to the system. The raised cost for excess waste is mainly due to more expensive labor and facility operation (for the collection and transportation of waste, and operation of the facility). In detail, such raised costs (economic penalties) are associated with: (i) increased collection cost for excess waste (i.e. longer time and more workers are required for collecting the surplus waste), (ii) increased transportation cost for shipping the excess waste to more remote facilities (when the capacities of local facilities are exhausted), (iii) increased operating costs for waste management facilities (i.e., extended working hours, more workers, and more expensive facilities), and (iv) extra expenses and/or fines caused by contingent events. Table 4 presents a number of violation levels on the objective function and constraints, which are normalized as percentages of fuzzy values. Four cases based on different violation levels for the different constraints are considered. For example, under case 1, violation levels for waste-generation rate, landfill capacity, incinerator capacity, and composting capacity are 2, 6, 4 and 4%, respectively. For each case, 10 violation levels for the objective function value are designed, where assume that decision makers expect a reduction of the system cost.

Table 1. Waste-Generation Rates

Level of waste generation	Probability	Waste-generation amount (t/d)		
		t = 1	t = 2	t = 3
Low (L)	0.2	410	440	475
Medium (M)	0.6	450	490	530
High (H)	0.2	500	550	585

Table 2. Costs for Facility-Capacity Expansion

	Planning period		
	t = 1	t = 2	t = 3
Fixed cost for facility development/expansion ($$10^6$)			
Landfill	3.09	2.94	2.79
Incinerator	5.42	5.15	4.89
Composting	4.51	4.28	4.07
Variable cost for facility development/expansion ($/t)			
Landfill	0.28	0.266	0.253
Incinerator	730.01	693.51	658.83
Composting	289.80	275.31	261.63

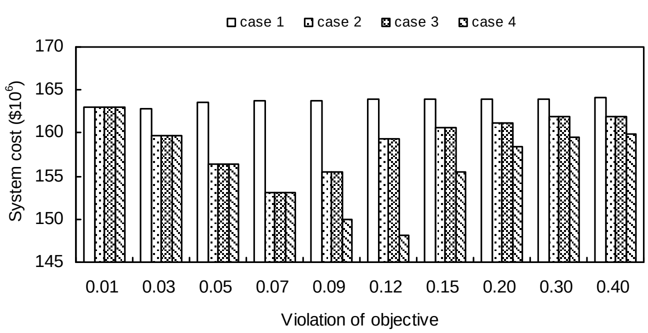

Figure 1. System costs under different cases.

4. Results and Discussion

In this study, four cases corresponding to different violation levels for the objective function and constraints were examined; they could help investigate the risks of violating the constraints and generate a range of decision alternatives. Figures 1 and 2 show the system costs and λ values under different violation levels (for the objective function and constraints). The results indicate that the system cost and λ value would vary with the violation level. The lowest system cost (i.e. $148.1 × 10^6) would be achieved under case 4 (when violation level for the objective function is 0.12), and the corresponding λ value would be 0.84. Moreover, the minimum disparity would exist under case 1 (range from $163.1 × 10^6 to $164.1 × 10^6), while the maximum disparity between the system cost and objective-violation level would exist under case 4 (range from $148.1 × 10^6 to $163.1 × 10^6); in comparison, the maximum disparity between the λ value and objective-violation level would exist under case 1, while the minimum disparity would exist under case 4. Under each case, the λ value would decrease as the violation level on the objective function is raised. This is because a high violation level corresponds to a reduced strictness for the objective constraint.

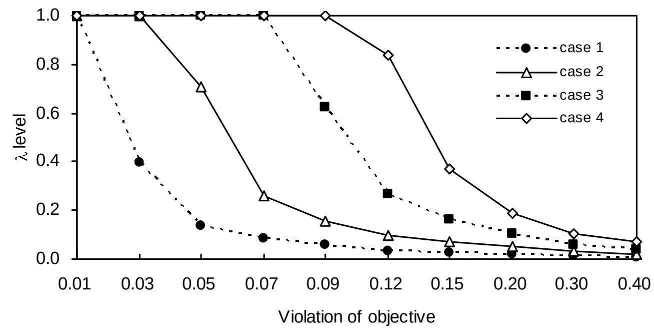

Figure 2. The λ levels under different cases.

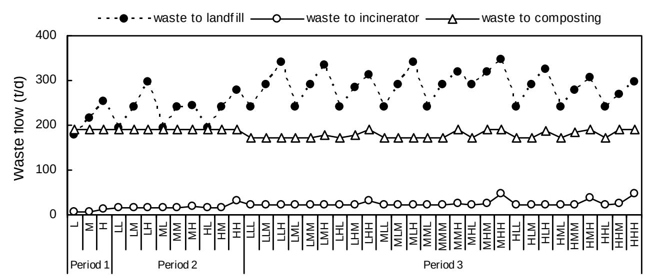

Figure 3. Waste-allocation pattern from FSPM-SC (with a minimum system cost).

Figure 3 provides the solutions for waste-allocation patterns obtained from the FSPM-SC with a minimum system cost; they include pre-regulated and excess waste flows from the city to the landfill, incinerator, and composting facility over the planning horizon. Excess waste could be generated if the pre-regulated waste flow is exceeded (i.e. excess waste = generated waste – optimal pre-regulated waste). Besides, a multilayer scenario tree with a branching structure of 1-3-3-3 was constructed for reflecting uncertainties (i.e. the scenario tree has one initial node at time 0 and 3 succeeding nodes in period 1; each node in period 1 corresponds to 3 succeeding ones in period 2, and so on for each node in period 3; these result in 27 nodes in period 3). The waste-allocation patterns would vary under different scenarios, due to the temporal and spatial variations of waste generation and management conditions. For example, when waste generation rates are low in all of the three periods, the total wastes shipped to the landfill would be 177.5 t/day in period 1,195.2 t/d in period 2, and 241.4 t/d in period 3; wastes treated at the incinerator would respectively be 5.9, 16.1 and 23.2 t/day in periods 1, 2 and 3; wastes to the composting facility would respectively be 192.2, 191.7 and 170.5 t/d in periods 1, 2 and 3 (where "t/d" is the abbreviate of "tonne/day"). In comparison, when waste generation rates are high over the planning horizon, the total wastes shipped to the landfill would respectively be 254.0, 279.4 and 297.2 t/d in periods 1 to 3; wastes trea-

Table 3. Transportation and Disposal Costs (unit: $/t)

	t = 1	t = 2	t = 3		t = 1	t = 2	t = 3
Normal cost for shipping allowable waste				Penalty for shipping excess waste			
To landfill	15.0	13.8	12.8	To landfill	22.4	21.1	19.1
To incinerator	11.2	10.6	9.6	To incinerator	17.9	16.9	15.3
To composting facility	14.1	13.3	12.0	To composting facility	21.7	20.5	18.5
Regular cost for treating allowable waste				Penalty for treating excess waste			
Landfill	37.5	35.4	32.0	Landfill	52.5	49.5	44.8
Incinerator	70.0	66.0	59.8	Incinerator	105.0	99.0	89.6
Composting facility	60.0	56.6	51.3	Composting facility	90.0	84.9	76.8
Regular cost for shipping allowable residue				Penalty for shipping excess residue			
Incinerator	5.5	5.2	4.7	Incinerator	8.3	7.9	7.1
Composting facility	10.3	9.7	8.8	Composting facility	15.4	14.5	13.2
Regular revenue generated by allowable waste				Excess revenue generated by excess waste			
Incinerator	20	18.8	17.1	Incinerator	20	18.8	17.1
Composting facility	12.5	11.8	10.7	Composting facility	12.5	11.8	10.7

Table 4. Violation Levels on System Objective and Constraints

Violation level (VL) for constraints (%):	Case 1	Case 2	Case 3	Case 4
Waste generation rate	2	4	7	10
Landfill capacity	6	8	13	20
Incinerator capacity	4	6	10	15
Composting capacity	4	6	10	15

Violation level (VL) for objective (%):									
1	3	5	7	9	12	15	20	30	40

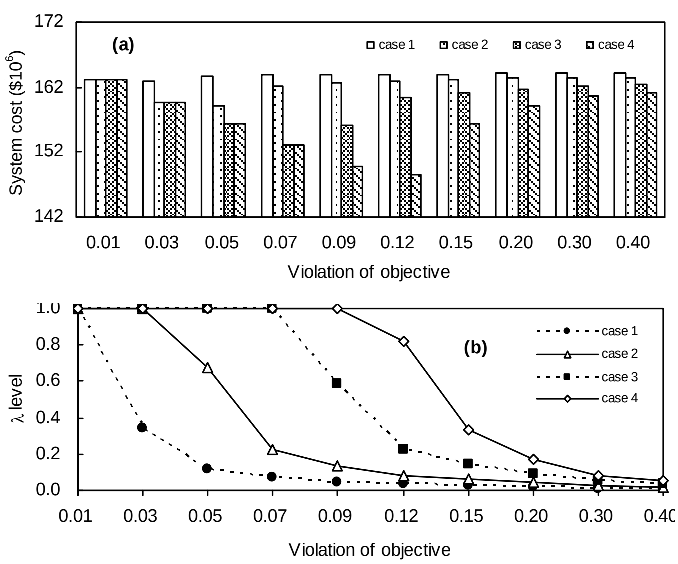

Figure 4. Variations of (a) system cost and (b) λ level with changed waste-generation rates.

ted at the incinerator would respectively be 11.8, 32.2 and 46.5 t/d in periods 1, 2 and 3; wastes to the composting facility would be same in the three periods (i.e. 192.2 t/d). The solutions under the other waste-generation scenarios can be similarly interpreted based on the results as shown in Figure 3.

In this study, the effects of violated waste-disposal-demand constraint on λ value and system cost were conducted (i.e. the violation level for facility-capacity constraint equal zero). Figure 4 shows the results for λ value and system cost under different cases. An increased violation level for waste-generation rate corresponds to decreased waste-disposal demand, excess waste, and expansion requirement, and thus leads to reduced normal cost, penalty and capital cost. Figure 5 presents the effects on system costs for changed waste-generation rates. The maximum differences would be 0.27, 0.37, 0.88 and 1.05 million dollars under cases 1 to 4, respectively. Moreover, the minimum system cost (associated with 0.82 of λ value) would be $148.6 $\times 10^6$ when violation level for the objective function is 0.12 (i.e. under case 4); in comparison, when constraints of waste-disposal demand and waste-disposal capacity are both violated, the minimum system cost would be $148.1 $\times 10^6$. This demonstrates that the violation of waste-disposal-demand constraint has significant effect on reducing system cost.

Figure 6 shows the comparison results of waste-allocation patterns from MSPM and FSPM-SC with a minimum cost. In MSPM, fuzzy information in waste-generation rates, waste-management-facility capacities, and waste-diversion goals were neglected (i.e. merely random uncertainties in the waste-generation amount were considered). In comparison, in FSPM-SC, a number of violation variables were introduced to soften the system constraints under fuzzy condition. The results indicate that the waste flow-allocation patterns obtained from MSPM and FSPM-SC would be different from each other, due to the temporal and spatial variations of waste-generation and management conditions (i.e. uncertain inputs). For example, when waste generation rates are medium in all of the three periods, the total wastes (obtained from MSPM) buried at the landfill would be 280, 250 and 285 t/d in periods 1, 2 and 3; wastes treated at the incinerator would be 60, 50 and 55 t/d in periods 1, 2 and 3; wastes allocated to the composting facility would be same over the planning periods (i.e.190 t/d). In comparison, the total wastes (obtained from FSPM-SC) allocated to the land-

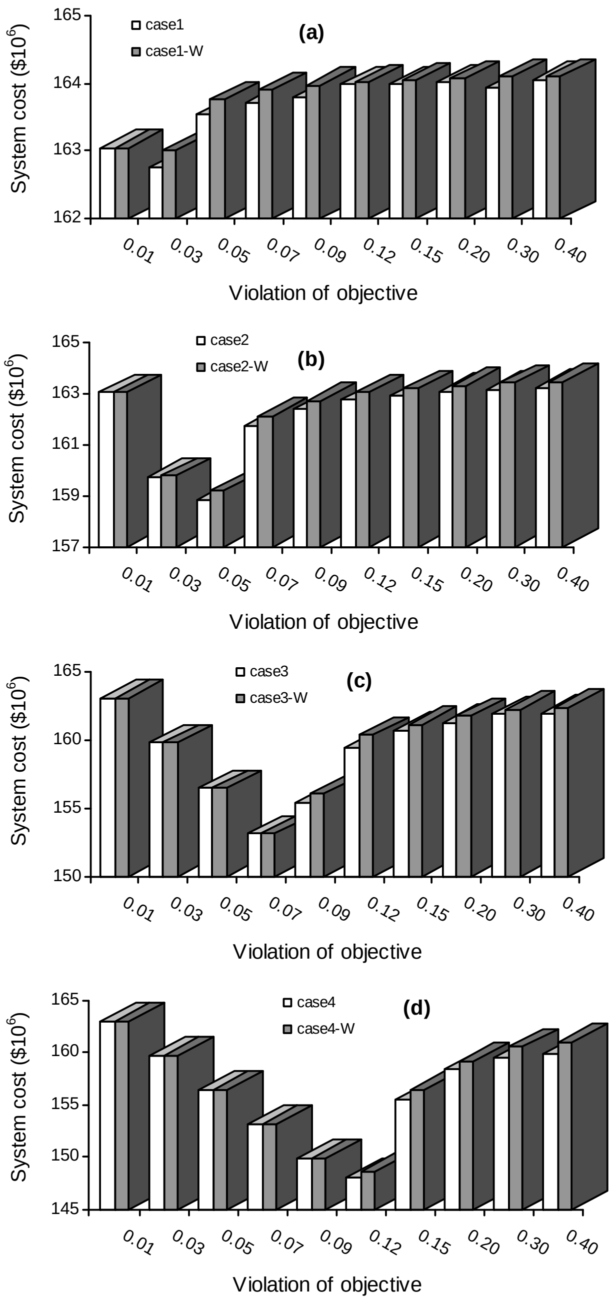

Figure 5. Effects on system costs for changed waste-generation rates (symbols of “case1-W”, “case2-W”, “case3-W” and “case4-W” denote that violation for facility-capacity constraint equal zero).

fill, incinerator and composting facility would be (i) 214.1, 5.9 and 192.2 t/d in period 1, (ii) 240.9, 16.1 and 191.7 t/d in period 2, and (iii) 291.7, 23.3 and 170.5 t/d in period 3.

The results also indicate that different violation levels lead to varied waste-disposal demands and capacities, and thus lead to different incremental requirements for waste-management-facility expansion. Figure 7 shows the solutions for landfill-capacity-expansion schemes through MSPM and FSPM-SC (with

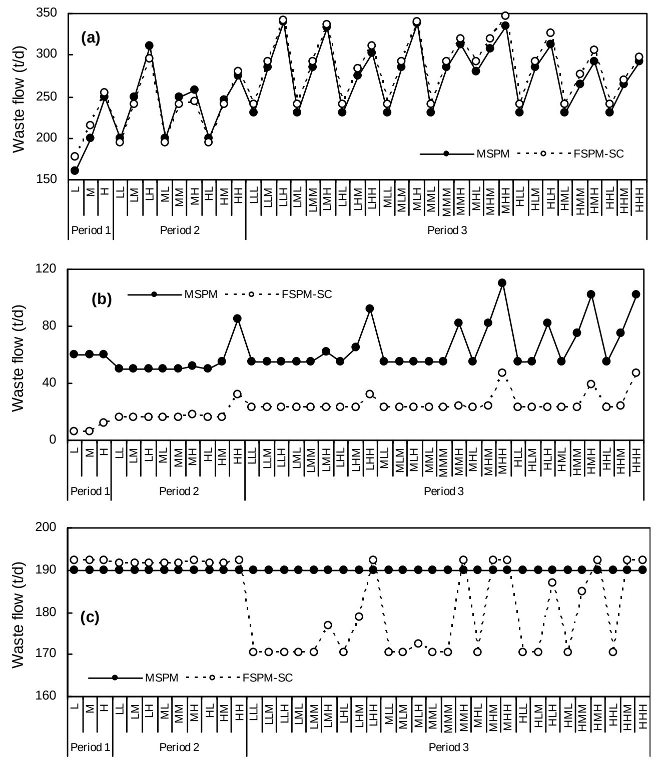

Figure 6. Comparison of waste-allocation patterns: (a) to landfill; (b) to incinerator; (c) to composting facility.

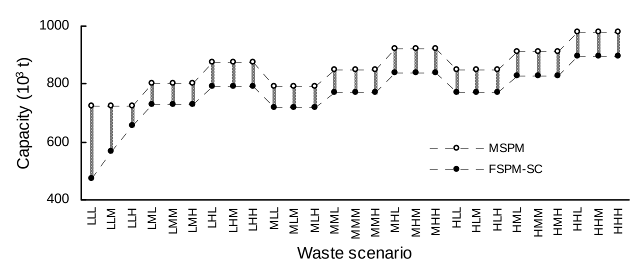

Figure 7. Expansion schemes for landfill.

a minimum cost). The results indicate that, under MSPM and FSPM-SC, the landfill would both be expanded at the start of period 2, while no expansion would be undertaken in periods 1 and 3. However, the amount of capacity expanded would be different from each other, as shown in Figure 7. In detail, when waste-generation rates are low in all of the three periods (denoted as symbol “LLL”), the landfill capacities expanded would be 721.9 × 10^3 tonnes (from MSPM) and 472.4 × 10^3 tonnes (from FSPM-SC); when waste-generation rates are high in all of the three periods (denoted as symbol “HHH”), the landfill capacities expanded would increase to 981.5 × 10^3 tonnes (from MSPM) and 896.6 × 10^3 tonnes (from FSPM-SC). The results indicate that there would be one expansion option for the composting facility over the planning horizon; this facility would be expanded at the start of period 1 with increments of 100 tonnes per day under MSPM and FSPM-SC. The incinerator would not be expanded due to its high capital cost for capacity expansion.

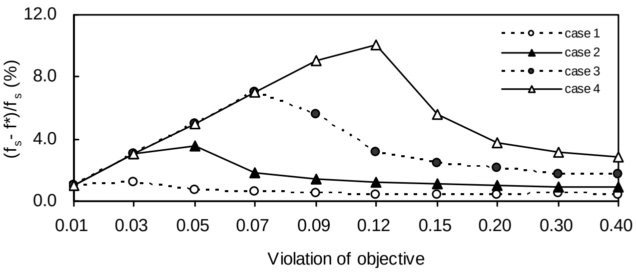

Figure 8. Relative difference of system costs.

Figure 8 presents the relative difference of system costs from MSPM and FSPM-SC [i.e. $[(f_s - f^*) / f_s]\%$] under different cases, where f^* are optimized system costs obtained from the FSPM-SC under various violation levels, and f_s is cost from the MSPM. The solution for system cost from MSPM would be $\$164.7 \times 10^6$; the lowest and highest system costs from FSPM-SC would be $\$148.1 \times 10^6$ and $\$164.1 \times 10^6$, respectively. Correspondingly, the minimum and maximum relative differences (of f^* and f_s) would be 0.38 and 10.09%. The main limitation of the MSPM is its incapability of reflecting fuzzy information; it can only reflect uncertainties expressed as probabilities. In comparison, the FSPM-SC can directly incorporate dual uncertainties of randomness and fuzziness within its optimization framework, and thus has advantages over the MSPM in reflecting these uncertainties. More importantly, a number of violation variables for the objective and constraints are introduced into the FSPM-SC, such that in-depth analyses of tradeoffs among system benefit and constraint-violation risk can be facilitated.

5. Conclusions

In this study, a multistage fuzzy-stochastic programming model with soft constraints (FSPM-SC) has been developed based on the techniques of fuzzy programming and stochastic programming. The developed method can deal with uncertainties expressed as probability distributions and fuzzy sets over a multistage context. A number of violation variables for the objective and constraints are introduced, such that in-depth analyses of tradeoffs among system cost, satisfaction degree, and constraint-violation risk can be facilitated. The developed FSPM- SC has been applied to a case study of municipal solid waste (MSW) under fuzzy and random uncertainties. The results indicate that solutions have been generated for binary and continuous decision variables under fuzzy and random uncertainties. The binary variable solutions represent the decisions of MSW management facility expansion, while the continuous variable solutions are related to decisions of waste-flow allocation. Besides, a number of violation analyses have been conducted to investigate the effects of violating the objective and constraints on system benefit and satisfaction degree. The results obtained are useful for generating alternatives under various system conditions.

This study is an attempt for planning MSW management system through the developed method; however, for large MSW management systems that involve multiple interactive and dynamic components, effective reflection of the system complexities becomes more difficult. For example, the allowable waste flow levels are dependent upon the existing and expanded capacities of waste-management facilities and the increasing rate of waste generation. There may exist multiple combinations of these uncertain parameters. Identification of an optimal allowable waste-flow level (i.e. the first-stage variable) which could help to minimize the sum of regular cost as well as the mean of recourse cost (i.e. penalty for excess waste disposal) is of challenge.

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