Journal of Environmental Informatics 20(1) 20-33 (2012)

Journal of Environmental Informatics

www.iseis.org/jei

Production Scheduling and Waste Disposal Planning for Oil Sands Mining Using Goal Programming

E. Ben-Awuah*, H. Askari-Nasab and K. Awuah-Offei

Mining Optimization Laboratory, School of Mining and Petroleum Engineering, Department of Civil and Environmental Engineering, 3-133 Markin/CNRL Natural Resource Engineering Facility, University of Alberta, Edmonton, Alberta T6G 2W2, Canada

Received 27 August 2011; revised 9 Feburary 2012; accepted 8 April 2012; published online 27 September 2012

ABSTRACT. In oil sands mining, timely provisions of ore and tailings containment with less environmental footprints are the main drivers of profitability and sustainability. The recent Alberta Energy Resources Conservation Board Directive 074 requires oil sands waste disposal planning to be an integral part of mine planning. This requires the development of a well integrated strategy of directional mining and tailings dyke construction for in-pit and ex-pit tailings storage management. The objectives of this paper are to: 1) determine the order and time of extraction of ore, dyke material and waste that maximizes the net present value; 2) determine the destination of dyke material that minimizes construction cost; and 3) minimize deviations from the production goals of the mining operation. We have developed, implemented, and verified a theoretical optimization framework based on mixed integer linear goal programming (MILGP) to address these objectives. This study presents an integration of mixed integer linear programming and goal programming in solving large scale mine planning optimization problems using clustering and pushback techniques. Application of the MILGP model was presented with an oil sands mining case. The MILGP model generated a smooth and uniform mining schedule that generates value and provides a robust framework for effective waste disposal planning. The results show that mining progresses with an ore to waste ratio of 1:1.5 throughout the mine life, generating an overall net present value of $14,237M. This approach improves the sustainable development of oil sands through better waste management.

Keywords: oil sands mining, production scheduling optimization, mixed integer linear goal programming, waste disposal planning, dyke construction

1. Introduction

Open-pit mining involves extracting blocks of material from the earth's surface to retrieve the ore contained in them or to access blocks of ore. This mining process causes the surface of the land to be continuously excavated causing an increasingly deeper pit to be formed until the end of the mine life (Hochbaum and Chen, 2000; Newman et al., 2010). Prior to the mining operation, the complex strategy of displacement of ore, waste, overburden, and tailings over the mine life need to be decided and this is known as mine planning. Open-pit mine planning can be defined as the process of finding a feasible block extraction sequence that generates the highest net present value (NPV) subject to operational and technical constraints (Whittle, 1989). Mine planning is done for different time horizons and these include short-term, medium-term, and long-term production scheduling. This paper focuses on the long-term production scheduling optimization process which is the backbone of the entire mining operation. In mining projects, deviations from optimal mine plans will result in significant financial losses, future financial liabilities, delayed reclamation, and resource sterilization.

The objective of this study is to develop a theoretical framework that maximizes the NPV of an oil sands mining operation, minimizes dyke construction cost for tailings containment and minimizes deviations from the production goals using a mixed integer linear goal programming (MILGP) model. The MILGP model incorporates multiple material types with multiple elements for multiple destinations in long-term production scheduling. Though operation research methods have been applied in mine production scheduling, very little work has been done in terms of oil sands mine planning, which has a unique scenario when it comes to waste management. Oil sands mining profitability depends on a carefully planned and integrated mine planning and waste management strategy that generates value and sustainability by maximizing NPV and creating timely tailings storage areas with less environmental footprints. Recent mining regulations by Alberta Energy Resources Conservation Board (Directive 074) (McFadyen, 2008) requires that oil sands mining companies develop an integrated mine planning and waste management strategy for their in-pit and external tailings facilities. This requires a new and more systematic approach in looking at the planning of oil sands mining operations.

* Corresponding author. Tel.: +1 780 4929188; fax: +1 780 4920249.
E-mail address: benawuah@ualberta.ca (E. Ben-Awuah).

ISSN: 1726-2135 print/1684-8799 online

The next section of this paper presents the problem definition and section 3 is on our conceptual mining model. Section 4 covers a literature review on linear programming (LP), mixed integer programming (MIP), mixed integer linear programming (MILP), goal programming (GP) and waste disposal planning. The application of MILGP to the long-term production planning (LTPP) problem is formulated in section 5. The formulation is applied to an oil sands mine planning and waste management case with an example and the results discussed in sections 6 and 7, respectively. Section 8 outlines the conclusions and future research direction.

2. Problem Definition

Mine management is always faced with the problem of achieving multiple goals with the available limited resources. In oil sands mining, due to the limitation of lease area, the pit phase advancement is carried out simultaneously with the construction of tailings dykes in the mined out areas of the pit and designated areas outside the pit. These dykes are constructed to hold tailings that are produced during the processing of the oil sands. Dykes with different configurations are required during the construction. Most of the materials used in constructing these dykes come from the oil sands mining operation. The dyke materials are comprised of overburden and interburden (OI) and tailings coarse sand (TCS). The material sent to the processing plant (ore) must have a specified minimum amount of bitumen and percentage fines, while material sent for dyke construction (dyke material) must meet the fines requirement for the dyke construction location. Any other material that does not meet the requirements of ore or dyke material is sent to the waste dump.

The main problem here has been categorized in three parts: 1) determining the order and time of extraction of ore, dyke material and waste to be removed from a predefined ultimate pit limit over the mine life, that maximizes the net present value of the operation; 2) determining the destination of dyke material that minimizes construction cost depending on the construction requirements of the various dykes as per their designs; 3) minimizing deviations from the production goals.

Before mine production scheduling, the material in the final pit limit is discretized into a three dimensional array called a block model. Figure 1 illustrates the scheduling of an oil sands ultimate pit block model containing K mining-cuts. Mining-cuts are clusters of blocks within the same level or mining bench that are grouped based on their attributes; location, rocktype and grade distribution (Askari-Nasab and Awuah-Offei, 2009; Ben-Awuah and Askari-Nasab, 2011). Each mining-cut k, is made up of ore o_k, OI dyke material d_k, and waste w_k. The material in each mining-cut is to be scheduled over T periods depending on the goals and constraints associated with the mining operation. OI dyke material scheduled, d_k^T, and TCS dyke material from the processed ore, l_k^T, must further be assigned to the dyke construction sites based on construction requirements. For period t_1, the dyke construction material required by site i is $dyKei$. Details of these notations can be found in the Appendix.

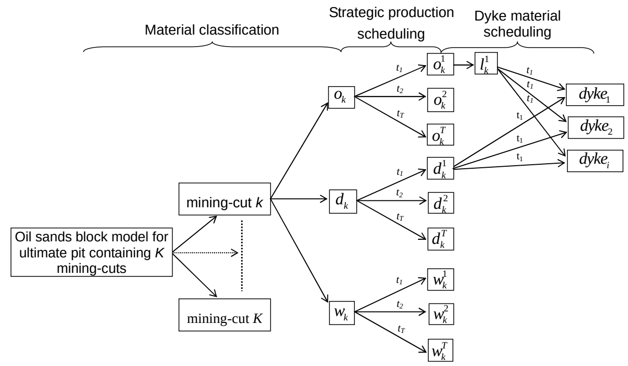

Figure 1. Schematic representation of the problem definition showing strategic production and dyke material scheduling modified after Ben-Awuah and Askari-Nasab (2011).

The strategic and dyke material production schedules to be developed are subject to a variety of economic, technical, and physical constraints. The constraints control the mining extraction sequence, ore and dyke material blending requirements and mining, processing, and dyke material goals. The mining, processing, and dyke material goals specify the quantities of material allowed for the mining operation, processing plant and dyke construction, respectively.

3. Conceptual Mining Model

The key drivers for oil sands mine planning are the provision of a processable blend of ore at the required grade and the provision of tailings containment at the right time. Figure 2 shows a conceptual mining model, consistent with practical oil sands mining and waste management, used to illustrate how the MILGP production scheduling model works. The mining model is made up of an oil sands deposit area which is to be mined and simultaneously used as an in-pit tailings storage area as mining progresses in a specified direction and the in-pit tailings dyke footprints are released. Each oil sands mining-cut is made up of ore, OI dyke material and waste. After processing the ore to extract bitumen, two main types of tailings are produced; fine and coarse tailings. The coarse tailings, also referred to as TCS dyke material, and OI dyke material are used in the construction of dykes for tailings facilities. The fine tailings form the slurry which needs to be contained in the tailings facilities.

3.1. Tailings Storage Management Strategy

Each tonne of ore is made up of bitumen, fines, sand, and water. Using the oil sands extraction process ore volume changes on the path from ore to tailings (Devenny, 2009), the volume of tailings to be produced can be calculated to plan an appropriate storage management strategy. In the conceptual mining model, the tailings storage volume required and the total in-pit tailings facilities volume available is used to calculate the external tailings facility (ETF) volume needed to support the mining operation.

The oil sands deposit area was divided into pushbacks, which coincide with the areas required by tailings dam engi-

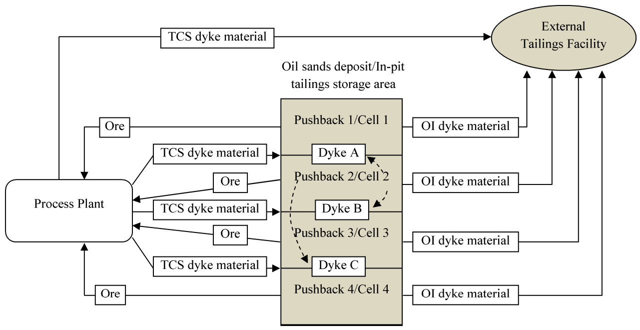

Figure 2. Conceptual mining model showing mining and waste management strategy modified after Askari-Nasab and Ben-Awuah (2011).

neers to set up in-pit tailings facility cells. In the case of our illustrative example in Figure 2, the deposit covers an area of 8 × 4 km with an average height of 75 m. Based on literature on oil sands mining operations with regards to standard sizes of ex-pit and in-pit tailings facility cells (Fort Hills Energy Corporation, 2009; Jackpine Mine, 2009; Kearl Oil Sands Project, 2009; Muskeg River Mine, 2009; Suncor Energy Incorporated Oil Sands, 2009; Syncrude Aurora North, 2009; Syncrude Aurora South, 2009; Syncrude Mildred Lake, 2009), it was decided to divide the mining area into four pushbacks which will result in four in-pit cells as shown in Figure 2. Each cell will have approximate dimensions of 2 × 4 km × 75 m. The mining operation will stay ahead of dyke construction by about 100 m. It is assumed that mining will start in pushback 1 and progress south. During the mining of pushback 1, all IO and TCS dyke material will be sent to the ETF for the construction of the ETF dyke. Fluid fine tailings produced from pushback 1 will be sent to the ETF after the key trench and starter dyke construction is completed. Once mining of pushback 1 is completed, the dyke ‘A’ footprint required to construct cell 1 becomes available. OI and TCS dyke material from pushback 2 will be used for the construction of dyke ‘A’ to enable in-pit tailings storage to start in cell 1.

As mining progresses to pushbacks 3 and 4, the OI and TCS dyke material produced can be used to construct dykes ‘B’ and ‘C’ to make available cells 2 and 3, respectively, for tailings storage. Any excess OI and TCS dyke material can be used for other purposes like shelling dumps, road construction, sand capping, and fines trapping as in non-segregating tailings. It is assumed that cell 4 will not be available for tailings storage until the end of the mine life; therefore it was not used for the volume balance calculations in the tailings storage management strategy. Table 1 shows estimates from the balancing of tailings storage requirements for the conceptual mining model. From the in-pit cell volumes generated for cells 1, 2, and 3, the required capacity of the ETF can be calculated and designed. The ETF was designed to cover an area of 1,600 ha with a height of 60 m resulting in a 13% excess containment capacity. The freeboard used for the designs is 5 m.

This tailings storage management strategy is based on the assumption that, all the available ore will be mined and processed. After the optimization of the production schedule, the actual mined ore tonnes can be used to reassess the tailings storage management strategy and appropriate modifications made. Further analysis of the conceptual mining model was done by starting the mining operation in pushback 4 and progressing north.

Table 1. Estimates for Tailings Storage Requirements for the Conceptual Mining Model

Material type	Oil sands deposit (Mtonnes)	Available dyke material (Mm^3)	Tailings/Waste produced (Mm^3)	Cells/ETF designed capacity (Mm^3)
Ore	2792.5	-	2251.1	Cell 1: 532
OI dyke	1697.8	797.6	-	Cell 2: 560
TCS dyke	2110.0	975.0	-	Cell 3: 560
Waste	375.9	-	179.0	ETF: 880

Table 2. Material Requirements for Dykes at Different Locations

Dyke location	OI and TCS dyke material required (Mm^3)		
	Key trench	Starter dyke	Main dyke
ETF dyke	1.96	20.58	507.63
Dykes A+B+C	1.38	10.80	304.95

3.2. Conceptual Dykes’ Designs

Simplified conceptual dyke designs were made for all the dykes and used as the basis for OI and TCS dyke material scheduling in all pushbacks. It was assumed that each dyke is made up of a key trench, a starter dyke and the main dyke as shown in Figure 3. The key trench and starter dyke will be constructed using OI dyke material and the main dyke will be constructed using TCS dyke material. Once construction of the key trench and starter dyke is complete, the tailings facility can be used whiles construction of the main dyke progresses. In line with the geology of the McMurray formation, it was assumed that the ETF dyke will be constructed, possibly, on a weak foundation and the in-pit cell dykes will be constructed on good foundation, thus requiring different side slopes. Table 2 shows the designed material requirements for the main dyke, starter dyke, and key trench at various destinations. The estimates are the minimum material required at the various destinations for dyke construction and any excess material can be used for other purposes.

4. Literature Review

Mining is the process of extracting a beneficial natural resource from the earth (Newman et al., 2010) and historical analysis of mineral resource evaluations has demonstrated the sensitivity of project profitability to decisions based on long-term mine production schedules. Long-term production planning (LTPP) problems have been a major research area for some time now and though major improvements have been made, the current dynamic mining environment brings about new and complex problems. Effective LTPP can increase the profitability and life of mine, considerably.

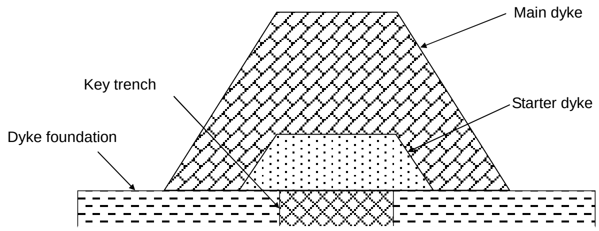

Figure 3. Schematic diagram showing cross section of a dyke (Askari-Nasab and Ben-Awuah, 2011).

Using mathematical programming models with exact optimization methods to solve the LTPP problem have proved to be robust. Mathematical programming models including LP, MIP, MILP and GP have the capability of considering multiple material types, elements, and destinations. Solving them with exact optimization methods result in solutions within known limits of optimality. As the solution gets closer to optimality, it leads to production schedules that generate higher NPV than those obtained from heuristic optimization methods. This has lead to extensive research on the application of mathematical programming models to the LTPP problem. Most of these models have been developed using LP, MIP and MILP. When these models are applied to the LTPP problem, they result in large scale optimization problems with numerous binary and continuous variables which become difficult to solve with the current state of hardware and software and may have lengthy solution times (Johnson, 1969; Gershon, 1983; Dagdelen, 1985; Dagdelen and Johnson, 1986; Akaike and Dagdelen, 1999; Caccetta and Hill, 2003). What makes these optimization problems more challenging are the large number of binary variables used to control the mining sequence, thus making the practical implementation of these models difficult.

Other mathematical programming modeling platforms that have been exploited in solving the LTPP problem includes GP. It can be said that in mining operations, one is faced with multiple objectives and in most cases it becomes necessary to trade off some targets for others. This is where GP becomes the appropriate modeling platform. GP allows for flexible formulation, specification of priorities among goals, and some level of interaction between the decision maker and the optimization process (Zeleny, 1980; Hannan, 1985). Against this background Zhang et al. (1993), Chanda and Dagdelen (1995) and Esfandiri et al. (2004) looked into the application of GP to the LTPP problem. They were however faced with the practical implementation of their models due to numerous constraints and size of the optimization problem.

Further researches have been conducted using MILP with block clustering techniques to reduce the size of the LTPP problem prior to optimization (Askari-Nasab et al., 2010; Askari-Nasab et al., 2011). These have been successfully implemented for some basic large scale production scheduling problems setting the stage for the practical implementation of mathematical programming models. These production scheduling models have been developed in isolation from other mine production systems. One of such systems is waste disposal planning. Waste disposal is an important part of the mining operation and when not well managed can result in mine closure or unbearable financial liabilities. In oil sands mining, waste disposal planning is even more closely connected to the mine planning system due to the mining strategy used and the regulatory requirements from the Alberta Energy Resources Conservation Board (Directive 074) (McFadyen, 2008; Askari-Nasab and Ben-Awuah, 2011; Ben-Awuah and Askari-Nasab, 2011). Consequently, the lack of an integrated mine production scheduling and waste disposal planning system in an optimization framework is worrisome.

Oil sands waste disposal planning is currently handled as a post production scheduling optimization activity (Fauquier et al., 2009). These are due to challenges that arise during the integration of such important major systems. These challenges include the size of the optimization problem resulting from scheduling different material types with multiple elements for multiple destinations. There is also the need to incorporate the availability of in-pit disposal areas with dyke construction planning on a continual basis throughout the mine life to support the tailings storage plan. Due to limited lease areas for oil sands operators, this is required to ensure the maximum use of in-pit and ex-pit tailings facilities for sustainable mining. Another challenge arises from competing objectives from such systems. Whilst production scheduling is driving NPV, waste disposal planning is driving sustainable mining and it becomes difficult to decide which targets must be traded off and at what cost.

The GP, LP, MIP, and MILP applications discussed lack the framework that can be used in solving the oil sands mine production scheduling and waste disposal problem. Some efforts have been made to combine GP and MILP models to solve some industrial problems because of the advantages of such hybrids. This hybridized model is referred to as MILGP. Using MILGP for oil sands production scheduling and waste disposal planning is appropriate because the structure enables the optimization solution to try achieving a set of goals where some goals can be traded off against one another depending on their priority. Hard constraints can also be converted to soft constraints which otherwise could lead to infeasible solutions. In simple terms, the advantage of using MILGP and deviational variables over other optimization formulations like LP, MIP or MILP is the fact that the deviational variables take values when an infeasible solution will otherwise have been returned. This allows an analyst to quickly pinpoint which goals are being relaxed. The analyst can then keep the results and change the input to obtain different results. In the case of an LP, MIP or MILP formulation, the optimizer will report infeasible solution and it may be difficult to understand which constraint is being violated and whether you can relax them or not.

The application of MILGP to the oil sands production scheduling and waste disposal planning problem as outlined in this paper has been setup in an optimization framework that integrates multiple material types, elements, and destinations. It includes large-scale optimization, directional mining, and integration of mine production planning and waste management. The practical implementation of the MILGP model and the generated production schedules are also highlighted.

5. MILGP Model for Open Pit Production Scheduling and Waste Disposal Planning

5.1. Notations

The notations used in the formulation of the oil sands strategic production and dyke material scheduling problem has been classified as sets, indices, subscripts, superscripts, parameters, and decision variables. Details of these notations can be found in the Appendix. In general, the MILGP formulation is for multiple material types and destinations as well as pushbacks which ties into the waste management strategy. The MILGP formulation framework was developed based on mining-cuts. This MILGP model is an extension of the oil sands mine planning formulation by Ben-Awuah and Askari-Nasab (2011).

5.2. Modeling of Economic Mining-cut Value

The objective function of the MILGP model for LTPP is to maximize the net present value of the mining operation and minimize the dyke construction cost and deviations from the mining goal, processing goal, OI dyke material goal, and TCS dyke material goal for all destinations. The concept of economic mining-cut value is based on ore parcels within mining-cuts which could be mined selectively. The profit from mining a mining-cut is a function of the value of the mining-cut based on the processing destination and the costs incurred in mining, processing, and dyke construction at a specified destination. The cost of dyke construction is also a function of the location of the tailings facility being constructed and the type and quantity of dyke material used. The discounted profit from mining-cut k is equal to the discounted revenue obtained by selling the final product contained in mining-cut k minus the discounted cost involved in mining mining-cut k as waste minus the extra discounted cost of mining OI dyke material minus the extra discounted cost of mining TCS dyke material. This has been simplified into Equations (1) to (5):

$$d_k^{u,t} = v_k^{u,t} - q_k^{u,t} - p_k^{u,t} - h_k^{u,t},\ \forall t \in \{1, \ldots, T\},\ u \in \{1, \ldots, U\},\ k \in \{1, \ldots, K\} \tag{1}$$

where:

$$v_k^{u,t} = \sum_{e=1}^{E} o_k \times g_k^e \times r^{u,e} \times \left(p^{e,t} - cs^{e,t} \right) - \sum_{e=1}^{E} o_k \times cp^{u,e,t},\ \forall t \in \{1, \ldots, T\},\ u \in \{1, \ldots, U\},\ k \in \{1, \ldots, K\} \tag{2}$$

$$q_k^{u,t} = \left(o_k + d_k + w_k \right) \times cm^{u,t},\ \forall t \in \{1, \ldots, T\},\ u \in \{1, \ldots, U\},\ k \in \{1, \ldots, K\} \tag{3}$$

$$p_k^{u,t} = d_k \times ck^{u,t},\ \forall t \in \{1, \ldots, T\},\ u \in \{1, \ldots, U\},\ k \in \{1, \ldots, K\} \tag{4}$$

$$h_k^{u,t} = l_k \times ct^{u,t},\ u \in \{1, \ldots, U\},\ k \in \{1, \ldots, K\} \tag{5}$$

5.3. The MILGP Model

Using multiple criteria decision making analysis, the objective functions of the MILGP model for production scheduling and waste disposal planning as applied in oil sands mining, can be formulated as: i) maximizing the NPV, ii) minimizing the dyke construction cost, and iii) minimizing deviations from the production goals. These are represented by Equations (6), (7), and (8), respectively:

$$\sum_{j=1}^{J} \{ Max \sum_{u=1}^{U} \sum_{t=1}^{T} [\sum_{k \in B_j} (v_k^{u,t} \times x_k^{u,t} - q_k^{u,t} \times y_k^{u,t})] \} \tag{6}$$

$$\sum_{j=1}^{J} \{ Min \sum_{u=1}^{U} \sum_{t=1}^{T} [\sum_{k \in B_j} (p_k^{u,t} \times z_k^{u,t} + h_k^{u,t} \times s_k^{u,t})] \} \tag{7}$$

$$\sum_{j=1}^{J} \{ Min \sum_{u=1}^{U} \sum_{t=1}^{T} [\sum_{k \in B_j} (P_1(a_1 d_1^{-,u,t}) + P_2(a_2 d_2^{-,u,t}) + P_3(a_3 d_3^{-,u,t} + a_3 d_3^{+,u,t}) + P_4(a_4 d_4^{-,u,t} + a_4 d_4^{+,u,t}))] \} \tag{8}$$

Equations (6) to (8) can be combined as a single objective function formulated as in Equation (9):

$$\sum_{j=1}^{J} \{ Max \sum_{u=1}^{U} \sum_{t=1}^{T} [\sum_{k \in B_j} [(v_k^{u,t} \times x_k^{u,t} - q_k^{u,t} \times y_k^{u,t}) - (p_k^{u,t} \times z_k^{u,t} + h_k^{u,t} \times s_k^{u,t}) - (P_1(a_1 d_1^{-,u,t}) + P_2(a_2 d_2^{-,u,t}) + P_3(a_3 d_3^{-,u,t} + a_3 d_3^{+,u,t}) + P_4(a_4 d_4^{-,u,t} + a_4 d_4^{+,u,t}))]] \} \tag{9}$$

The MILGP model goal functions and constraints can be formulated as:

Goal functions:

$$\sum_{j=1}^{J} (\sum_{k \in B_j} (o_k + w_k + d_k) \times y_k^{u,t}) + d_1^{-,u,t} = T_m^{u,t},\ \forall t \in \{1, \ldots, T\},\ u \in \{1, \ldots, U\} \tag{10}$$

$$\sum_{j=1}^{J} (\sum_{k \in B_j} (o_k \times x_k^{u,t})) + d_2^{-,u,t} = T_p^{u,t},\ \forall t \in \{1, \ldots, T\},\ u \in \{1, \ldots, U\} \tag{11}$$

$$\sum_{j=1}^{J} (\sum_{k \in B_j} (d_k \times z_k^{u,t})) + d_3^{-,u,t} - d_3^{+,u,t} = T_d^{u,t},\ \forall t \in \{1, \ldots, T\},\ u \in \{1, \ldots, U\} \tag{12}$$

$$\sum_{j=1}^{J} (\sum_{k \in B_j} (l_k \times s_k^{u,t})) + d_4^{-,u,t} - d_4^{+,u,t} = T_l^{u,t},\ \forall t \in \{1, \ldots, T\},$$

$u \in \{1, \ldots, U\}$ (13)

Constraints:

$$\sum_{j=1}^{J}\sum_{k\in B_j} g_k^e \times o_k \times x_k^{u,t} - \overline{g}^{u,t,e}\sum_{j=1}^{J}\sum_{k\in B_j} o_k \times x_k^{u,t} \le 0,\ \forall t \in \{1, \ldots, T\},\ u \in \{1, \ldots, U\},\ e \in \{1, \ldots, E\} \tag{14}$$

$$\sum_{j=1}^{J}\sum_{k\in B_j} g_k^e \times o_k \times x_k^{u,t} - \underline{g}^{u,t,e}\sum_{j=1}^{J}\sum_{k\in B_j} o_k \times x_k^{u,t} \ge 0,\ \forall t \in \{1, \ldots, T\},\ u \in \{1, \ldots, U\},\ e \in \{1, \ldots, E\} \tag{15}$$

$$\sum_{j=1}^{J}\sum_{k\in B_j} f_k^e \times o_k \times x_k^{u,t} - \overline{f}^{u,t,e}\sum_{j=1}^{J}\sum_{k\in B_j} o_k \times x_k^{u,t} \le 0,\ \forall t \in \{1, \ldots, T\},\ u \in \{1, \ldots, U\},\ e \in \{1, \ldots, E\} \tag{16}$$

$$\sum_{j=1}^{J}\sum_{k\in B_j} f_k^e \times o_k \times x_k^{u,t} - \underline{f}^{u,t,e}\sum_{j=1}^{J}\sum_{k\in B_j} o_k \times x_k^{u,t} \ge 0,\ \forall t \in \{1, \ldots, T\},\ u \in \{1, \ldots, U\},\ e \in \{1, \ldots, E\} \tag{17}$$

$$\sum_{j=1}^{J}\sum_{k\in B_j} f_k^d \times d_k \times z_k^{u,t} - \overline{f}^{u,t,d}\sum_{j=1}^{J}\sum_{k\in B_j} d_k \times z_k^{u,t} \le 0,\ \forall t \in \{1, \ldots, T\},\ u \in \{1, \ldots, U\},\ e \in \{1, \ldots, E\} \tag{18}$$

$$\sum_{j=1}^{J}\sum_{k\in B_j} f_k^d \times d_k \times z_k^{u,t} - \underline{f}^{u,t,d}\sum_{j=1}^{J}\sum_{k\in B_j} d_k \times z_k^{u,t} \ge 0,\ \forall t \in \{1, \ldots, T\},\ u \in \{1, \ldots, U\},\ e \in \{1, \ldots, E\} \tag{19}$$

$$\sum_{u=1}^{U}\left(x_k^{u,t} + z_k^{u,t}\right) \le \sum_{u=1}^{U} y_k^{u,t},\ \forall t \in \{1, \ldots, T\}, k \in \{1, \ldots, K\} \tag{20}$$

$$\sum_{u=1}^{U} s_k^{u,t} \le \sum_{u=1}^{U} x_k^{u,t},\ \forall t \in \{1, \ldots, T\}, k \in \{1, \ldots, K\} \tag{21}$$

$$\sum_{u=1}^{U}\sum_{t=1}^{T} x_k^{u,t} \le 1,\ \forall k \in \{1,..,K\} \tag{22}$$

$$\sum_{u=1}^{U}\sum_{t=1}^{T} z_k^{u,t} \le 1,\ \forall k \in \{1,..,K\} \tag{23}$$

$$\sum_{u=1}^{U}\sum_{t=1}^{T} s_k^{u,t} \le 1,\ \forall k \in \{1,..,K\} \tag{24}$$

$$b_k^t - \sum_{u=1}^{U}\sum_{i=1}^{t} y_s^{u,i} \le 0,\ \forall t \in \{1, \ldots, T\}, k \in \{1, \ldots, K\}, s \in C_k(L) \tag{25}$$

$$b_k^t - \sum_{u=1}^{U}\sum_{i=1}^{t} y_r^{u,i} \le 0,\ \forall t \in \{1, \ldots, T\}, k \in \{1, \ldots, K\}, r \in M_k(P) \tag{26}$$

$$b_k^t - \sum_{u=1}^{U}\sum_{i=1}^{t} y_h^{u,i} \le 0,\ \forall t \in \{1, \ldots, T\}, k \in \{1, \ldots, K\}, h \in N_k(H) \tag{27}$$

$$\sum_{u=1}^{U}\sum_{i=1}^{t} y_k^{u,i} - b_k^t \le 0,\ \forall t \in \{1,...,T\},\ k \in \{1,...,K\} \tag{28}$$

$$b_k^t - b_k^{t+1} \le 0,\ \forall t \in \{1,...,T-1\},\ \ k \in \{1,...,K\} \tag{29}$$

$$d_1^{-,t},\ d_2^{-,u,t},\ d_3^{-,u,t},\ d_3^{+,u,t},\ d_4^{-,u,t},\ d_4^{+,u,t} \ge 0,\ \forall t \in \{1, \ldots, T\}, u \in \{1, \ldots, U\} \tag{30}$$

$$P_1 > P_2 > P_3 > P_4 \tag{31}$$

Equations (10) to (13) are the goal functions which define the mining, processing, OI dyke material, and TCS dyke material goals that are required for all destinations. Equations (14) to (19) specify the limiting grade requirements for ore bitumen, ore fines, and OI dyke material fines for all destinations. Equation (20) ensures that the total material mined in each period for all destinations does not exceed the sum of the ore and OI dyke material mined. Equation (21) states that the fraction of TCS dyke material mined in each period should be less or equal to the fraction of ore material mined for all destinations. Equations (22) to (24) ensure that the total fractions of mining-cut k sent to all destinations in all periods is less or equal to one. Equations (25) to (29) check the set of immediate predecessor mining-cuts that must be mined prior to mining mining-cut k for all periods and destinations. These equations control the vertical and horizontal block extraction sequence. They ensure that mining proceeds in the specified mining direction as the mine goes deeper. Equation (30) ensures that the negative and positive deviations from the targeted mining, processing, OI dyke material, and TCS dyke material goals are always positive for all periods and destinations. Equation (31) states the order of prioritization associated with achieving the goals. The model assumes that there exists a pre-emptive priority structure among the goals and this can be changed depending on the mining operation and aim of optimization.

Using mathematical programming models like the MILGP formulation for mine optimization usually result in large-scale optimization problems. A commercial optimization solver capable of handling such problems is ILOG CPLEX (ILOG Inc., 2007). This optimization solver uses branch and cut algorithm and makes the solving of large-scale problems possible for the MILGP model. Branch and cut is a method of combinatorial optimization for solving integer programming problems. This algorithm is a hybrid of branch-and-bound and cutting plane methods (Horst and Hoang, 1996; Wolsey, 1998).

The MILGP model solver in this research is TOMLAB/CPLEX (Holmström, 2009). The user sets an optimization termination criterion in CPLEX known as the gap tolerance (EPGAP). The EPGAP, which is a measure of optimality, sets an absolute tolerance on the gap between the best integer objective and the objective of the best node remaining in the branch and cut algorithm. It instructs CPLEX to terminate once a feasible integer solution within the set EPGAP has been found (ILOG Inc., 2007).

6. Implementing the MILGP Model for Production Scheduling and Waste Disposal Planning

In implementing the MILGP model, the size of the mining-cuts used for production scheduling must be carefully selected to ensure that it is comparable to the selective mining units of the operation in practice. The proposed MILGP model uses continuous decision variables, $y_k^{u,t}$, $x_k^{u,t}$, $z_k^{u,t}$ and $s_k^{u,t}$ to model mining, processing, OI dyke material and TCS dyke material requirements, respectively, for all destinations. Binary integer decision variable b_k^t is used to control precedence of mining-cuts extraction. Continuous deviational variables $d_1^{-,t}$, $d_2^{-,u,t}$, $d_3^{-,u,t}$, $d_3^{+,u,t}$, $d_4^{-,u,t}$ and $d_4^{+,u,t}$ have been defined to support the goal functions that control mining, processing, OI and TCS dyke material for all destinations. The deviational variables make available a continuous range of units (tonnes) that the optimizer chooses from to satisfy the set goals and these deviational variables are minimized in the objective function. The objective function also contains deviational penalty cost and priority parameters, which are important aspects of this formulation. The deviational penalty cost parameters a_1, a_2, a_3 and a_4 penalizes the NPV for any deviation from the set goals. This parameter forces the optimizer to meet the set goals to avoid penalizing the NPV. The priority parameters P_1, P_2, P_3 and P_4 are used to place emphasis on the goals that are more important. This parameter is also set up to penalize the NPV more if the most important set goal is not met.

In setting up these parameters, the modeler needs to monitor how smooth mining proceeds from one period to another and the uniformity of tonnages mined per period; as well as the corresponding NPV generated in other to keep track of the impact of any parameter change on these key performance indicators. In some cases, the extent of setting the priority or penalty cost depends on the extent to which the modeler wants to trade off NPV to meet the set goals. A higher priority or penalty may enforce a goal to be met whilst reducing the NPV of the operation. A case showing this trend has been analyzed.

7. Results and Discussions

The performance of the proposed MILGP model was analyzed based on NPV, mining production goals, smoothness and practicality of the generated schedules and the availability of tailings containment areas at the required time. The formulation was verified by numerical experiments on a synthetic and an oil sands data set. The model was implemented on a Dell Precision T3500 computer at 2.4 GHz, with 3GB of RAM.

Table 3. Oil Sands Final Pit and Production Scheduling Information

Description	Value
Total tonnage of rock (Mt)	4,866.2
Total ore tonnage (Mt)	2,792.5
Total OI dyke material tonnage (Mt)	1,697.8
Total TCS dyke material tonnage (Mt)	2,110.0
Total waste tonnage (Mt)	375.9
Number of blocks	61,490
Block dimensions (m x m x m)	50 × 50 × 15
Number of benches	5
Bench height (m)	15
Bench elevations (m)	265-325
Number of scheduling periods (years)	20

Table 4. Details for Each Pushback to be Used for Production Scheduling and Waste Disposal Planning

Description	Pushback Value			
	1	2	3	4
Number of blocks	14,535	16,433	16,559	13,963
Number of mining-cuts	971	970	977	999
Tonnage of rock (Mt)	1,144.6	1,303.9	1313.2	1104.5
Ore tonnage (Mt)	631.1	758.7	775.7	627.0
OI dyke material tonnage (Mt)	432.4	434.2	435.6	395.7
TCS dyke material tonnage (Mt)	479.4	568.0	587.0	475.5
Average ore bitumen grade (wt%)	11.7	11.5	11.6	11.6
Average ore fines (wt%)	8.6	9.7	8.9	8.7
Average OI dyke material fines (wt%)	4.1	5.8	5.1	4.6

Further implementation of the MILGP model was done for a large scale oil sands deposit covering an area of 8 × 4 km, which is similar to the conceptual mining model. The rock types in the area are Pleistocene, Clearwater, Upper McMurray, Middle McMurray and Lower McMurray formations. Table 3 shows details of the oil sands final pit and the material contained in it. The deposit is to be scheduled over 20 periods equivalent to 20 years.

The designed final pit block model was divided into 4 pushbacks that are consistent with the conceptual mining model. The sizes of the pushbacks are determined in consultation with tailings dam engineers and are based on the required cell capacities and the timeliness required in making the cell areas available for tailings containment. The blocks within each pushback are clustered into mining-cuts using fuzzy logic clustering algorithm (Kaufman and Rousseeuw, 1990) to reduce the number of decision variables required in the MILGP model. Clustering of blocks into mining-cuts ensures the MILGP scheduler generates a mining schedule at a selective mining unit that is practical from mining operation point of view. The material in the designed final pit is to be scheduled for the processing plant

Table 5. Ore and OI Dyke Material Grade Constraints for the MILGP Model for 20 Periods

Ore bitumen grade (wt%)		Ore fines (wt%)		OI dyke material fines (wt%)	
$\underline{g}^{u,t,e}$	$\overline{g}^{u,t,e}$	$\underline{f}^{u,t,e}$	$\overline{f}^{u,t,e}$	$\underline{f}^{u,t,d}$	$\overline{f}^{u,t,d}$
7	16	0	30	0	50

Table 6. Mining, Processing, OI and TCS Dyke Material Goals for the MILGP Model for 20 Periods

Mining goal (Mt)	Processing goal (Mt)	OI dyke material goal (Mt)	TCS dyke material goal (Mt)
244	140	70	106

Table 7. Results from the MILGP Model in Terms of the NPV and Dyke Construction Cost for all Pushbacks and Destinations

Pushback #	NPV ($M)	Dyke construction cost ($M)	EPGAP (%)
Pushback 1	6,493.77	714.44	2.0
Pushback 2	4,695.34	524.20	2.0
Pushback 3	3,184.72	312.74	1.7
Pushback 4	1,588.65	174.39	1.1

and four dyke construction destinations with the objective of maximizing the NPV of the mining operation and minimizing the dyke construction cost. An EPGAP of 2% was set for the optimization of all pushbacks. A summary of the details for each pushback used for production scheduling are shown in Table 4.

For processing plant feed and dyke construction, bitumen grade and fines percent need to be controlled within an acceptable range for all pushbacks and destinations. This requirement has been summarized in Table 5. Mining will proceed south starting from pushback 1 to 4. When mining of pushback 1 starts, the OI and TCS dyke material will be used in constructing the key trench, starter dyke, and main dyke of the ETF where the initial fluid fine tailings will be stored. When pushback 1 is completely mined, cell 1 area becomes available and OI and TCS dyke material from pushback 2 can be used in constructing dyke 'A' about 100 m from the mine face to create cell 1 for in-pit tailings containment to start. This mining and tailings storage management strategy similar to the conceptual mining model will be utilized until all pushbacks are mined.

The aim is to generate a uniform schedule and a smooth mining sequence based on the availability of material, the plant processing capacity, and dyke construction requirements. The dyke construction material scheduled should meet the minimum requirements of material for the specified destination with any excess material being available for other purposes. Further to this, to ensure that the mining equipment capacity is well utilized throughout the mine life, we intend to keep a uniform stripping ratio when the mining of ore starts. Table 6 shows the input mining, processing and dyke material goals for the MILGP model for 20 periods.

Some of the important features that make this MILGP formulation a robust and flexible platform for mine planning are

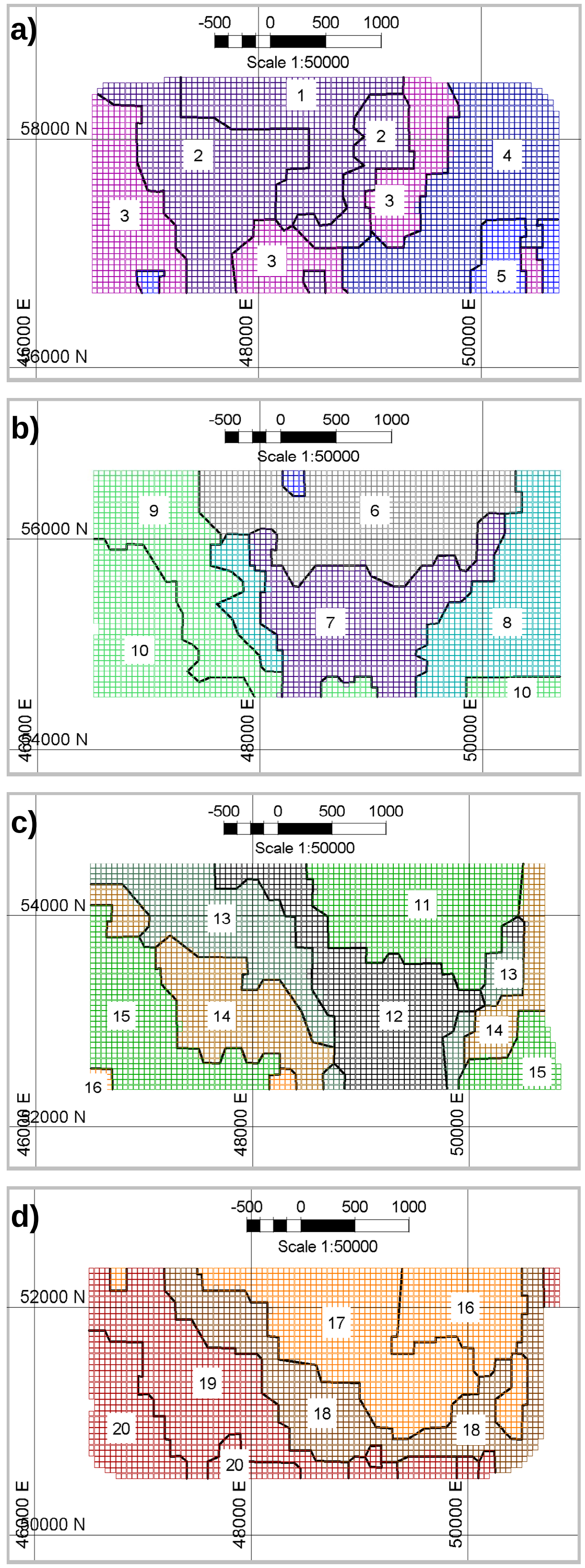

Figure 4. Pushback mining sequence at level 295 m: a) Pushback 1; b) Pushback 2; c) Pushback 3; d) Pushback 4.

that, the planner can decide on tradeoffs between NPV maximization or dyke construction cost minimization and goals achievement using the penalty and priority functions. Apart from maximizing NPV and minimizing dyke construction cost, the planner has control over the setting of goals and their deviational variables and the upper and lower limits of grades in each period for all pushbacks and destinations. An advantage of the MILGP model and deviational variables over other optimization formulations like LP, MIP or MILP is the fact that the deviational variables take values when an infeasible solution will otherwise have been returned. The planner can then quickly look for the goals that are being relaxed and then change them to obtain different results. The penalty cost and priority parameters used in the MILGP model for this optimization were: 0 for mining; 20 for processing; 30 for OI dyke material; and 30 for TCS dyke material. The magnitude of these values should be calibrated based on the objectives of management. More weight should be assigned to a goal that has higher priority for the management. These generated the required tonnages at the various production destinations. Table 7 summarizes the results from the MILGP model in terms of the NPV and dyke construction cost generated after optimization. The four pushbacks were optimized separately over a total of 20 periods. The overall NPV generated including the dyke construction cost for all pushbacks and destinations is $14,237 M.

Figures 4a to 4d show the mining sequence at level 295 m for all pushbacks with a north-south mining direction. The numbering and outlines in the figures show the periods each block of material is mined. The MILGP model generated a practical mining sequence that is smooth and consistent with the mining of oil sands. A smooth mining sequence implies that mining progresses continuously in the specified direction from one area to the next without jumping around randomly. Mining proceeds in the specified direction to ensure least mobility and increased utilization of loading equipment. This is very important in the case of oil sands mining where large cable shovels are used. The size of the mining-cuts in each period enables good equipment maneuverability and the number and size of active bench phases in each period also reduces the number of loading equipments required as well as providing alternative loading points if needed. Another strategic aspect of mining in the specified direction within each pushback is to ensure that the dyke footprints are released on time as the mining proceeds to enable in-pit dyke construction for tailings containment to start. This is an important integral part of the waste management strategy for oil sands mining operations, and a key driver for profitability and sustainable operations. This also reduces the environmental footprints of the ETF.

The results from Figure 5 shows a uniform mining, processing, OI and TCS dyke material schedules, which ensures effective utilization of mining fleet and processing plant throughout the mine life. The schedule ensures that apart from meeting the processing plant requirements to maximize NPV, the required quality and quantity of dyke material needed to build the dykes of the ETF, cells ‘A’, ‘B’, and ‘C’ are provided in a timely manner at a minimum cost for tailings containment. The schedule provides the minimum dyke material requirements of each dyke construction destination as per the conceptual dykes’ designs and any excess material can be used for other purposes.

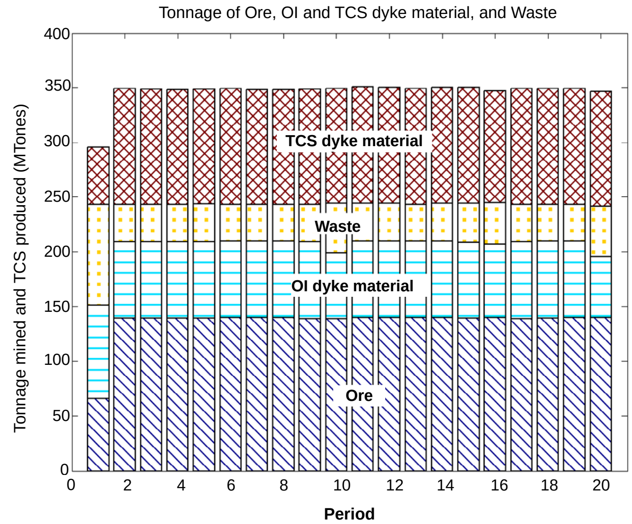

Figure 5. Schedules for ore, OI and TCS dyke material, and waste tonnages produced over 20 periods.

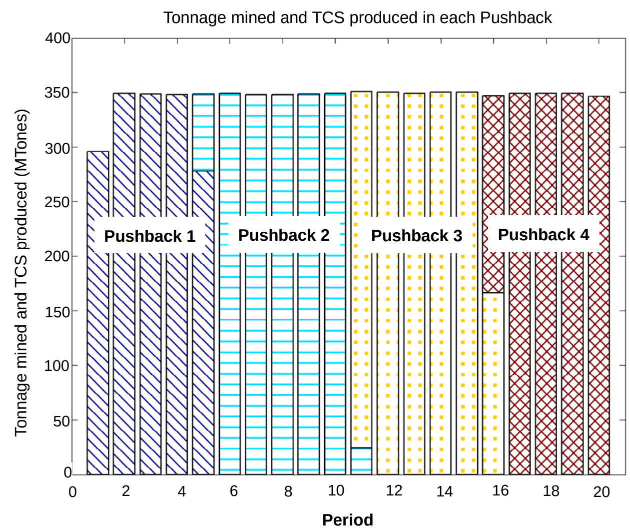

Figure 6. Material mined and TCS dyke material tonnage produced in each pushback for 20 periods.

During the first year, due to the requirements of the ETF dyke construction material, less ore and more OI dyke material is mined to facilitate the construction of the key trench and starter dyke and then subsequently, TCS dyke material can be used to continue constructing the main dyke as planned in the conceptual dyke design. This ensures that tailings containment area is created in time for the storage of fluid fine tailings. Ore becomes available at full processing plant capacity from year 2 until the end of the mine life and subsequently TCS dyke material. The OI dyke material supply was also maintained at a uniform rate throughout the mine life. Figure 5 shows the schedules for ore, OI and TCS dyke material, and waste tonnages generated for 20 periods. Figure 6 shows the material mined

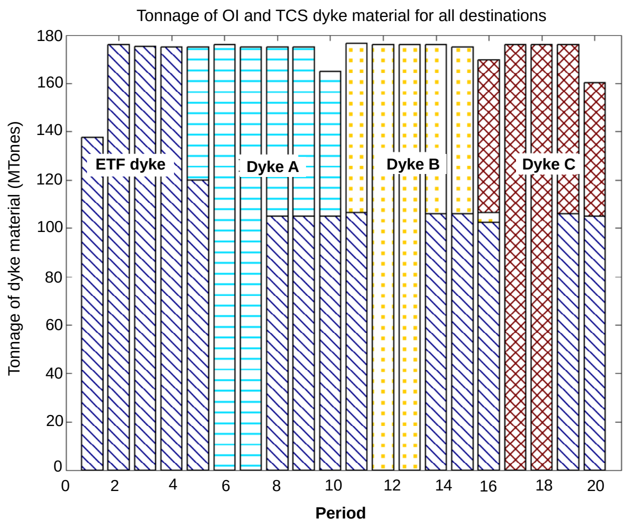

Figure 7. Dyke material tonnage sent to the various dyke construction destinations for 20 periods.

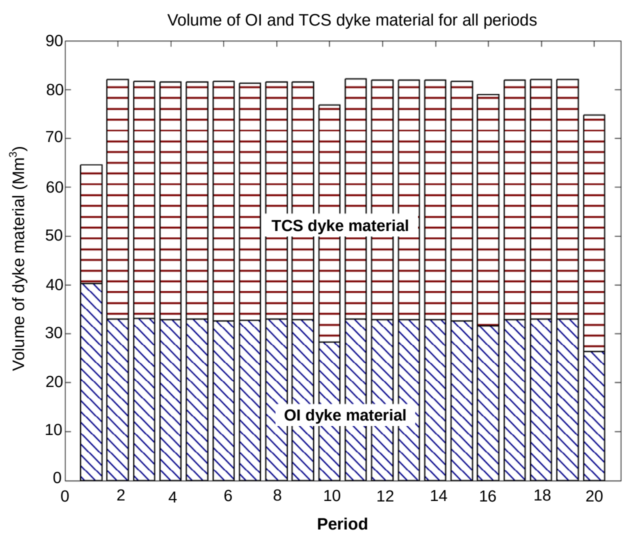

Figure 8. OI and TCS dyke material volume scheduled for 20 periods.

and TCS dyke material tonnage produced in each pushback for 20 periods. Figure 7 shows the dyke material tonnage sent to the various dyke construction destinations for 20 periods and Figure 8 shows the OI and TCS dyke material volume scheduled for 20 periods. It can be seen from Table 2 that 23Mm^3 of OI dyke material is required for the ETF key trench and starter dyke construction and this material requirement has been adequately catered for by scheduling 40 Mm^3 of OI dyke material in period 1 as shown in Figure 8.

The total material mined was 4,866.2 Mt. This is made up of 2,720.4 Mt of ore and 1,386.7 Mt of OI dyke material whilst 2,055.2 Mt of TCS dyke material was generated. A total of 1,602.1 Mm^3 of dyke material was scheduled. The schedules give the planner good control over dyke material and provides a robust platform for effective dyke construction planning and tailings storage management.

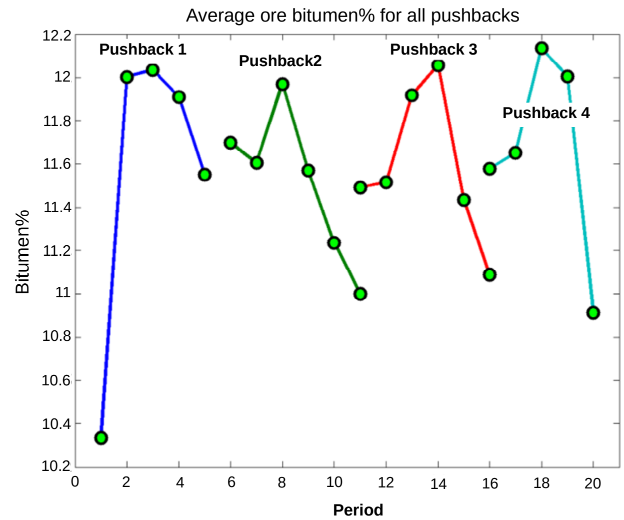

Figure 9. Average ore bitumen grade for all pushbacks.

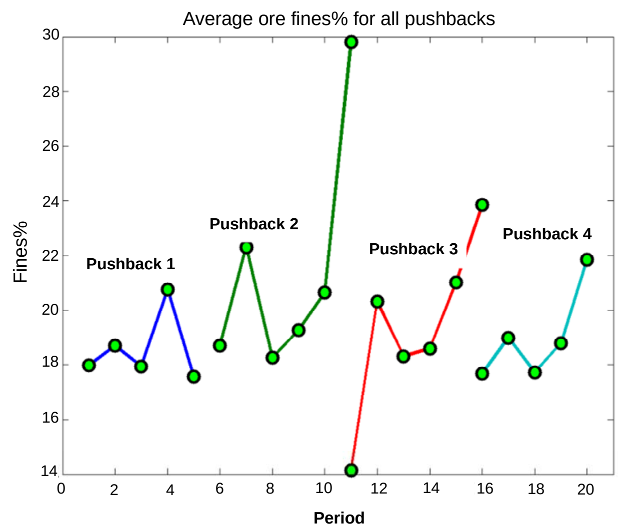

Figure 10. Average ore fines percent for all pushbacks.

There is also an inherent task of blending the run-of-mine materials to meet the quality and quantity specifications of the processing plant and dyke construction. The blending problem becomes more prominent as more detailed planning is done in the medium to short term. The processing plant head grade and OI dyke material grade that was set were successfully achieved in all periods for all destinations. It was our target to reduce the periodic grade variability in each pushback by setting tighter lower and upper grade bounds. The grades in each period are obtained from a blend of ore or dyke material required to meet the grade and tonnage requirements for that period. Depending on the processing plant or dyke construction requirements, the periodic grades in each pushback can be varied appropriately while ensuring a feasible solution is obtained. With the exception of period 1, the scheduled average ore bitumen grade was between 10.9 and 12.2%. The average ore bitumen grade for period 1 was 10.3% basically due to the emphasis placed on mining OI dyke material for the ETF key trench and starter dy-

Table 8. Results for Supplementary Experiments Showing that Run 3 Generates the Highest NPV and Best Schedule

Run #	Total Cuts	Mining direction	P_1a_1	P_2a_2	P_3a_3	P_4a_4	Runtime (minutes)	Overall NPV ($M)	Dyke material (Mt)	Schedule uniformity & smoothness ranking
1	1977	NS	0	20	30	30	105	13,810	3315	3
2	1977	SN	0	20	30	30	17	10,713	3012	4
3	3917	NS	0	20	30	30	288	14,237	3442	1
4	3917	NS	0	20	60	30	59	14,121	3460	2

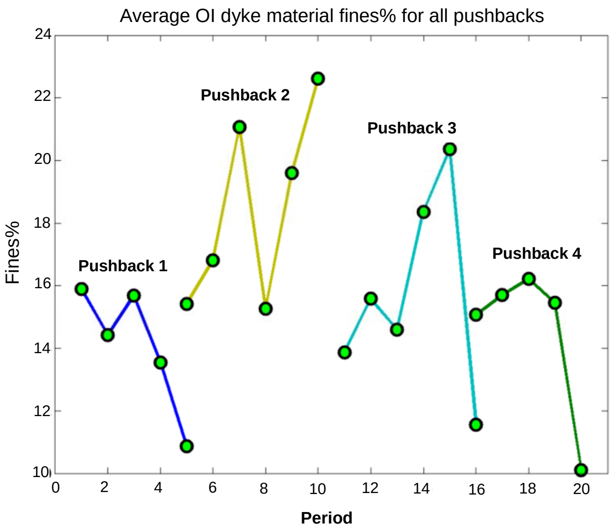

Figure 11. Average OI dyke material fines percent for all pushbacks.

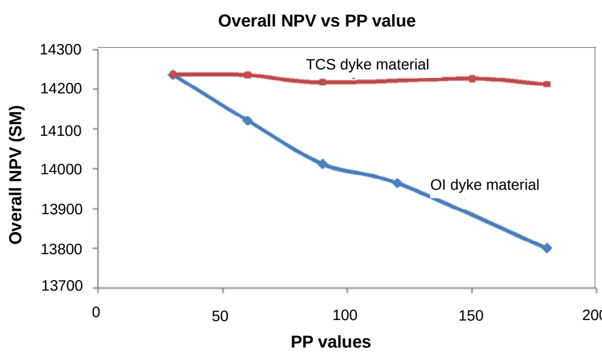

Figure 12. General trend of overall NPV with PP values of dyke material.

ke construction. This was required to construct the initial tailings containment when ore processing starts. The average ore and OI dyke material fines percent were between 14 and 30%, and 10 and 23% respectively. Figures 9 and 10 show the average ore bitumen grade and ore fines percent for all pushbacks respectively. Figure 11 shows the average OI dyke material fines percent for all pushbacks.

7.1. Waste Disposal Planning and the Environment

Using the conceptual mining model, the MILGP model framework has illustrated how production scheduling can be effectively integrated with waste disposal planning for oil sands mining. Based on dyke construction requirements, schedules are generated to provide the required dyke materials. Providing appropriate dyke material to support engineered dyke construction will help in reducing environmental and public concerns related to the risk of tailings dam failure, seepage, potential water contamination and intergenerational transfer of liability (Devenny, 2009). This will be due to the improved integrity of constructed dykes for tailings containment.

The directional pushback mining ensures that timely in-pit tailings storage areas are made available for tailings storage thereby reducing the footprints of the ETF and tailings containment in general. This will also help in reducing environmental and public concerns related to large scarred areas, lack of progressive reclamation and return of the land to traditional use (Devenny, 2009) since less effort will be required to reclaim a smaller disturbed landscape. Using the MILGP model framework therefore results in better environmental management and sustainable oil sands mining.

7.2. Supplementary Experiments

The data shown in Table 8 represents the summary of results for other optimization experiments that were conducted prior to selecting the illustration presented in this paper. The illustration corresponds to run 3 on the table. These experiments were designed to highlight some of the basic properties of the MILGP model. The experiments were ranked based on how smooth the mining proceeds from one period to another and the uniformity of tonnages mined per period. The initial optimization experiment conducted was run 1 which schedules for a north-south mining direction. Further work was done by optimizing with a south-north mining direction (run 2) which yielded a lower NPV and a lower dyke material tonnage. The lower NPV results from mining pushbacks with lower economic block values in the early years. Less ore was mined and a less uniform schedule was produced due to the mining direction.

Further investigations were conducted by increasing the number of mining cuts as in run 3. This resulted in an increase in NPV resulting from an increase in the resolution of the optimization problem. The increased resolution increases the flexibility of the problem as well as the number of decision variables thereby increasing the optimization runtime. A smooth and uniform schedule was generated. Another experiment (run 4) was done to test the MILGP model in terms of placing a higher penalty cost and priority (PP) value on one goal as compared to the others. The increased PP value for OI dyke material further constrains the optimization problem decreasing the ore to dyke material ratio and causing a decrease in the overall NPV which includes dyke construction cost. The dyke material tonnes increases and hence the dyke construction cost. As illustrated in Figure 12, in general within the set mining constraints, as the PP values for dyke material increases, the NPV decreases as a result of a reduction in ore tonnes and/or an increase in dyke material tonnes. This approach is useful when more dyke material is required for tailings containment construction to enable a sustainable mining operation.

Comparing these experiments, run 3 was selected because it generates the best overall NPV as well as a good schedule and the required dyke material tonnage.

8. Conclusions and Future Work

The developed model was able to create value and a sustainable operation by generating a practical, smooth and uniform schedule for ore and dyke material using mining-cuts from block clustering techniques. The schedule gives the planner good control over dyke material and provides a robust platform for effective dyke construction and waste disposal planning. The schedule ensures that the key drivers for oil sands profitability and sustainability, which is maximizing NPV whilst creating timely tailings storage areas are satisfied within an optimization framework. The planner also has the flexibility of choosing goal deviational variables, penalty costs and priorities to achieve a uniform schedule and improved NPV. Similarly, tradeoffs between achieving goals and maximizing NPV or minimizing dyke construction cost can be made.

The overall NPV generated including the dyke construction cost for all pushbacks and destinations is \$14,237 M. The scheduled average ore bitumen grade was between 10.9 and 12.2%. The average ore and OI dyke material fines percent were between 14 and 30%, and 10 and 23% respectively. The total material mined was 4,866.2 Mt. This is made up of 2,720.4 Mt of ore and 1,386.7 Mt of OI dyke material whilst 2,055.2 Mt of TCS dyke material was generated.

Future research will focus on developing more efficient mathematical formulation techniques for the MILGP model that will reduce the solution time for large-scale open pit production scheduling and waste disposal planning problems.

Appendix

Notations

Sets:

$\boldsymbol{K} = \{1, ..., K\}$: set of all the mining-cuts in the model.

$\boldsymbol{J} = \{1, ..., J\}$: set of all the phases (push-backs) in the model.

$\boldsymbol{U} = \{1, ..., U\}$: set of all the possible destinations for materials in the model.

$C_k(L)$: for each mining-cut *k*, there is a set $C_k(L) \subset \boldsymbol{K}$ defining the immediate predecessor mining-cuts above mining-cut *k* that must be extracted prior to extraction of mining-cut *k*, where *L* is the total number of mining-cuts in the set $C_k(L)$.

$M_k(P)$: for each mining-cut *k*, there is a set $M_k(P) \subset \boldsymbol{K}$ defining the immediate predecessor mining-cuts in a specified horizontal mining direction that must be extracted prior to extraction of mining-cut *k* at the specified level, where *P* is the total number of mining-cuts in the set $M_k(P)$.

$N_k(H)$:for each phase *j*, there is a set $N_k(H) \subset \boldsymbol{K}$ defining the mining-cuts within the immediate predecessor pit phases (push-backs) that must be extracted prior to extracting phase *j*, where *H* is an integer number representing the total number of mining-cuts in the set $N_k(H)$.

$B_j(Q)$: for each phase *j*, there is a set $B_j(Q) \subset J$ defining the immediate predecessor pit phases (push-backs) that must be extracted prior to extracting phase *j*, where *Q* is an integer number representing the total number of phases in the set $B_j(Q)$.

Indices, Subscripts and Superscripts:

A parameter, *f*, can take indices, subscripts, and superscripts in the format $f_{k,j}^{u,e,t}$. Where:

$t \in \{1, ..., T\}$: index for scheduling periods.

$k \in \{1, ..., K\}$: index for mining-cuts.

$e \in \{1, ..., E\}$: index for element of interest in each mining-cut.

$j \in \{1, ..., J\}$: index for phases.

$u \in \{1, ..., U\}$: index for possible destinations for materials.

d, *l*, *m*, *p*: subscripts and superscripts for overburden and interburden dyke material, tailings coarse sand dyke material, mining and processing respectively.

Parameters:

$d_k^{u,t}$: the discounted profit obtained by extracting mining-cut *k* and sending it to destination *u* in period *t*.

$v_k^{u,t}$: the discounted revenue obtained by selling the final products within mining-cut *k* in period *t* if it is sent to destination *u*, minus the extra discounted cost of mining all the material in mining-cut *k* as ore and processing at destination *u*.

$p_k^{u,t}$: the extra discounted cost of mining all the material in mining-cut *k* in period *t* as overburden and interburden dyke material for construction at destination *u*.

$h_k^{u,t}$: the extra discounted cost of mining all the material in mining-cut *k* in period *t* as tailings coarse sand dyke material for construction at destination *u*.

$q_k^{u,t}$: the discounted cost of mining all the material in mining-cut *k* in period *t* as waste and sending it to destination *u*.

g_k^e : the average grade of element *e* in ore portion of mining-cut *k*.

$\underline{g}^{u,t,e}$: the lower bound on the required average head grade of element *e* in period *t* at processing destination *u*.

$\overline{g}^{u,t,e}$: the upper bound on the required average head grade of element *e* in period *t* at processing destination *u*.

f_k^e : the average percent of fines in ore portion of mining-cut *k*.

$\underline{f}^{u,t,e}$: the lower bound on the required average fines percent of ore in period *t* at processing destination *u*.

$\overline{f}^{u,t,e}$: the upper bound on the required average fines percent of ore in period t at processing destination u.

f_k^d : the average percent of fines in overburden and interburden dyke material portion of mining-cut k.

$\underline{f}^{u,t,d}$: the lower bound on the required average fines percent of overburden and interburden dyke material in period t at dyke construction destination u.

$\overline{f}^{u,t,d}$: the upper bound on the required average fines percent of overburden and interburden dyke material in period t at dyke construction destination u.

o_k : the ore tonnage in mining-cut k.

w_k: the waste tonnage in mining-cut k.

d_k: the overburden and interburden dyke material tonnage in mining-cut k.

l_k: the tailings coarse sand dyke material tonnage in mining-cut k.

$T_m^{u,t}$: the mining goal (tonnes) in period t at destination u.

$d_1^{-,u,t}$: the negative deviation from the mining goal (tonnes) in period t at destination u.

$d_1^{+,u,t}$: the positive deviation from the mining goal (tonnes) in period t at destination u.

$T_p^{u,t}$: the processing goal in period t at destination u (tonnes).

$d_2^{-,u,t}$: the negative deviation from the processing goal in period t at destination u (tonnes).

$d_2^{+,u,t}$: the positive deviation from the processing goal in period t at destination u (tonnes).

$T_d^{u,t}$: the overburden and interburden dyke material goal in period t at destination u (tonnes).

$d_3^{-,u,t}$: the negative deviation from the overburden and interburden dyke material goal in period t at destination u (tonnes).

$d_3^{+,u,t}$: the positive deviation from the overburden and interburden dyke material goal in period t at destination u (tonnes).

$T_l^{u,t}$: the tailings coarse sand dyke material goal in period t at destination u (tonnes).

$d_4^{-,u,t}$: the negative deviation from the tailings coarse sand dyke material goal in period t at destination u (tonnes).

$d_4^{+,u,t}$: the positive deviation from the tailings coarse sand dyke material goal in period t at destination u (tonnes).

$r^{u,e}$: the proportion of element e recovered (processing recovery) if it is processed at destination u.

$p^{e,t}$ the price of element e in present value terms per unit of product.

$cs^{e,t}$: the selling cost of element e in present value terms per unit of product.

$cp^{u,e,t}$: the extra cost in present value terms per tonne of ore for mining and processing at destination u.

$ck^{u,t}$: the cost in present value terms per tonne of overburden and interburden dyke material for dyke construction at destination u.

$ct^{u,t}$: the cost in present value terms per tonne of tailings coarse sand dyke material for dyke construction at destination u.

$cm^{u,t}$: the cost in present value terms of mining a tonne of waste in period t and sending it to destination u.

P_1: the priority level associated with minimizing the deviations from the mining goal.

P_2: the priority level associated with minimizing the deviations from the processing goal.

P_3: the priority level associated with minimizing the deviations from the overburden and interburden dyke material goal.

P_4: the priority level associated with minimizing the deviations from the tailings coarse sand dyke material goal.

a_1: the penalty paid per tonne in deviating from the mining goal.

a_2: the penalty paid per tonne in deviating from the processing goal.

a_3: the penalty paid per tonne in deviating from the overburden and interburden dyke material goal.

a_4: the penalty paid per tonne in deviating from the tailings coarse sand dyke material goal.

Decision Variables

$x_k^{u,t} \in [0,1]$: a continuous variable representing the portion of mining-cut k to be extracted as ore and processed at destination u in period t.

$z_k^{u,t} \in [0,1]$: a continuous variable representing the portion of mining-cut k to be extracted as overburden and interburden dyke material and used for dyke construction at destination u in period t.

$s_k^{u,t} \in [0,1]$: a continuous variable representing the portion of mining-cut k to be extracted as tailings coarse sand dyke material and used for dyke construction at destination u in period t.

$y_k^{u,t} \in [0,1]$: a continuous variable representing the portion of mining-cut k to be mined in period t and sent to destination u, which includes both ore, overburden and interburden dyke material and waste.

$b_k^t \in [0,1]$: a binary integer variable controlling the precedence of extraction of mining-cuts. b_k^t is equal to one if the extraction of mining-cut k has started by or in period t, otherwise it is zero.

References

Akaike, A., and Dagdelen, K. (1999). A strategic production scheduling method for an open pit mine. Proc. of the 28th International Symposium on the Application of Computers and Operations Research in the Mineral Industry, Littleton, 729-738.

Askari-Nasab, H., and Awuah-Offei, K. (2009). Mixed integer linear programming formulations for open pit production scheduling, Edmonton, University of Alberta, Mining Optimization Laboratory Report One.

Askari-Nasab, H., Awuah-Offei, K., and Eivazy, H. (2010). Large-scale open pit production scheduling using mixed integer linear programming. *Int. J. Min. Miner. Eng.*, 2(3), 185-214. http://dx.doi.org/10.1504/IJMME.2010.037624

Askari-Nasab, H., and Ben-Awuah, E. (2011). Integration of oil sands mine planning and waste management using goal programming. Proc. of the 35th International Symposium on the Application of Computers and Operations Research in the Mineral Industry, Wollongong, 329-350.

Askari-Nasab, H., Pourrahimian, Y., Ben-Awuah, E., and Kalantari, S. (2011). Mixed integer linear programming formulations for open pit production scheduling. *J. Min. Sci.*, 47(3), 338-359. http://dx.doi.org/ 10.1134/S1062739147030117

Ben-Awuah, E. and Askari-Nasab, H. (2011). Oil sands mine planning and waste management using mixed integer goal programming. *Int. J. Min. Reclam. Environ.*, 1-22. http://dx.doi.org/10.1080/17480930.2010.549656

Caccetta, L., and Hill, S. P. (2003). An application of branch and cut to open pit mine scheduling. *J. Global Optimiz.*, 27(2-3), 349-365. http://dx.doi.org/10.1023/A:1024835022186

Chanda, E.K.C., and Dagdelen, K. (1995). Optimal blending of mine production using goal programming and interactive graphics systems. *Int. J. Min. Reclam. Environ.*, 9(4), 203-208. http://dx.doi.org/10.1080/09208119508964748

Dagdelen, K. (1985). Optimum multi-period open pit mine production scheduling by Lagrangian parameterization. PhD Dissertation, School of Mines, University of Colorado, Golden, Colorado.

Dagdelen, K., and Johnson, T. B. (1986). Optimum open pit mine production scheduling by Lagrangian parameterization. Proc. of the 19th International Symposium on the Application of Computers and Operations Research in the Mineral Industry, Littleton, 127-142.

Devenny, D.W. (2009). Overview of oil sands tailings, Calgary, Alberta Energy Research Institute.

Esfandiri, B., Aryanezhad, M.B., and Abrishamifar, S.A. (2004). Open pit optimization including mineral dressing criteria using 0-1 non-linear goal programming. Transactions of the Institutions of Mining and Metallurgy: Section A. 113(1), 3-16.

Fauquier, R., Eaton, T., Bowie, L., Treacy, D., and Horton, J. (2009). In-pit dyke construction planning, Ft. McMurray, Shell Upstream Americas.

Fort Hills Energy Corporation (2009). Annual tailings managment plan, Calgary, Suncor Energy.

Gershon, M.E. (1983). Optimal mine production scheduling: evaluation of large scale mathematical programming approaches. *Int. J. Min. Miner. Eng.*, 1(4), 315-329. http://dx.doi.org/10.1007/BF00881548

Hannan, E.L. (1985). An assessment of some criticisms of goal programming. *Comput. Oper. Res.*, 12(6), 525-541. http://dx.doi.org/10.1016/0305-0548(85)90052-8

Hochbaum, D.S., and Chen, A. (2000). Performance analysis and best implementations of old and new algorithms for the open-pit mining problem. *Oper. Res.*, 48(6), 894-914. http://dx.doi.org/10.1287/opre.48.6.894.12392

Holmström, K. (2009). TOMLAB /CPLEX (Version 11.2), Pullman, WA, USA, Tomlab Optimization.

Horst, R., and Hoang, T. (1996). Global optimization: Deterministic approaches. Springer.

ILOG Inc. (2007). ILOG CPLEX, Ver. 11.0. Pullman, WA, USA.

Jackpine Mine (2009). Tailings managment plan, Calgary, Shell Canada Limited.

Johnson, T.B. (1969). Optimum open-pit mine production scheduling. Proc. of the 8th International Symposium on Computers and Operations Research, Utah, 539-562.

Kaufman, L., and Rousseeuw. (1990). Finding groups in data: An introduction to cluster analysis. J. Wiley.

Kearl Oil Sands Project (2009). Annual tailings plan submission, Calgary, Imperial Oil Resources.

McFadyen, D. (2008). Directive 074, Calgary, Energy Resources Conservation Board.

Muskeg River Mine (2009). Tailings management plan, Calgary, Shell Canada Limited.

Newman, A.M., Rubio, E., Caro, R., Weintraub, A., and Eurek, K. (2010). A review of operations research in mine planning. *Interfaces*, 40(3), 222-245. http://dx.doi.org/10.1287/inte.1090.0492

Suncor Energy Incorporated Oil Sands (2009). Annual tailings managment plan, Calgary, Suncor Energy.

Syncrude Aurora North (2009). Annual tailings plan submission, Fort McMurray, Syncrude Canada Limited.

Syncrude Aurora South (2009). Annual tailings plan submission, Fort McMurray, Syncrude Canada Limited.

Syncrude Mildred Lake (2009). Annual tailings plan submission, Fort McMurray, Syncrude Canada Limited.

Whittle, J. (1989). The facts and fallacies of open pit design, North Balwyn, Victoria, Australia, Whittle Programming Pty Ltd.

Wolsey, L. A. (1998). Integer programming. J. Wiley.

Zeleny, M. (1980). Multiple objectives in mathematical programming: letting the man in. *Comput. Oper. Res.*, 7(1-2), 1-4. http://dx.doi.org/10.1016/0305-0548(80)90010-6

Zhang, Y.D., Cheng, Y.P., and Su, J. (1993). Application of goal programming in open pit planning. *Int. J. Min. Reclam. Environ.*, 7(1), 41-45. http://dx.doi.org/10.1080/09208119308964683