

A Hesitant Heterogeneous Approach for Environmental Impact Significance Assessment

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ABSTRACT. Projects and industrial activities developed by human beings generally affect their surroundings. For this reason, the efficacy of Environmental Impact Significance Assessment (EISA) method is increasingly demanded. There are multiple criteria involved in EISA problems that interact each other and may have either quantitative or qualitative nature. Classical approaches for EISA are not efficient in managing either uncertainty or different types of information, and the results obtained are numerical values difficult to interpret. The complexity of such problems and the uncertain information involved imply that experts sometimes hesitate among several values to express their assessments over criteria and they do not want to provide just one value, because it cannot reflect their hesitation. This brings about incomplete data in the experts' assessments. In order to deal with such situations, the concepts of hesitant fuzzy sets and hesitant fuzzy linguistic term sets have recently been introduced in quantitative and qualitative contexts respectively. Therefore, the aim of this paper is to define an EISA approach that allows managing heterogeneous information, including hesitant information. This approach provides a flexible evaluation framework in which experts can express their assessments, using different information domains that are unified in a linguistic domain by the 2-tuple linguistic model. It also obtains accurate results, which are easy to understand and interpret.

Keywords: Environmental Impact Significance Assessment, multicriteria decision making, heterogeneous information, hesitant information

1. Introduction

Projects and activities developed by human beings inevitably cause positive or negative changes over the environment. Therefore, project approval decision making implies complex interdisciplinary processes to optimize resources in a sustainable way. An environmental impact assessment is a systematic process of determining and managing the potential impacts of proposed or existing human actions (projects, plans, programs, legislations, activities) and their alternatives on the environment (Lawrence, 2013).

Impact assessments have been used for quantifying impacts, either by using indicators to obtain the difference of environmental quality between situations “with project” and “without project” (a quantitative assessment based on magnitude), or according to the experts' judgements, commonly expressed in ordinal scales (a qualitative assessment based on significance) (Blanco et al., 2009; Conesa, 1997). Our interest is focused on the latter perspective, i.e., the Environmental Impact

Significance Assessment (EISA) which is the process for determining the importance of the project's impacts on the affected environmental factors, considering subjective judgements provided in a qualitative or quantitative way.

There are two main problems related to EISA: (i) the selection of the best project from a set of alternatives, and (ii) the evaluation of environmental impacts of a single project. Both problems have a multi-criteria character since they are undertaken considering different interacting elements, and it is impossible to reduce these problems to one single criterion (Cloquell et al., 2007).

A Multi-Criteria Decision Making (MCDM) problem consists of selecting the best alternative from a set of possible alternatives according to a set of criteria (Ishizaka and Nemery, 2013; Pedrycz et al., 2011). In recent decades the interest in the application of MCDM methods in EISA has been renewed, since it produces transparent environmental impact assessment results and allows the consolidation of the interdisciplinary information into a unified decision making framework (Bojórquez-Tapia et al., 2005; Shepard, 2005).

An EISA problem can be intuitively modeled as a MCDM problem in which a project is evaluated through the impacts caused by the effect of its interactions with the surrounding environmental factors, and such impacts are assessed by one or more experts considering multiple criteria. It usually man-

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ages different features: first, large amounts of data that might be of poor quality because of uncertainty, measurement errors and even absence of such data; second, different spatial and temporal scales of impacts; and third, the concurrence of teams from several disciplines or areas of knowledge (Vrana and Aly, 2011; Onar, et al., 2014).

A key issue in EISA is to maximize the accuracy of the assessments while the results are kept understandable for stakeholders (Ijas et al., 2010). Given the complexity in EISA problems and the vagueness of the information, the preliminary judgements and the recognition of non-professional knowledge have already led to an increasingly formal (explicit, structured, documented) incorporation of experts' assessments in EISA models (Krueger et al., 2012). It demands a broad and inclusive concept of expertise that includes professionals such as scientists, managers of technical agencies, government reviewers, experienced members of the public and local community groups or individuals, and so on. Environmental assessments are subjective expressions of societal, group, and individual values and opinions which are not objective or directly measurable (Shepard, 2005); therefore, EISA methods should effectively deal with such feelings, beliefs, and imprecise concepts (Ahmadi et al., 2015; Li et al., 2011).

In an EISA problem, there are many criteria involved and they can be different in nature, either qualitative or quantitative. Nevertheless, one of the main limitations in traditional EISA methods is concerned to the incapacity of handling suitably information of qualitative and quantitative nature, because they indistinctively model the information into a numerical scale. There is another limitation in classical EISA methods (Conesa, 1997; Pastakia and Jensen, 1998) that consists in using precise numerical values to assess criteria, despite the inherent ambiguities and imprecisions of impact assessment data. Moreover, these methods provide numerical results difficult to interpret in an environmental framework.

The complexity of EISA problems not only implies the necessity of a heterogeneous context of definition (Carrasco and Villar, 2012; Herrera et al., 2005; Li et al., 2010) in which different types of information can model knowledge, the expertise and the uncertainty related to the information, but also makes experts provide vague and imprecise information and hesitate about their assessments.

For instance, if an expert has to provide his/her assessment about the recoverability of an impact, he/she might use a numerical value, because it is possible to obtain a precise reconstruction time of the affected factor by human intervention. However, if an expert has to provide his/her assessment about the intensity of an environmental impact, it is much more difficult to provide precise numerical values because this criterion is qualitative in nature and it seems more suitable to use linguistic terms, such as "low", "medium", "high" that better model the qualitative nature and inherent subjectivity than a numerical value.

Sometimes, due to the lack of information and/or knowledge about the EISA problem, experts might hesitate among several values to elicit their assessments. In such cases, the use

of just one value could not be enough to reflect their hesitation in a precise way, and it might occur that experts do not provide any value, thus causing incomplete data. In order to cope with the uncertainty provoked by hesitancy, in quantitative contexts, Torra introduced the notion of Hesitant Fuzzy Sets (HFS) (Torra, 2010) and in qualitative contexts, Rodríguez et al. presented the concept of Hesitant Fuzzy Linguistic Terms Set (HFLTS) (Rodríguez et al., 2012). For instance, if an expert hesitates in determining if the intensity of an impact is "low" or "medium", he/she could elicit that the intensity is "between low and medium".

The aim of this paper is to propose a new EISA method that attempts to overcome the limitations of the traditional EISA methods. This proposal is able to deal with a heterogeneous context in which different types of information, such as numerical, linguistic, interval-valued, HFS and HFLTS, can be used to assess the criteria defined in an EISA problem, according to the criteria nature and experts' hesitation. Moreover, the use of hesitant information allows to reflect the experts' hesitation in a proper way. The proposed model conducts the heterogeneous information in a linguistic domain by means of the 2-tuple linguistic model (Herrera and Martínez, 2000; Martínez and Herrera, 2012), which accomplishes the computing with words (CWW) processes in a symbolic and precise way, obtaining linguistic results that are easy to understand by experts involved in EISA problems.

The remainder of the paper is organized as follows. Section 2 reviews a classical method for EISA problems and the management of hesitant information. Section 3 extends the 2-tuple based heterogeneous approach by including hesitant information. Section 4 presents the novel hesitant heterogeneous approach for EISA. Section 5 shows an example to illustrate the usefulness of the proposed approach. Section 6 highlights the main advantages of the proposal, and some conclusions are pointed out in Section 7. Appendix contains some necessary concepts to easily understand the proposed approach.

2. Preliminaries

This section revises a general method for EISA and shows its limitations to solve EISA problems in complex and uncertain contexts. The need of dealing with hesitant situations in which experts hesitate among several values to express their opinions has driven to the introduction of the HFS and HFLTS in the proposed model, thus they will be revised too.

2.1. Classical Method for EISA Problems

As discussed earlier, the EISA can be naturally modeled as a MCDM problem considering the following elements:

- A set of environmental factors $F = \{f_i | i \in \{1, \dots, m\}\}$ affected by different actions.
- A set of actions $A = \{a_j | j \in \{1, \dots, n\}\}$ executed to accomplish the project.
- A set of impacts $I = \{I_{ij} | i \in \{1, \dots, m\}, j \in \{1, \dots, n\}\}$ caused by the interaction of factors and actions.
- A set of criteria $C = \{c_r | r \in \{1, \dots, t\}\}$ characterizing these im-

pacts.

- One or more experts $E = \{e_p \mid p \in \{1, \dots, q\}\}$ providing information about each impact over the set of criteria.

Matrix-based methods (Bojórquez-Tapia et al., 1998; Conesa, 1997; Pastakia and Jensen, 1998), which have a MCDM character, are the most frequently used to solve EISA problems because they are simple, inexpensive, and quick to apply (Bojórquez-Tapia et al., 1998; Toro et al., 2013).

A general method for EISA was introduced by Conesa (Conesa, 1997) based on the Battelle-Columbus Laboratories' method (Dee et al., 1973). It is briefly revised below and later on some of its limitations are pointed out.

2.1.1. The General Method for EISA

This method accomplishes the following steps:

1. Constructing the EISA matrix: A double-entry matrix relates the set of factors and the set of actions, and their intersections represents the possible impacts.

2. Assessing criteria for impacts: Once the impacts are identified, they are evaluated according to the criteria and numerical scales (Morón et al., 2009; Conesa, 1997). In (Conesa, 1997), it was also defined the nature as the beneficial (+) or harmful (-) nature of the interaction of an action over an environmental factor.

3. Computing significance of impacts: For each identified impact, the value of significance is computed by using the following mathematical expression: $Y_{ij} = \pm(3IN_{ij} + 2EX_{ij} + MO_{ij} + PE_{ij} + RV_{ij} + SY_{ij} + AC_{ij} + EF_{ij} + PR_{ij} + RC_{ij})$

The conventional definition of criteria and acronyms used in the expression can be consulted in (Morón et al., 2009; Conesa, 1997). The result, independently of the nature, is a numerical value in the interval [13,100], which is classified as follow:

- Irrelevant or compatible, if $Y_{ij} \in [13, 25]$,
- Moderate, if $Y_{ij} \in [25, 50]$,
- Severe, if $Y_{ij} \in [50, 75]$ and
- Critical, if $Y_{ij} \in [75, 100]$.

4. Updating the EISA matrix: Significance values obtained in the previous step are used to update the cross-boxes with the corresponding Y_{ij} .

5. Obtaining qualitative assessment indicators: Throughout algebraic sums by columns and rows respectively, the significance of actions and factors is calculated as well as the total significance of the project:

- Total significance of impacts over each factor: $Y_i = \sum_j Y_{ij}$
- Total significance of impacts caused by each action: $Y_j = \sum_i Y_{ij}$
- Total significance: $Y = \sum_i \sum_j Y_{ij}$

2.1.2. Main Limitations

Even though this EISA general method has been widely applied in the European Union, as well as in Central and South

America because of its versatility, simplicity, and cost-effectiveness (Toro et al., 2013), the analysis of its performance reveals some important weaknesses:

- Inconsistencies and inflexibility in the mathematical expression for the calculation of significance: On the one hand, criteria and their weights are fixed, although the environmental impact assessment process varies greatly. The effectiveness of an EISA is often highly dependent on how well the process fits the context (Lawrence, 2013).
- No uncertainty modeling: As the EISA of any project requires the evaluation of the likely positive and/or negative effects of very diverse actions on many and various environmental factors, the uncertainty and inaccuracy is inherent in the process. However, using solely numerical precise values for assessing criteria means such inherent uncertain nature of EISA is not considered.
- Incapacity for dealing with heterogeneous information: Regardless their qualitative or quantitative character, criteria are assessed by means of precise numerical values or linguistic descriptors, that are conducted into numerical values to compute the significance value. Therefore, it seems more appropriate to use different types of information such as, numerical, linguistic, interval-valued and so on, according to the nature of the criteria for their assessment (Herrera and Martínez, 2001).
- Incapacity for dealing with experts' hesitation: The classical model does not provide any mechanism for dealing with the hesitant information that often appears in these problems.
- Low interpretability of results: Outcomes are numerical values that are not always easy to interpret by experts involved in decision making based on EISA.

2.2. Management of Hesitant Information

An EISA problem is complex because deals with a large amount of information which might be vague and imprecise due to the measurements errors, missing data, etc. Its complexity, the lack of information or even the lack of experts' knowledge on it, implies that experts might hesitate among several values when they express their assessments about each impact and criterion. In these hesitant situations, the use of just one value is not enough to reflect their assessments and often experts do not provide any value because of their hesitation. In order to deal with this type of uncertainty provoked by hesitancy, in quantitative contexts, Torra introduced the concept of HFS (Torra, 2010) as an extension of fuzzy sets.

Definition 1. (Torra, 2010) Let X be a reference set, a HFS on X is a function η that returns a non-empty subset of values in $[0, 1]$:

$$\eta : X \rightarrow \rho([0, 1]) \tag{1}$$

Therefore, an HFS allows considering several possible values to fix the membership degree of an element to a fuzzy set. In the seminal paper (Torra, 2010), Torra proposed some basic operations for HFSs based on intervals and intuitionistic

fuzzy sets. Some more operations can be found in Pei and Yi, (2015).

Similarly to the hesitant situations managed by means of HFS, in qualitative contexts, it may occur that experts hesitate among several linguistic terms to assess a linguistic variable. To deal with such situations Rodríguez et al. (Rodríguez et al., 2012) proposed the concept of HFLTS which is based on the fuzzy linguistic approach (Zadeh, 1975) and extends the concept of HFS to linguistic contexts.

Definition 2. (Rodríguez et al., 2012) Let $S = \{s_0, \dots, s_g\}$ be a linguistic term set, a HFLTS H_s , is defined as an ordered finite subset of consecutive linguistic terms of S :

$$H_S = \{s_i, s_{i+1}, \dots, s_j\} \text{ such that } s_k \in S, k \in \{i, \dots, j\} \quad (2)$$

Some basic operations for HFLTS can be found in (Rodríguez et al. (2012).

In real-world problems experts commonly use linguistic expressions to provide their assessments and opinions instead of multiple linguistic terms. Therefore, to improve the elicitation of linguistic information, Rodríguez et al. (Rodríguez et al., 2013) proposed the use of context-free grammars to generate expressions close to human beings' reasoning. A context-free grammar G_H , was defined to generate comparative linguistic expressions similar to the expressions used by experts in real-world decision making problems.

Definition 3. (Rodríguez et al., 2013) Let G_H be a context-free grammar, and $S = \{s_0, \dots, s_g\}$ be a linguistic term set. The elements of $G_H = (V_N, V_T, I, P)$ are defined as follows:

$$\begin{aligned} V_N &= \{\langle \text{primary term} \rangle, \langle \text{composite term} \rangle, \langle \text{unary relation} \rangle, \langle \text{binary relation} \rangle, \langle \text{conjunction} \rangle\}, \\ V_T &= \{\text{greater than, lower than, at least, at most, between, and, } \\ & s_0, \dots, s_g\}, \\ I &\in V_N. \\ P &= \{I ::= \langle \text{primary term} \rangle | \langle \text{composite term} \rangle \\ & \langle \text{composite term} \rangle ::= \langle \text{unary relation} \rangle \langle \text{primary term} \rangle | \\ & \langle \text{binary relation} \rangle \langle \text{primary term} \rangle \langle \text{conjunction} \rangle \langle \text{primary term} \rangle \\ & \langle \text{primary term} \rangle ::= s_0 | s_1 | \dots | s_g \\ & \langle \text{unary relation} \rangle ::= \text{greater than} | \text{lower than} | \text{at least} | \text{at most} \\ & \langle \text{binary relation} \rangle ::= \text{between} \\ & \langle \text{conjunction} \rangle ::= \text{and} \} \end{aligned}$$

The comparative linguistic expressions can be represented into a HFLTS applying a transformation function E_{G_H} defined as follows:

Definition 4. (Rodríguez et al., 2013) Let E_{G_H} be a function that transforms comparative linguistic expressions S_H obtained from a context-free grammar G_H , into HFLTS H_S , where S is the linguistic term set used by G_H , and S_H is the set of linguistic expressions generated by G_H .

$$E_{G_H} : S_H \rightarrow H_S \quad (3)$$

The linguistic expressions generated by the production rules will be transformed into HFLTS in different ways depend-

ing on their meaning:

- $E_{G_H}(s_i) = \{s_i / s_i \in S\}$
- $E_{G_H}(\text{lower than } s_i) = s_i / s_j \in S \text{ and } s_j < s_i$
- $E_{G_H}(\text{greater than } s_i) = s_i / s_j \in S \text{ and } s_j > s_i$
- $E_{G_H}(\text{at least } s_i) = s_i / s_j \in S \text{ and } s_j \geq s_i$
- $E_{G_H}(\text{at most } s_i) = s_i / s_j \in S \text{ and } s_j \leq s_i$
- $E_{G_H}(\text{between } s_i \text{ y } s_j) = s_k / s_k \in S \text{ and } s_i \leq s_k \leq s_j$

3. Extension of the Heterogeneous Approach to Deal with Hesitant Information

A common situation in problems with multiple criteria is the appearance of information of different nature that might be modeled with heterogeneous information such as numerical, linguistic, interval-valued and so on. There are different heterogeneous approaches (Carrasco and Villar, 2012; Herrera et al., 2005; Li et al., 2010) that manage these types of information. Nevertheless, none of them deals with hesitant information. Despite the concept of hesitant information has been recently introduced (Torra, 2010; Rodríguez et al., 2012), it has been successfully applied in different fields, such as decision making (Liao and Xu, 2013; Liu and Rodríguez, 2014; Wei et al., 2015), evaluation (Yu et al., 2013), clustering (Zhang and Xu, 2015), etc.

In this section we extend the heterogeneous approach (Herrera et al., 2005) revised in Appendix by including hesitant information (HFS and HFLTS). To do so, different transformation functions have been defined to conduct the hesitant information into a linguistic domain as depicts Figure 1. These functions are defined as follows.

• Transforming HFS into a linguistic domain

A HFS cannot be directly transformed into a 2-tuple linguistic value, therefore, the unification phase is divided into three steps:

1. *Obtaining an interval:* A numeric interval is built by using the lower and upper bounds defined for a HFS.

Definition 5. Let h_1 be a HFS, the interval of the h_1 is:

$$h_{v_1} = [h_1^-, h_1^+]$$

being $h_1^- = \min\{\gamma \mid \gamma \in h\}$ and $h_1^+ = \max\{\gamma \mid \gamma \in h\}$.

2. *Transforming into fuzzy sets:* A transformation function $\tau_{v_{S_T}} : V \rightarrow F(S_T)$ which transforms an interval h_i into a fuzzy set in S_T is applied (see Appendix, Definition 13).

3. *Transforming into 2-tuple linguistic values:* The fuzzy set $F(S_T)$, is converted into a 2-tuple linguistic value by using the transformation function $\chi(\cdot)$ (see Appendix, Definition 15).

• Transforming HFLTS into a linguistic domain

Due to a HFLTS is compounded of several linguistic terms, to transform it into a linguistic domain, such linguistic ter-

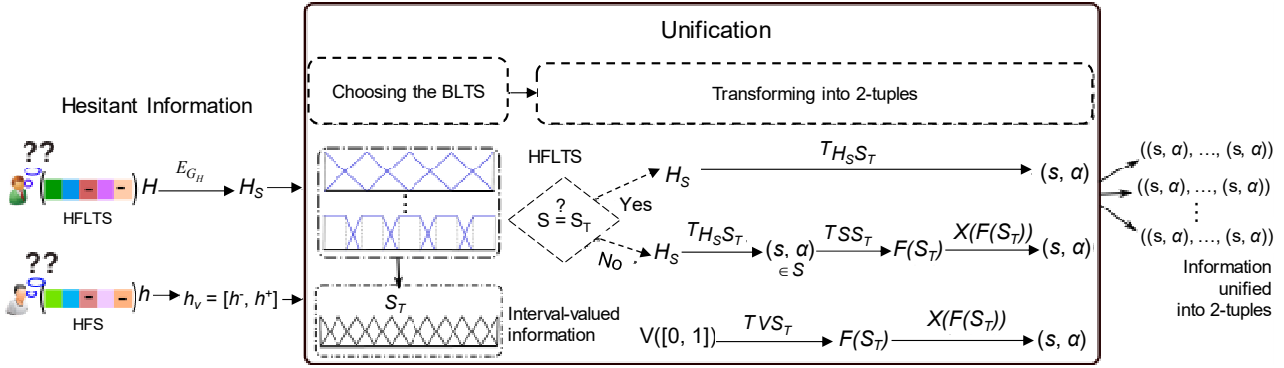


Figure 1. Scheme to manage hesitant information.

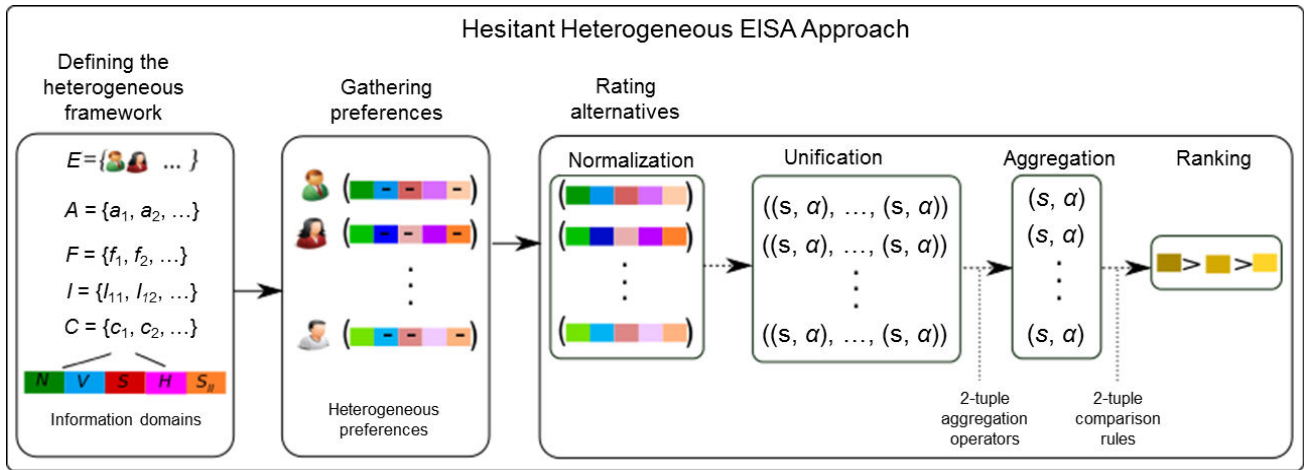


Figure 2. The general structure of the hesitant heterogeneous EISA approach.

ms are aggregated and the result is represented by means of a 2-tuple linguistic value.

In the unification process of HFLTS into 2-tuple linguistic values, there are two different situations to be considered: (i) the linguistic term set S , used by the grammar G_H is selected as the BLTS, (ii) the linguistic term set S , is different to the BLTS, which means that S_T is a linguistic term set with a granularity greater than S . Consequently the unification has to regard both options.

- If $S = S_T$:

Definition 6. Let $H_{S_i} = \{s_1, \dots, s_j\}$ be a HFLTS, the transformation function $\tau_{H_S S_T} : H_S \rightarrow S_T \times [-0.5, 0.5]$ is defined as follows:

$$\tau_{H_S S_T}(H_{S_i}) = \Delta \left(\sum_{k=i}^j w_k * k \right) \quad (4)$$

where $w_k \in [0, 1]$, $\sum_{k=i}^j w_k = 1$ and $k = \{i, \dots, j\}$.

- If $S \neq S_T$:

Once the 2-tuple linguistic value is obtained from the HFLTS, it is transformed into S_T applying the transformation function for linguistic domain (see Appendix Definition 14), that means that the 2-tuple linguistic value on S will be transferred

into a fuzzy set in S_T . Afterwards, the result is transformed into 2-tuple on S_T by using the transformation function $\chi(\cdot)$.

4. A Hesitant Heterogeneous Approach for EISA

Here it is presented a novel hesitant heterogeneous approach for EISA which is able to manage numerical, linguistic, interval-valued, HFS and HFLTS domains in the assessment process according to the criteria nature and the experts' hesitation. It provides linguistic results accurate and easy to understand by experts involved in the EISA problem. In order to tackle the difficulty of dealing with this hesitant heterogeneous context in an EISA problem in a rational and well-organized manner, the proposed approach follows the classical decision analysis scheme (Clemen, 1996) which is divided into three main phases: (i) Defining the heterogeneous framework, (ii) Gathering preferences and (iii) Rating alternatives. Figure 2 shows the general structure of the approach.

As the criteria proposed by different authors for impact assessments, differ in number as in character, a suitable EISA evaluation framework enables the gathering of such heterogeneous information depending on the criteria nature. The first phase specifies the structure and representation of the input data including the information domains in which the experts will provide their preferences about impacts considering dif-

ferent criteria. Such preferences are collected in the second phase. The third phase accomplishes the CWW processes to obtain linguistic significance values easy to interpret (Martínez et al., 2010).

To do so, the heterogeneous information is conducted into a common 2-tuple linguistic domain by using the transformation functions introduced in Section 3 and Appendix. The linguistic information is then aggregated in a multiple-step process to obtain the significance values which are compared to obtain final ranking of impacts, factors and actions. All phases are described in further detail in the following subsections.

4.1. Defining the Heterogeneous Framework

In this phase the elements of the EISA MCDM problem are defined according to Section 2.1. Additionally, let $W = (w_r | r \in \{1, \dots, t\})$, $w_r \in [0, 1]$ with $\sum_{r=1}^t w_r = 1$ be the weighting vector for the criteria, and $U = \{u_{ij} | i \in \{1, \dots, m\}, j \in \{1, \dots, n\}\}$, $u_{ij} \in \{-1, 1\}$ be the set for representing the *nature* of impacts, where -1 and 1 stand for the negative and positive impacts respectively.

Since criteria represent different dimensions of the impacts, they may conflict with each other (Triantaphyllou, 2000) originating the division of C into two subsets: C^1 with benefit criteria and C^2 with cost criteria. It means that for the benefit criteria, the more the better, and for the cost criteria, the less the better. Furthermore $C = C^1 \cup C^2$ and $C^1 \cap C^2 = \emptyset$ where \emptyset is an empty set.

The preference provided by expert $e_p \in E$ about the impact of action $a_j \in A$ over factor $f_i \in F$ according to the criteria $c_r \in C$ is represented by x_{ij}^{rp} .

Criteria could be assessed using the following information domains, $O = \{N, V, S, H, S_H\}$:

- Numerical values (N): $x_{ij}^{rp} = v_{ij}^{rp} \in [0, 1]$.
- Interval values (V): $x_{ij}^{rp} = V([0, 1]) = [a_{ij}^{rp}, b_{ij}^{rp}]$ with $a_{ij}^{rp}, b_{ij}^{rp} \in [0, 1]$ and $a_{ij}^{rp} \leq b_{ij}^{rp}$.
- Linguistic values (S): $x_{ij}^{rp} = s_{ij}^{rp} \in S = \{s_0, \dots, s_g\}$ being $g + 1$ the cardinality of the linguistic term set S .
- HFS (H): $x_{ij}^{rp} = h_{ij}^{rp} \in \rho([0, 1])$.
- Comparative linguistic expressions S_H : $x_{ij}^{rp} = ll_{ij}^{rp}$ generated by using context-free grammar G_H revised in Section 2.2.

4.2. Gathering Preferences

Once the heterogeneous framework has been defined, each expert provides her/his preferences about impacts I_{ij} by means of assessment vectors: $X_{ij}^p = (x_{ij}^{1p}, \dots, x_{ij}^{tp})$.

4.3. Rating Alternatives

Rating alternatives is critical in the resolution process, because in this phase the heterogeneous preferences are synthesized into full-valued significance assessments. Therefore, the four steps presented in Figure 2 are carried out. (i) The heterogeneous information is normalized to eliminate cost/benefit criteria conflicts. (ii) The information is conducted into a unique linguistic domain to facilitate the aggregation of preferen-

ces. (iii) The preferences are aggregated to obtain linguistic significance values for impacts, factors and actions; as well as an interpretable full-value global significance of the project. (iv) Finally, a ranking of impacts, factors and actions is obtained.

4.3.1. Normalization

Due to the fact that the attributes are either cost or benefit ones, their values will be normalized as benefit criteria according to their type of information. Therefore, from a gathered cost preference x_{ij}^{rp} , its correspondent benefit preference, x_{ij}^{-rp} , is expressed as:

- Numerical domain

$$x_{ij}^{-rp} = \begin{cases} 1 - x_{ij}^{rp} & \text{if } c_r \in C^2 \\ x_{ij}^{rp} & \text{otherwise} \end{cases} \quad (5)$$

- Interval-valued domain

$$x_{ij}^{-rp} = \begin{cases} [1 - b_{ij}^{rp}, 1 - a_{ij}^{rp}] & \text{if } c_r \in C^2 \\ [a_{ij}^{rp}, b_{ij}^{rp}] & \text{otherwise} \end{cases} \quad (6)$$

- Linguistic domain

$$x_{ij}^{-rp} = \begin{cases} \text{Neg}_S(s_{ij}^{rp}) & \text{if } c_r \in C^2 \\ s_{ij}^{rp} & \text{otherwise} \end{cases} \quad (7)$$

where Neg is a linguistic negation operator such that $\text{Neg}(s_i) = s_{g-i}$ (Herrera and Martínez, 2000).

- HFS domain

$$x_{ij}^{-rp} = \begin{cases} [1 - h_{ij}^{+rp}, 1 - h_{ij}^{-rp}] & \text{if } c_r \in C^2 \\ [h_{ij}^{-rp}, h_{ij}^{+rp}] & \text{otherwise} \end{cases} \quad (8)$$

where $h^+ = \max\{\gamma | \gamma \in h\}$ and $h^- = \min\{\gamma | \gamma \in h\}$.

- Comparative linguistic expressions domain

$$x_{ij}^{-rp} = \begin{cases} \text{Neg}_{H_S}(E_{G_H}(ll_{ij}^{rp})) & \text{if } c_r \in C^2 \\ E_{G_H}(ll_{ij}^{rp}) & \text{otherwise} \end{cases} \quad (9)$$

where Neg is a linguistic negation operator over an H_S such that $\text{Neg}_{H_S}(H_S) = \text{Neg}_{H_S}(\{s_i, s_{i+1}, \dots, s_j\}) = \{g - j, g - (j - 1), \dots, g - i\}$.

4.3.2. Unification into a Linguistic Domain

Given that we assume a heterogeneous framework in the EISA problem, it is then necessary to make the information

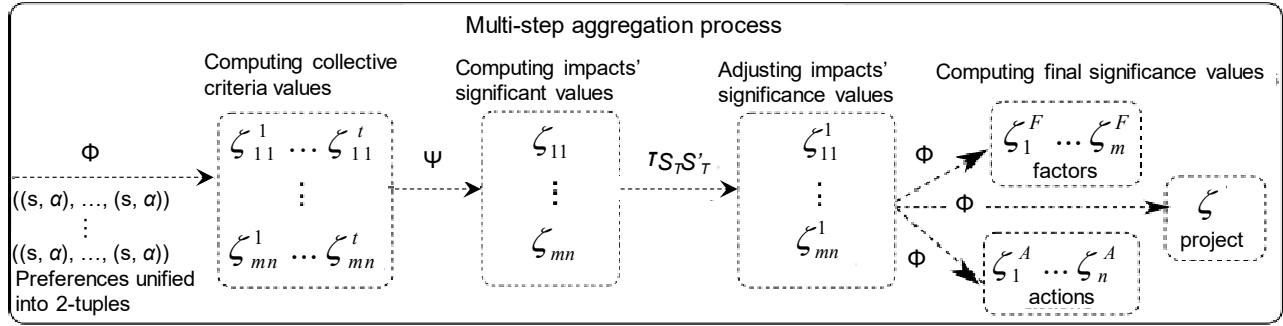


Figure 3. Schema of the multi-step aggregation process.

uniform before applying the multi-step aggregation process. Each normalized heterogeneous preference is conducted into a 2-tuple linguistic value:

$$\hat{x}_{ij}^{rp} = \begin{cases} \chi(\tau_{NS_T}(\bar{x}_{ij}^{rp})) & \text{if } \bar{x}_{ij}^{rp} \in N \\ \chi(\tau_{VS_T}(\bar{x}_{ij}^{rp})) & \text{if } \bar{x}_{ij}^{rp} \in V \\ \chi(\tau_{SS_T}(\bar{x}_{ij}^{rp})) & \text{if } \bar{x}_{ij}^{rp} \in S \\ \chi(\tau_{HS_T}(\bar{x}_{ij}^{rp})) & \text{if } \bar{x}_{ij}^{rp} \in H_S \end{cases} \quad (10)$$

4.3.3. Multi-step Aggregation Process

In a MCDM problem, the aggregation process computes intermediate and global assessments on the set of alternatives. In the MCDM EISA problem, due to the multiple alternatives considered, that includes not only individual impacts but also the environmental factors, the actions and the project, it is necessary to perform the aggregation in a multi-step process to generate intermediate and global significance values.

In order to obtain interpretable significance results close to human beings' cognitive model and taking into account the linguistic modeling of the unified information, the proposed EISA approach carries out the CWW processes in a multi-step aggregation process which is supported by aggregation operators for 2-tuple linguistic values, such as the ones defined in (Herrera and Martínez, 2000; Liu et al., 2013; Merigó et al., 2010; Wan, 2013).

Once the preferences have been conducted into 2-tuple linguistic values, the multiple aggregation step is performed as follows (see Figure 3).

1. Computing collective criteria values:

The assessments of all experts about each criterion c_r , for each impact I_{ij} , is denoted as ζ_{ij}^r , and it is computed with a 2-tuple aggregation operator, Φ :

$$\zeta_{ij}^r = \Phi(\hat{x}_{ij}^{r1}, \dots, \hat{x}_{ij}^{rq}) \quad (11)$$

2. Computing impacts' significance values:

Using a 2-tuple weighted aggregation operator Ψ , with the weighting vector W , the significance value ζ_{ij} of each impact I_{ij} is computed as:

$$\zeta_{ij} = \Psi(\zeta_{ij}^1, \dots, \zeta_{ij}^t) \quad (12)$$

However, a linguistic value ζ_{ij} , can not tell us if an impact will be positive or negative. It is necessary to adjust the significance according to the given nature to specify this fact, so-called adjusting significance.

3. Adjusting impacts' significance values:

In traditional numerical methods for EISA, positive numbers represent benefits caused by the project, whereas negative numbers indicate harm caused by it. From the original impact significance value, two new values (one positive and one negative) are obtained. To do so, it is used the 2-tuple based model to deal with multiple linguistic scales (see Appendix).

In order to solve our problem, a two-level LH should be constructed, with the BLTS, ST, at level 1 and a new Adjusted Linguistic Term Set (ALTS), generated as $l(t, n(t)) \rightarrow l(t+1, 2*n(t)-1)$, denoted as S'_T following the linguistic hierarchy basic rules introduced in Appendix. Once we have the ALTS at level 2, to generate its semantic, a transformation function is defined:

Definition 7. Let $LH = Ul(t, n(t))$ be a linguistic hierarchy whose term sets are

$$\begin{aligned} l(1, n(t)) & S_T \\ l(2, 2*n(t)-1) & S'_T \end{aligned}$$

and let us consider the 2-tuple linguistic model. The transformation function $\tau_{S_T S'_T} : l(1, n(t)) \rightarrow l(2, 2*n(t)-1)$ of a 2-tuple linguistic value on $S_T, (s_k, \alpha)_{ij}$, into S'_T , according to the nature u_{ij} of the impact ζ_{ij} , is defined as:

$$\tau_{S_T S'_T}((s_k, \alpha)_{ij}) = \begin{cases} \Delta^{-1} \left(\Delta(s_k, \alpha)_{ij} + \frac{2*n(t)-1}{2} \right) & \text{if } u_{ij} = 1 \\ \Delta^{-1} \left(\frac{2*n(t)-1}{2} - \Delta(s_k, \alpha)_{ij} \right) & \text{if } u_{ij} = -1 \end{cases} \quad (13)$$

Finally all values ζ_{ij} , are adjusted as:

$$\zeta'_{ij} = \tau_{S_T S'_T}(\zeta_{ij}) \quad (14)$$

As can be seen on Figure 4, the transformation function $\tau_{S_T S'_T}(\cdot)$, allows to outcome impacts' significance values on a

new linguistic terms set $S'_T = \{s'_0, \dots, s'_{g'}\}$, with $g' = 2 * g$ and its granularity $g' + 1$. On the ALTS, S'_T , the term $s'_{g'/2}$ represents the indifference, while the terms situated on its left side and on its right side represent the negative and the positive impacts respectively.

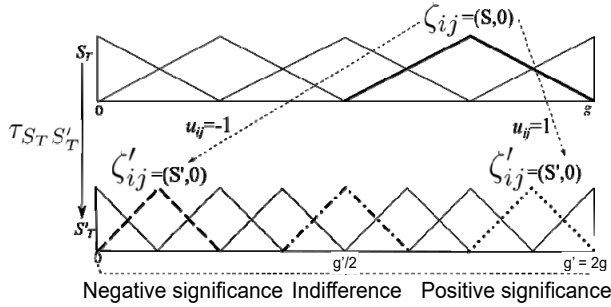


Figure 4. Transformation function for adjusting impact significance values.

4. Computing final significance values:

Using again a suitable problem-dependent 2-tuple aggregation operator Φ , final indicators are generated: Factor's Significance (ζ_i^F), Action's Significance (ζ_i^A) and Global Significance (ζ):

$$\zeta_i^F = \Phi(\zeta_{i1}^T, \dots, \zeta_{in}^T) \tag{15}$$

$$\zeta_j^A = \Phi(\zeta_{1j}^T, \dots, \zeta_{mj}^T) \tag{16}$$

$$\zeta = \Phi(\zeta_{11}^T, \dots, \zeta_{mn}^T) \tag{17}$$

4.3.4. Ranking Alternatives

This phase transforms the global information about the alternatives into global rankings. Therefore, impacts, factors and actions are ordered according to their significance values. As they are expressed into 2-tuple linguistic values, the global rankings are obtained applying the comparison operation for 2-tuple (Herrera and Martínez, 2000) introduced in Appendix.

The larger the values, the better, therefore the more affected factors and the more aggressive actions, the significance values are lower.

5. Illustrative Example

In this section the functionality of the hesitant heterogeneous approach for EISA is illustrated by means of the project for limestone mining in “El Cacao” deposit in Cuba (adapted from (Hernández et al., 2011)).

5.1. Description of the Problem

The implementation of a mining project includes a set of actions that affect the environment. These actions should be

studied for each type of mineral and method of exploitation. The high demand of building materials in Cuba requires an increasing exploitation in the whole country which must be carried out in a sustainable way.

The development of new tools for the mining tasks together the environmental constraints force to use methods of exploitation that cause the least possible environmental impact, as well as reduce the production costs. In Cuba there are environmental regulations for all activities that provoke damage environmental. There is an environmental Law which establishes the requirement of minimizing the negative effects on the environment. The failure to fulfill of this law has sometimes led that the exploited areas are not properly rehabilitated after finishing the mining tasks. An example of this type of projects is the limestone mining in “El Cacao” deposit, that is located to the South of Jiguaní Municipality, Granma Province and it has been exploited for more than 40 years. In this place, limestone of high quality is extracted and during the implementation of the mining project have occurred ecological devastation.

Applying to this example the proposed approach for EISA, it is obtained a ranking of impacts, factors, actions and the overall impact of the project. Such results are accurate and linguistic easy to understand by human beings.

5.2. Solving Process

In order to solve the example, the resolution scheme depicted in Figure 2 is followed.

5.2.1. Defining the Heterogeneous Framework

This phase defines the EISA framework:

- Actions: land clearing (a_1), raw material extracting (a_2) and drilling and blasting (a_3).
- Environmental factors: water (f_1), flora (f_2) and local infrastructures (f_3). Although the selected actions interact with other environmental factors, such as soil, air, fauna, landscape and economy, we have selected water, flora and infrastructures, for the sake of simplicity and to better illustrate the different positive or negative nature of impacts caused by the project.
- The identified impacts are $I = \{I_{11}, I_{12}, I_{13}, I_{21}, I_{33}\}$.
- The nature of each impact is given by $U = \{-1, -1, -1, -1, 1\}$.
- Information Domains:
 - N: $[0, 1]$.
 - H: $\rho([0, 1])$
 - S_H : Linguistic expressions generated by the G_H introduced in Definition 3 using the linguistic term set of five terms S^5 , shown in Figure 5.
- Impacts will be evaluated according to five criteria whose names and expression domains are:
 - c_1 , Intensity, S_H
 - c_2 , Extension, H
 - c_3 , Moment, N
 - c_4 , Persistence, S_H
 - c_5 , Reversibility, S_H

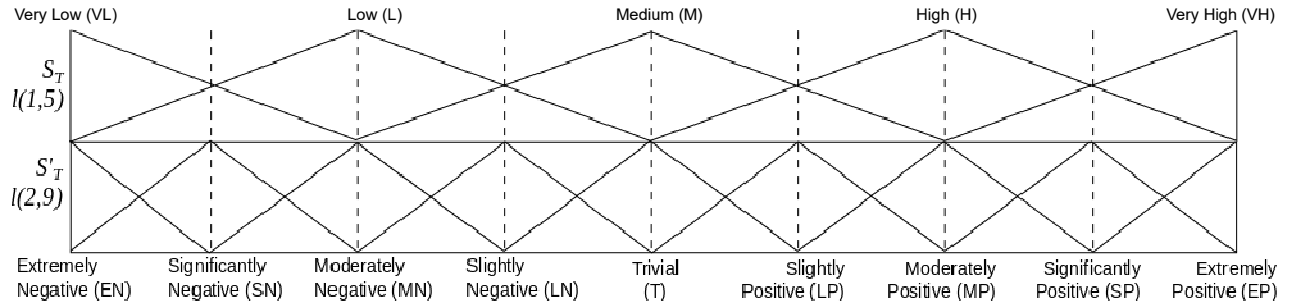


Figure 5. Linguistic set of nine terms addressing negative and positive significance values.

The weights of criteria are $W = (0.3, 0.3, 0.1, 0.15, 0.15)$.

- Selection of information domains to assess criteria: The expression domains used to assess the criteria depends on the criteria nature (qualitative or quantitative). For example, the moment of an impact c_3 , is assessed using numerical values on $[0, 1]$, because it represents the time between the start of the action and the start of the impact. But, due to the complexity of the problem, experts might have hesitation when they express their assessments and they do not want to provide just one value, because the use of only one value cannot reflect their hesitation about their assessment. Therefore, in order to avoid missing values in the gathering preferences, we use hesitant information. In this example, we have considered that the criteria $\{c_1, c_2, c_4, c_5\}$ are assessed by using hesitant information, but it is worthy to note that the use of this type of information will depend on the experts and criteria.

5.2.2. Gathering Preferences

In this phase every impact is submitted to experts evaluation. The gathered heterogeneous preferences are shown in Table 1.

5.2.3. Rating Alternatives

This phase of the EISA approach computes a collective significance value for each item according to the information gathered in the previous phase.

1. Normalization

Just point out that c_3 is a cost criterion, so it is normalized according to Equation (5), for example: $x_{11} = 1 - x_{11}^{c_3} = 1 - 0.3 = 0.7$.

2. Unification into a linguistic domain

Once all preferences are expressed in the same direction, they are unified into 2-tuple linguistic values. The first step is to select the BLTS S_T , that in this case is $S_T = S^5$ which is the linguistic domain used to provide preferences. The unification is then conducted according to the information domain of data. In order to clarify the transformation of linguistic expressions into 2-tuple linguistic values, we show the transformation of the expression “greater than Low” provided by expert e_1 for impact I_{11} about criterion c_1 :

$$\tau_{H,S_T}(\{M,H,VH\}) = \Delta\left(\frac{1}{4}\Delta^-(VH,0) + \frac{1}{4}\Delta^-(H,0) + \frac{1}{2}\Delta^-(M,0)\right) = (H,-0.25)$$

The OWA weights were generated using the method proposed by Liu and Rodríguez (Liu and Rodríguez, 2014). Table 2 shows the unified 2-tuple linguistic values.

3. Multi-step Aggregation Process

Following the Conesa's method in this step, the aggregation operators, 2-tuple arithmetic mean and 2-tuple weighted mean are applied (see Appendix).

(a) Computing collective criteria values:

From linguistic assessments, the criteria collective values are then computed for each impact using Eq. (11). The results are shown in Table 3. The collective value of the criterion c_1 for the impact I_{11} is computed as follows:

$$\zeta'_{I_1} = \Phi((H,-0.25),(M,0),(L,0.25)) = \Delta\left(\frac{2.75+2+1.25}{3}\right) = (M,0)$$

(b) Computing impacts' significance:

The significance values of impacts are generated by aggregating the previous collective criteria values obtained through Equation (12). The results are shown in Table 4, column 2. The significance value of impact I_{11} is computed as:

$$\zeta_{I_1} = \Psi((M,0),(M,0.19)(H,-0.07),(M,0.27),(M,-0.42)) = \Delta(0.3 * 2 + 0.3 * 2.19 + 0.1 * 2.93 + 0.15 * 2.27 + 0.15 * 1.58) = (M,0.13)$$

(c) Adjusting significance:

The results are represented by 2-tuple linguistic values on S^5 and they do not tell us if the impact is positive or negative, therefore, such values must be adjusted. Taking the BLTS $S_T = S^5$ as level 1 of the LH, we generate the level 2 obtaining an LH whose linguistic term sets are:

$$S_T: l(1, 5) \quad \{s_0^5, s_1^5, s_2^5, s_3^5, s_4^5\}$$

$$S'_T: l(2, 9) \quad \{s_0^9, s_1^9, s_2^9, s_3^9, s_4^9, s_5^9, s_6^9, s_7^9, s_8^9\}$$

Once the syntax of the ALTS $S^*_T = S^9$ is obtained, its se

Table 1. Heterogeneous Preferences Provided by Experts.

I_{ij}	c_1	c_2	c_3	c_4	c_5
e_1					
I_{11}	gr than L	{0.3, 0.4, 0.5}	0.3	gr than L	bt L & M
I_{12}	H	{0.4, 0.5, 0.6}	0.1	M	bt M & H
I_{13}	M	{0.6, 0.7}	0.1	M	bt M & H
I_{21}	bt H & VH	{0.9, 1.0}	0	gr than H	H
I_{23}	gr than M	{0.3, 0.4, 0.5}	0	lw than M	lw than M
e_2					
I_{11}	M	{0.4, 0.5, 0.6}	0.2	at least M	M
I_{12}	at least H	{0.6, 0.7, 0.8}	0.2	M	H
I_{13}	M	{0.5, 0.6, 0.7}	0.2	M	bt M & H
I_{21}	at least H	{0.8, 0.9}	0.1	bt M & H	H
I_{23}	gr than M	{0.3, 0.4, 0.5}	0.3	gr than L	gr than L
e_3					
I_{11}	at most M	{0.7, 0.8}	0.3	at least L	at most M
I_{12}	H	{0.4, 0.5, 0.6}	0.1	bt M & H	M
I_{13}	M	{0.5, 0.6, 0.7}	0.1	M	bt M & H
I_{21}	at least H	{0.8, 0.9, 1.0}	0	at most H	H
I_{23}	at most H	{0.5, 0.6}	0	L	L

Note: bt stands for between, gr stands for greater and lw stands for lower.

Table 2. Unified Preferences

I_{ij}	c_1	c_2	c_3	c_4	c_5
e_1					
I_{11}	(H, -0.25)	(M, -0.44)	(H, -0.2)	(H, -0.25)	(M, -0.5)
I_{12}	(H, 0)	(M, 0)	(VH, -0.4)	(M, 0)	(H, -0.5)
I_{13}	(M, 0)	(H, -0.4)	(VH, -0.4)	(M, 0)	(H, -0.5)
I_{21}	(VH, -0.5)	(VH, -0.28)	(VH, 0)	(VH, 0)	(H, 0)
I_{23}	(VH, -0.25)	(M, -0.44)	(VH, 0)	(VL, 0.25)	(VL, 0.25)
e_2					
I_{11}	(M, 0)	(M, 0)	(H, 0.2)	(H, -0.25)	(M, 0)
I_{12}	(VH, -0.25)	(H, -0.22)	(H, 0.2)	(M, 0)	(H, 0)
I_{13}	(M, 0)	(M, 0.44)	(H, 0.2)	(M, 0)	(H, -0.5)
I_{21}	(VH, -0.25)	(H, 0.42)	(VH, -0.4)	(H, -0.5)	(H, 0)
I_{23}	(VH, -0.25)	(M, -0.44)	(H, -0.2)	(H, -0.25)	(H, -0.25)
e_3					
I_{11}	(L, 0.25)	(H, 0)	(H, -0.2)	(L, 0.32)	(L, 0.25)
I_{12}	(H, 0)	(M, 0)	(VH, -0.4)	(H, -0.5)	(M, 0)
I_{13}	(M, 0)	(M, 0.44)	(VH, -0.4)	(M, 0)	(H, -0.5)
I_{21}	(VH, -0.25)	(VH, -0.44)	(VH, 0)	(H, -0.32)	(H, 0)
I_{23}	(H, -0.32)	(M, -0.28)	(VH, 0)	(L, 0)	(L, 0)

mantic is defined according to the significance scale proposed by (Pastakia and Jesen, 1998) (see Figure 5).

Applying Equation (14) is obtained the adjusted significance values of impacts in S^g , (see Table 4, column 3).

(d) Computing final significance values:

The significance values for factors and actions are individually computed using Equations (15) and (16). And the global significance value of the project is computed using Equation (17) (see Table 5).

4. Ranking alternatives

Finally, the rankings of impacts, factors and actions are obtained by using the comparison operation of 2-tuple:

- Impacts' ranking: $I_{33} > I_{11} > I_{13} > I_{12} > I_{21}$.
- Factors' ranking: $f_3 > f_1 > f_2$.
- Actions' ranking: $a_3 > a_2 > a_1$.

The linguistic value for the global significance of the project means that the overall impact caused by the implementation of the mining project in “El Cacao” deposit is **Moderately Negative**.

Table 3. Collective Criteria Values.

I_{ij}	c_1	c_2	c_3	c_4	c_5
I_{11}	(M, 0)	(M, 0.19)	(H, -0.07)	(M, 0.27)	(M, -0.42)
I_{12}	(H, 0.25)	(M, 0.26)	(H, 0.47)	(M, 0.17)	(H, -0.50)
I_{13}	(M, 0)	(M, 0.49)	(H, 0.47)	(M, 0)	(H, -0.50)
I_{21}	(VH, -0.33)	(VH, -0.43)	(VH, -0.13)	(H, 0.06)	(H, 0)
I_{23}	(H, 0.39)	(M, -0.39)	(VH, -0.40)	(L, 0.33)	(L, 0.33)

Table 4. Significance Values for Impacts

I_{ij}	ζ_{ij} Non adjusted significance values	ζ'_{ij} Adjusted significance values
I_{11}	(M, 0.13)	(MN, -0.13)
I_{12}	(H, -0.30)	(SN, 0.30)
I_{13}	(M, 0.37)	(MN, -0.37)
I_{21}	(H, 0.47)	(SN, -0.47)
I_{23}	(M, 0.26)	(MP, 0.26)

Table 5. Significance Values for Factors, Actions and the Project

	f_1	f_2	f_3
ζ_i^F	(MN, -0.4)	(MN, -0.4)	(MN, -0.4)
ζ_j^A	a_1 project	a_2 project	a_3 project
ζ	(MN,0.33)	(MN,0.33)	(MN,0.33)

Table 6. Comparison of Approaches

	Crisp matrix	Hesitant heterogeneous EISA
Uncertainty modeling	No	Yes
Expression domains	Fixed numerical scales	Numerical, linguistic, interval values, HFS and HFLTS
Results	Numeric	Linguistic expressed into 2-tuple
Interpretability	Low, due to it is difficult to represent qualitative subjects through precise numeric values	High, due to there is an unique bipolar linguistic scale which synthesizes understandable significance of impacts
Impact significance calculation	Fixed formula	Aggregation using 2-tuple operators, depending on problem features and requirements of decision makers

6. Advantages of the Proposed Approach

As mentioned in Section 2, the classical crisp matrix for EISA has some disadvantages that we have attempted to overcome with the new hesitant heterogeneous EISA approach. It

not only preserves the traditional use of numerical values, but also improves the evaluation framework, including other information domains to assess the criteria according to their nature and improve the uncertainties modeling and data representation in EISA.

Additionally, the use of hesitant information allows experts to be comfortable when they provide their opinions and hesitate about them, because they can use hesitant information to reflect their hesitation in a flexible way.

Another important feature of this proposal is the use of the 2-tuple linguistic model to represent both the heterogeneous and the hesitant information, and to carry out the CWW processes. This allows to apply different 2-tuple aggregation operators, according to the specific problem and the requirements of decision makers. It also obtains linguistic results in each step, which are easier to understand by human beings than numerical values.

Table 6 summarizes a comparison of the EISA approaches addressed in this paper.

7. Conclusions

EISA problems consider multiple criteria whose nature can be quantitative or qualitative. However, classical approaches for EISA do not manage heterogeneous information efficiently and the results are numerical values difficult to interpret by stakeholders. Therefore, in this paper a new approach for EISA has been proposed, which deals with hesitant heterogeneous information including not only numerical, interval-valued and linguistic domains, but also hesitant information which allows to model the hesitancy and uncertainty in qualitative and quantitative contexts. This approach conducts heterogeneous assessments to 2-tuple linguistic values in order to accomplish the processes of CWW and to obtain easy-to-understand linguistic results. The approach also enables calculating the significance values for impacts that are aggregated to obtain significance values for actions and factors, as well as a global significance value for the project impact. An illustrative example about the limestone mining in “El Cacao” has been presented to show the performance of the proposed approach.

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Appendix

This appendix revises some basic and necessary concepts

to understand the proposed approach for EISA. It reviews the 2-tuple linguistic model, its computational model and some of its extensions, such as the heterogeneous approach to deal with different types of information and the multigranular model which manages different linguistic scales.

2-Tuple Linguistic Model

The 2-tuple linguistic model was proposed by Herrera and Martínez to improve the accuracy of the computing with words (CWW) processes and avoid the loss of information (Herrera and Martínez, 2000; Rodríguez and Martínez, 2013) keeping the linguistic basis (semantics and syntax). This model represents the linguistic information by means of a pair of values called 2-tuple $(s_i, \alpha) \in \bar{S} = S \times [-0.5, 0.5]$, where $s_i \in S$ is a linguistic term and $\alpha \in [-0.5, 0.5]$ is a numerical value that represents the symbolic translation.

Definition 8 (Herrera and Martínez, 2000): The symbolic translation is a numerical value assessed in $[-0.5, 0.5]$ that supports the “difference of information” between a counting of information assessed in the interval of granularity $[0, g]$ of the linguistic term set S , and the closest value in $\{0, \dots, g\}$ which indicates the index of the closest linguistic term in S .

This representation model defines the functions Δ and Δ^{-1} to facilitate the CWW processes (Herrera and Martínez, 2000).

Definition 9 (Martínez and Herrera, 2012) Let $S = \{s_0, \dots, s_g\}$ be a set of linguistic terms and $\beta \in [0, g]$ a value supporting the result of a symbolic aggregation operation. A 2-tuple linguistic value that expresses the equivalent information to β is obtained as follows:

$$\Delta: [0, g] \rightarrow \bar{S}$$

$$\Delta(\beta) = (s_i, \alpha), \text{ with } \begin{cases} i = \text{round}(\beta), \\ \alpha = \beta - i, \end{cases} \quad (18)$$

being round the round operation, i the index of the closest label s_i , to β and α the symbolic translation.

We note that Δ is a bijective function and $\Delta^{-1}: \bar{S} \rightarrow [0, g]$ is defined by $\Delta^{-1}(s_i, \alpha) = i + \alpha$.

The 2-tuple linguistic model defined a computational model based on the functions Δ and Δ^{-1} and introduced the comparison between two 2-tuples linguistic values and several aggregation operators (Herrera and Martínez, 2000).

Let us suppose two 2-tuple linguistic values, (s_k, α_1) and (s_l, α_2) , the comparison is as follows:

- if $k < l$ then $(s_k, \alpha_1) < (s_l, \alpha_2)$.
- if $k = l$ then
 - if $\alpha_1 = \alpha_2$ then $(s_k, \alpha_1) = (s_l, \alpha_2)$;
 - if $\alpha_1 < \alpha_2$ then $(s_k, \alpha_1) < (s_l, \alpha_2)$;
 - if $\alpha_1 > \alpha_2$ then $(s_k, \alpha_1) > (s_l, \alpha_2)$.

In the literature can be found different aggregation operators defined for 2-tuple linguistic values (Herrera and Martínez, 2000; Liu et al., 2013; Merigó et al., 2010; Wan, 2013). We only revise the arithmetic mean and the weighted mean because they

are used in the illustrative example shown in Section 5.

Definition 10 (Herrera and Martínez, 2000): Let $x = \{(s_1, \alpha_1), \dots, (s_m, \alpha_m)\}$ be a set of 2-tuple linguistic values, the 2-tuple arithmetic mean is the function $\Phi: \bar{S}^m \rightarrow \bar{S}$ defined as:

$$\Phi(x) = \Delta \left(\frac{1}{n} \sum_{i=1}^m \Delta^{-1}(s_i, \alpha_i) \right) \quad (19)$$

Definition 11 (Herrera and Martínez, 2000): Let $x = \{(s_1, \alpha_1), \dots, (s_m, \alpha_m)\}$ be a set of 2-tuple linguistic values, and $W = (w_1, \dots, w_m), w_i \in [0, 1]$ be a weighting vector such that $\sum_{i=1}^m w_i = 1$, the 2-tuple weighted mean operator associated with W is the function $\Psi: \bar{S}^m \rightarrow \bar{S}$ defined as:

$$\Psi(x) = \Delta \left(\frac{\sum_{i=1}^m w_i * \Delta^{-1}(s_i, \alpha_i)}{\sum_{i=1}^m w_i} \right) \quad (20)$$

2-Tuple Based Model to Deal with Heterogeneous Information

Usually, in problems with multiple criteria and/or experts is common the appearance of different types of information that might be modeled with heterogenous information such as numerical, interval-valued, linguistic and forth. For this type of framework was introduced in (Herrera et al., 2005) a 2-tuple based model to manage and operate with this type of information. It conducts the heterogeneous information into 2-tuple linguistic values to facilitate the computations and obtain results easy to understand for experts involved in the problem. It consists of three steps.

1. *Choosing the Basic Linguistic Term Set (BLTS)*: The selected BLTS, $S_T = \{s_0, \dots, s_g\}$, must have the maximum granularity to maintain the uncertainty degree associated to each expert, as well as the ability of discrimination to express the preference values. To achieve both purposes some suggestions are detailed in (Herrera et al., 2005).

2. *Transforming into fuzzy sets*: Each value is then transformed into a fuzzy set on $S_T, F(S_T)$, using one of the following functions:

(a) Numerical domain

Definition 12 (Herrera et al., 2005) Let $\mathcal{G} \in [0, 1]$ be a numerical value and $S_T = \{s_0, \dots, s_g\}$ be a linguistic term set. The transformation function $\tau_{NS_T}: [0, 1] \rightarrow F(S_T)$ defined by $\tau_{NS_T}(\mathcal{G}) = \sum_{i=0}^g s_i / \gamma_i$ transforms a numerical value into a fuzzy set in S_T :

$$\gamma_i = \mu_{s_i}(\mathcal{G}) = \begin{cases} 0, & \mathcal{G} < a \text{ or } \mathcal{G} > c, \\ (\mathcal{G} - a) / (b - a), & a < \mathcal{G} < b, \\ 1, & b \leq \mathcal{G} \leq d, \\ (c - \mathcal{G}) / (c - d), & d < \mathcal{G} < c, \end{cases} \quad (21)$$

being $F(S_T)$ the set of fuzzy sets on $S_T, \gamma_i = \mu_{s_i}(\mathcal{G}) \in [0, 1]$ the

membership degree of ϑ to $s_i \in S_T$, and (a, b, d, c) a parametric membership function.

(b) Interval domain

Definition 13 (Herrera et al., 2005) Let $V = [\bar{a}, \underline{a}]$ be an interval in $[0,1]$, the transformation function $\tau_{V S_T} : V \rightarrow F(S_T)$ defined by $\tau_{V S_T}(V) = \sum_{i=0}^g s_i / \gamma_i$ transforms an interval V into a fuzzy set S_T :

$$\gamma_i = \max_{\min_y} \{ \mu_{\nu_i}(y), \mu_{s_i}(y) \}, i = \{0, \dots, g\} \tag{22}$$

where $F(S_T)$ is the set of fuzzy sets on S_T , and μ_{ν_i} and μ_{s_i} the membership functions of the fuzzy sets associated to the interval V and the terms $s_i \in S_T$, respectively.

(c) Linguistic domain

Definition 14 (Herrera et al., 2005) Let $S_T = \{s_0, \dots, s_g\}$ be a linguistic term set, the transformation function $\tau_{S S_T} : S \rightarrow F(S_T)$ defined by $\tau_{S S_T}(s_j) = \sum_{i=0}^g s_i / \gamma_i$ transforms a linguistic term into a fuzzy set in S_T :

$$\gamma_i = \max_{\min_y} \{ \mu_{s_j}(y), \mu_{s_i}(y) \}, i = \{0, \dots, g\} \tag{23}$$

being $F(S_T)$ the set of fuzzy sets on S_T , μ_{s_j} and μ_{s_i} the membership functions of the fuzzy sets associated to the terms $s_j \in S$ and $s_i \in S_T$ respectively.

3. *Transforming into linguistic 2-tuple values:* Finally the fuzzy sets are transformed into linguistic 2-tuple values over the BLTS, using the function $\chi(\cdot)$.

Definition 15 (Martínez and Herrera, 2012) Let $F(S_T)$ be a fuzzy set in S_T , the function $\chi: F(S_T) \rightarrow \bar{S}$ is defined as:

$$\chi(F(S_T)) = \Delta \left(\sum_{j=0}^g j \gamma_j / \sum_{j=0}^g \gamma_j \right) = \Delta(\beta) = (s_l, \alpha) \tag{24}$$

where the fuzzy set $F(S_T)$ can be obtained from $\tau_{NS_T}, \tau_{V S_T}$ or $\tau_{S S_T}$, respectively.

2-Tuple Based Model to Manage Multigranular Linguistic Information. Linguistic Hierarchies

Sometimes, it is necessary to deal with linguistic frameworks in which the linguistic information can belong to linguistic term sets with different granularity. In (Herrera and Martínez, 2001) was presented an approach to manage multigranular linguistic information in a symbolic and precise way by means of Linguistic Hierarchies (LH). This approach builds a structure so-called *linguistic hierarchy*, and a computational symbolic model based on the 2-tuple linguistic model is defined over it to accomplish the CWW processes.

A LH is the union of all levels $t: LH = \cup_t l(t, n(t))$, where each level t of a LH corresponds to a linguistic term set with a granularity of uncertainty of $n(t)$ denoted as: $S^{n(t)} = \{s_0^{n(t)}, \dots, s_{n(t)-1}^{n(t)}\}$ (Herrera and Martínez, 2001).

The construction of LH must satisfy a pair of rules, so-called

linguistic hierarchy basic rules (Herrera and Martínez, 2001):

- To preserve all former modal points of the membership functions of each linguistic term from one level to the following one.
- To make smooth transitions between consecutive levels.

The goal is to add a new linguistic term set $S^{n(t+1)}$ by adding a new linguistic term between each pair of terms belonging to the linguistic term set of the previous level t . To do so, it is necessary to reduce the support of the linguistic labels to keep place for the new one located in the middle of them. Therefore, a linguistic term set in the level $t + 1$ is obtained from its predecessor as $l(t, n(t)) \rightarrow l(t + 1, 2 * n(t) - 1)$. Figure 5 shows a LH according to the rules mentioned.

A transformation function was defined to transform a linguistic term in level t to its correspondent linguistic term in level $t + 1$ following the linguistic hierarchy basic rules (Herrera and Martínez, 2001).

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