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A Basic Hierarchical Graph Model for Conflict Resolution with Weighted Preference

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ABSTRACT. A novel hierarchical graph model for conflict resolution in which preferences are determined by weighting component graphs is proposed. This weighted hierarchical model contains three decision makers (DMs), one common decision maker (CDM) appearing in two local graphs, each with one local decision maker. Reachable lists and unilateral improvements for DMs are represented by matrices, which can be used to calculate stability results. Theorems reveal the relationship between the stability results in the hierarchical graph and in each local graph. Algorithms are designed to capitalize on these relationships in the calculation of stability. A case study of water diversion conflicts in China is provided to show how the new methodology can be applied in practice. The weighted hierarchical graph model improves the modeling of hierarchical conflicts by providing more flexibility in describing the preference of the CDM, who is the key decision-maker.

Keywords: hierarchical graph model, graph model for conflict resolution (GMCR), matrix representation, stability definitions, water diversion conflicts

1. Introduction

Decision makers are involved in a conflict when they make choices in pursuit of different objectives. Various methodologies have been used for analyzing strategic conflicts, including Game Theory (Von Neumann and Morgenstern, 1944), Metagame Analysis (Howard, 1971), Conflict Analysis (Fraser and Hipel, 1984), Drama Theory (Howard, 1999), and the Graph Model for Conflict Resolution (GMCR) (Fang et al., 1993). These methodologies can provide strategies for mediators to balance conflicting demands from the stakeholders (Wang et al., 2003; Hipel et al., 2008;). The study on environmetal management are implemented by multidisciplinary approaches (Fan and Huang, 2012; Gunalay et al., 2012; Barbalios et al., 2013; Xu and Qin, 2013). The environmental conflicts often take place when stakeholders compete over limited natural resources.

The study on environmental management are implemented by multidisciplinary approaches (Fan and Huang, 2012; Gunalay et al., 2012; Barbalios et al., 2013; Xu and Qin, 2013). The environmental conflicts often take place when stakeholders compete over limited natural resources. GMCR is a methodology that can flexibly analyze strategic conflicts and provide meaningful analytical results (Kilgour and Hipel, 2005; Hipel et al., 2008; Hipel and Walker, 2011). GMCR is a major expansion and improvement of game theory and conflict analysis (Kilgour and Hipel, 2010). Game theory has been widely applied to real world conflicts. However, DMs in game theory make decisions based on simple solution concepts, completely ignoring counteractions by other DMs (Madani and Hipel, 2011). Thus, less restrictive stability definitions, such as General Metarationality, and Symmetric Metarationality have been proposed in metagame analysis (Howard, 1971). These solution concepts reflect the behaviors of DMs and are more reliable in indicating equilibria in many real world conflicts (Madani, 2013). Conflict analysis methodology is an expansion of metagame analysis by introducing sequential stability. GMCR is an improvement of conflict analysis approach by using graphs to represent moves (Kilgour and Hipel, 2010). It uses all stability definitions in conflict analysis, extends to graph model context, and includes limited moves and non-myopic stability to reflect a greater foresight for DMs. A decision support system, called GMCR II, has been designed to carry out calculation for stabilities (Fang et al., 2003a, b).

A graph model contains decision makers (DMs), states, transitions of states controlled by each DM, and preference relations for each DM. A DM can be an individual or a group with common interests in a conflict. A DM can have one or more choices in a strategic conflict, called options. A state is a selection of options by all DMs. A state is stable for a DM if he or she does not have any advantageous move to other states. An equilibrium, a state stable for all of the DMs, indicates possible resolutions of a conflict. The stability of a state can be

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described in different solution concepts, such as Nash stability (R) (Nash, 1950, 1951), sequential stability (SEQ) (Fraser and Hipel, 1984), general metarationality (GMR) (Howard, 1971), symmetric metarationality (SMR) (Howard, 1971), and limited move stability (Brams and Wittman, 1981; Zagare, 1984; Fang et al., 1993). These solution concepts differ in the foresights of conflicts by DMs and their perception of risks. Studies within the graph model paradigm include coalition analysis (Kilgour et al., 2001; Inohara and Hipel, 2008a, b), preference uncertainty (Li et al., 2005), fuzzy preferences (Bashar et al., 2014), strength of preference (Hamouda et al., 2004), and the matrix representation of a conflict (Xu et al., 2010a, b, 2018).

In particular, the matrix representation is a more efficient and convenient approach for calculating stabilities in the graph model. The unilateral moves (UMs) and unilateral improvements (UIs) for DMs can be expressed using matrices. An UM refers to a move controlled by a DM from one state to another, by changing its one or more options. An UI is an UM for the focal DM resulting in a more preferred state. The solution concepts in a graph model can be represented in matrices which are the function of UMs, UIs, and preferences for a focal DM and other DMs. The matrix representation methodology can be used for predicting equilibria of real world conflicts with easy calculation procedure. Preference uncertainty and strength have also been expressed in matrices (Xu et al., 2013; 2018).

Hierarchical structures in game theory have been studied in cooperative game theory (Gilles, 2010). Players in a game have been defined with different seniorities by weights (Gvozdeva et al., 2012). However, this approach requires data to determine permission values. Stackelberg games are hierarchical conflicts with one leader and several followers (Von Stackelberg, 1934; Simaan and Cruz, 1973), in which information about the game is asymmetric. GMCR has been used for modelling strategic conflicts with hierarchical structure (He et al., 2013, 2014a). Compared with other methodologies, a graph model requires only relative preferences (Hipel, 2002). It can also handle irreversible moves.

In a hierarchical conflict, one or more DMs, defined as common DMs (CDMs), participate in all the smaller conflicts, while other DMs only appear in just one subconflict. A basic hierarchical graph model has been proposed containing only one CDM and two local graphs. This model is applied to water diversion conflicts in China (He et al., 2012, 2013, 2014a). The stability results of the water diversion conflicts were calculated using matrices (He et al., 2014b). Preference relations for CDM are constructed using the lexicographical order (He et al., 2013). CDM only cares about the more important subconflict when making a move. However, in the real world, CDM often weights up situations in all subconflicts before making a decision. To describe the preferences for CDM with more accuracy, a hierarchical graph model with weighted preference for CDM is proposed. The importance of each subconflict in this model is indicated by a weight. Relations between stability results in the hierarchical graph and the local graphs are also discussed. Algorithms for calculating stability results in the hierarchical model are designed based on these relations.

This novel methodology has been applied to water diversion conflicts in China. Stability results calculated by the algorithms indicate possible resolutions for DMs in these conflicts.

The upcoming sections in this paper are arranged as follows. The hierarchical graph model methodologies are introduced in Sections 2 to 5. The case study is presented in Section 6. In Section 7, the comparison with other hierarchical graph model methodologies is discussed. The conclusion and further studies are provided in Section 8. Representative proofs for some of the theorems are placed in Supplementary Material.

In more detail, the process of developing the new methodologies in this paper is depicted in Figure 1. Each step of the procedure is denoted by a rounded rectangle. Hollow arrows between two steps show the direction of the process. A parallelogram represents definitions or theorems used in each step. Slim arrows mean the information in the starting object (rounded rectangle or parallelogram) may be used in the object to which the arrow points.

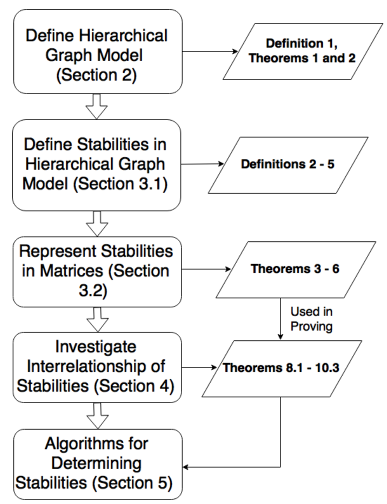

Figure 1. Development of Hierarchical Graph Model Methodologies.

The basic hierarchical graph model with weighted preference is designed in Definition 1 in Section 2. To represent the basic hierarchical graph model using matrices, the reachable matrices for DMs are constructed in Theorems 1 and 2. The stabilities in the basic hierarchical model are discussed in Section 3. Formal definitions of stabilities are shown in Section 3.1. These stabilities are represented in matrices using Theorems 3 to 6 in Section 3.2, which are also used to prove theorems in Section 4. Interrelationships between the stabilities in the hierarchical model and the local models are investigated in Section 4 using Theorems from 8.1 to 10.3. Algorithms to determine stabilities in the hierarchical graph model in Section 5 are designed using these theorems.

2. The Basic Hierarchical Graph Model with Weighted Preferences

2.1. Structure of a Graph Model

A graph model for a strategic conflict contains a finite set of DMs, N, a finite set of states, S, and a preference relation on S for each DM $i \in N$. A directed arc connecting the two states denotes a move for each DM i, given as $A_i \subseteq S \times S$. In a graph model, a node is used to represent a state.

The moves for a DM in one step from a specified starting state s form its reachable list from $s \in S$. These moves are called unilateral moves (UMs). From state s, the reachable list for DM i is denoted as $R_i(s) = \{s' \in S, (s, s') \in A_i\}$. In a graph model, any two feasible states can be compared according to the preference information for each DM. A DM's preference over S is a complete binary relation on S. Between two states $s' \in S$, $s \succ_i s'$, indicates that DM i prefers s to s'. Likewise, $s \prec_i s'$ and $s \sim_i s'$ mean state s' is more preferred and the two states are equally preferred. The relation $s \geq_i s'$ denotes that s is more or equally preferred to s' for DM i. The relation $s \leq_i s'$ can be defined analogously. Four properties should be satisfied in the preference structure (Kilgour and Hipel, 2005; 2010):

1) $\succ_i$ is asymmetric, which means that $s \succ_i s'$ and $s' \succ_i s$ cannot hold true at the same time.
2) $s \sim_i s$ is reflexive, which indicates that $s \sim_i s$ for any $s \in S$.
3) $\sim_i$ is symmetric, which means if $s \sim_i s'$ then $s' \sim_i s$.
4) $\{\succ_i, \sim_i\}$ is strongly complete, which indicates exactly one of $s \succ_i s'$, $s \prec_i s'$ and $s \sim_i s'$ is true.

The structure of a graph model can be written.

Suppose $N = \{1, 2, \dots, n\}$ denote the set of DMs, $S = \{s_1, s_2, \dots, s_m\}$ represent the set of feasible states. D_i is DM i's directed graph and $A_i \subseteq S \times S$ is the set of directed arcs in D_i, a graph model is defined as: $D_i = < N, S, \{A_i : i \in N\}, \{\succsim_i, i \in N\} >$, where $\succsim_i$ denotes the preference relations for DM i.

The unilateral improvement (UI) list is the set of all reachable states from a specified state for a given DM. The UI list from a state $s \in S$ for DM $i \in N$ is marked as $R_i^+(s) = \{s' \in R_i(s) : s' \succ_i s\}$.

2.2. Formal Definition

A hierarchical graph model for a strategic conflict contains smaller graph models, called local graph models. These local models feature one or more common DMs (CDMs) who appear in each of the local graphs. Local DMs (LDMs) appear only in one local graph. A basic hierarchical graph model has been defined based on Section 2.1 by He et al. (2013). A basic hierarchical graph model contains two smaller graph models with only one CDM.

Definition 1 (**Basic Hierarchical Graph Model**) (He et al., 2013): Suppose there are three DMs, consisting of CDM, LDM_1 and LDM_2 and two local graph models:

$$G_1 = \langle \{CDM, LDM_1\}, S_1, \{AC_1, AL_1\}, \{\succsim_{C_1}, \succsim_{L_1}\} \rangle,$$

where $AC_1 \subseteq S_1 \times S_1$, $AL_1 \subseteq S_1 \times S_1$, and $\succsim_{C_1}$ and $\succsim_{L_1}$ are preference relations on S_1 for CDM and LDM_1, respectively;

$$G_2 = \langle \{CDM, LDM_2\}, S_2, \{AC_2, AL_2\}, \{\succsim_{C_2}, \succsim_{L_2}\} \rangle,$$

where $AC_2 \subseteq S_2 \times S_2$, $AL_2 \subseteq S_2 \times S_2$ and $\succsim_{C_2}$ and $\succsim_{L_2}$ are preference relations on S_2.

Then the graph model:

$$G = \langle \{CDM, LDM_1, LDM_2\}, \ S = S_1 \times S_2, \{AC, AL_1^u, AL_2^u\}, \{\succsim_C, \succsim_{L_1}^u, \succsim_{L_2}^u\} \rangle$$

is a basic hierarchical graph model based on G_1 and G_2.

The structure of the basic hierarchical graph model can be better understood by Example 1 illustrated below.

Example 1 (States and Moves in a Basic Hierarchical Graph Model): Suppose that in a basic hierarchical graph model G containing CDM, LDM_1 and LDM_2, there are three states in G_1: $S_1 = \{a, b, c\}$ and two states in G_2: $S_2 = \{A, B\}$. Then the set of states in G can be written as $S = S_1 \times S_2 = \{aA, aB, bA, bB, cA, cB\}$.

The moves for CDM and LDM_1 in G_1 and G_2 are assumed in Figure 2, where each node represents a state and a directed arrow denotes a move from one state to another. According to Figure 2, CDM can move from state a to b in G_1 and state A to B in G_2. LDM_1 can move from state b to c in G_1.

Then, in the basic hierarchical graph model, G, the moves for CDM from state $aA \in S$ and the move for LDM_1 from state $bB \in S$ are different. According to Definition 1, from state aA, CDM can move to state aB, bA, or bA. From state bB, LDM_1 can only move to state cB.

Note that in the hierarchical graph model G, the set of states S is the Cartesian Product of the sets of the component states S_1 and S_2. The moves for LDM_2 are not listed because they are analogous to LDM_1's.

To determine the preference structure for G, each local graph is assigned with a weight.

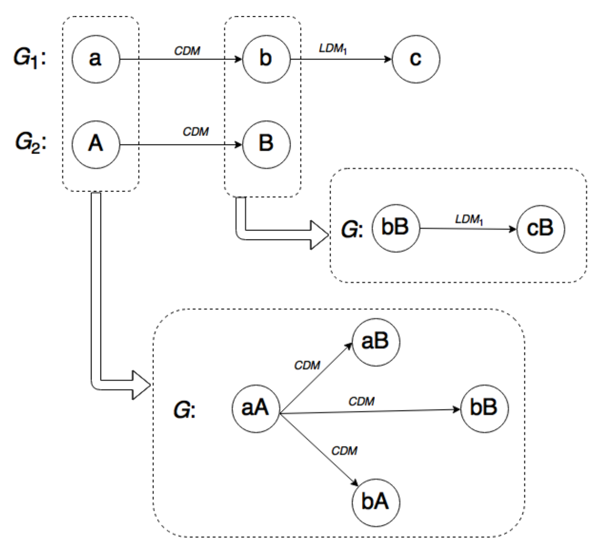

Figure 2. Demonstration of Moves in G.

2.3. Weighted Preference Structure

As demonstrated by He et al. (2013), if all the features of the two local graphs are known, the preference relations for

CDM in the basic hierarchical graph model can be partially determined. In this paper, the preference structure for CDM are constructed using the option prioritization method (Fang et al., 2003a, b).

Suppose two states $s_a, s_b \in S$ in a basic hierarchical graph model G consisting of G_1 and G_2, where $S_a = (S_{1a}, S_{2a})$ and $S_b = (S_{1b}, S_{2b})$, the preferences for a DM are represented by a list of preference statements ordered from the most important at the top to the least important at the bottom. As each DM has at least one option in a strategic conflict, the options for all DMs in G are numbered. Each preference statement is expressed by option numbers connected by logical symbols, such as "& (AND)", "| (OR)", and "IF".

Let $\left\{\Omega_1^{(k)}, \Omega_2^{(k)}, \ldots, \Omega_{h_k}^{(k)}\right\}$ be the set of statements in G_k, $k = 1, 2$. In local graph G_k, a score $\Psi_{j_k}^{(k)}\left(s_{ka}\right)$ is assigned to state s_{ka} according to its true values when the statements are applied (Peng et al., 1997), $0 < j_k < h_k$. Then,

$$\Psi_{j_k}^{(k)}\left(s_{ka}\right) = \begin{cases} 2^{h_k - j_k} & \text{if } \Omega_{j_k}^{(k)}\left(s_{ka}\right) = T \\ 0 & \text{otherwise} \end{cases} \tag{1}$$

and

$$\Psi^{(k)}\left(s_{ka}\right) = \sum_{j_k = 1}^{h_k} \Psi_{j_k}^{(k)}\left(s_{ka}\right) \tag{2}$$

The importance of each local graph for CDM is denoted by a weight w_k ($w_k > 0$), where $w_1 + w_2 = 1$. Thus, the score for state s_a in G is defined as:

$$\Psi\left(s_a\right) = \Psi^{(1)}\left(s_{1a}\right) w_1 + \Psi^{(2)}\left(s_{2a}\right) w_2 \tag{3}$$

The score on s_b for CDM can be similarly obtained as $\Psi\left(s_b\right)$, then, s_a and s_b can be compared:

$$s_a \succ s_b \text{ if } \Psi\left(s_a\right) > \Psi\left(s_b\right);$$

$$s_a \prec s_b \text{ if } \Psi\left(s_a\right) < \Psi\left(s_b\right);$$

$$s_a \sim s_b \text{ } if \text{ } \Psi\left(s_a\right) = \Psi\left(s_b\right).$$

2.4. Reachable Matrix

The weighted basic hierarchical graph model can be represented by matrices. The reachable matrix denoting the reachable list for a given DM in the hierarchical model can be constructed by the corresponding reachable matrices in the local graphs. The reachable matrix for CDM in the hierarchical model is a Tensor product of the reachable matrices in the two local graphs.

Theorem 1 (Reachable Matrix for CDM): Suppose $J_{CDM}^{(1)}$ is the $m \times m$ reachable matrix for CDM in G_1 and $J_{CDM}^{(1)}$ is the $n \times n$ reachable matrix for CDM in G_2, I_n is an identity matrix of n scale, then the $mn \times mn$ hierarchical reachable matrix for CDM, J_{CDM} in G, is written as:

$$J_{CDM}^{(2)} = \begin{pmatrix} J_{CDM}^{(1)}\left(1,1\right) \otimes_r J_{CDM}^{(2)} & \cdots & J_{CDM}^{(1)}\left(1,m\right) \otimes_r J_{CDM}^{(2)} \\ \vdots & \ddots & \vdots \\ J_{CDM}^{(1)}\left(m,1\right) \otimes_r J_{CDM}^{(2)} & \cdots & J_{CDM}^{(1)}\left(m,m\right) \otimes_r J_{CDM}^{(2)} \end{pmatrix} \tag{4}$$

where $J_{CDM}^{(1)}\left(s_1, q_1\right)$ is an entry in $J_{CDM}^{(1)}$ $(s_1, q_1 = 1, \ldots, m)$ and

$$J_{CDM}^{(1)}\left(s_1, q_1\right) \otimes_r J_{CDM}^{(2)} = \begin{cases} J_{CDM}^{(2)} & s_1 = q_1 \\ J_{CDM}^{(1)}\left(s_1, q_1\right)\left(I_n + J_{CDM}^{(2)}\right) & s_1 \neq q_1 \end{cases} \tag{5}$$

The reachable matrix for a LDM in the hierarchical graph is the Kronecker Product of the reachable matrix in the local graph and an identity matrix.

Theorem 2 (Reachable Matrix for LDM): Suppose that states $s, q \in S$ are two states in G, where $s = \left(s_1, s_2\right)$ and $q = (q_1, q_2)$ ($s_1, q_1 = 1, 2, \ldots, m$; $s_2, q_2 = 1, 2, \ldots, n$). Let J_{L_1} denotes the $m \times m$ reachable matrix for LDM_1 in G_1 and $J_{L_2}^u$ denotes the $n \times n$ reachable matrix for LDM_2 in G_2, I_m and I_n represent identity matrices of m and n scales respectively, and $\otimes$ means the Kronecker Product of two matrices, then the $mn \times nm$ hierarchical reachable matrices $J_{L_1}^u$ and $J_{L_2}^u$ for LDM_1 and LDM_2 are expressed as:

$$J_{L_1}^u = J_{L_1} \otimes I_n = \begin{pmatrix} J_{L_1}\left(1,1\right) I_n & \cdots & J_{L_1}\left(1,m\right) I_n \\ \vdots & \ddots & \vdots \\ J_{L_1}\left(m,1\right) I_n & \cdots & J_{L_1}\left(m,m\right) I_n \end{pmatrix} \tag{6}$$

$$J_{L_2}^u = I_m \otimes J_{L_2} = \begin{pmatrix} J_{L_2} & & \\ & \ddots & \\ & & J_{L_2} \end{pmatrix} \tag{7}$$

The proof of Theorem 2 is analogous to Theorem 1.

2.5. UI Matrix

The UIs for DMs in the weighted hierarchical graph model can be expressed in matrix. In particular, suppose $s, q \in S$ for $s = (s_1, s_2)$ and $q = \left(q_1, q_2\right)$, the entries for CDM in the UI matrix J_{CDM}^{+} can be written as:

$$J_{CDM}^{+}\left(s, q\right) = \begin{cases} 1 & s_1 \prec_C q_1 \\ 0 & other \end{cases} \tag{8}$$

The entries for LDMs in their UI matrices $J_{L_1}^{u+}$ and $J_{L_2}^{u+}$ can be expressed as:

$$J_{L_1}^{u+}\left(s, q\right) = \begin{cases} 1 & s_1 \prec_{L_1} q_1 \quad \text{and} \quad s_2 = q_2 \\ 0 & \text{other} \end{cases} \tag{9a}$$

and

$$J_{L_2}^{u+}(s,q)=\begin{cases}1 & s_2 \succ_{L_2} q_2 \text{ and } s_1=q_1\\ 0 & \text{other}\end{cases} \tag{9b}$$

2.6. Joint Movement and Improvement Matrices

In a graph model containing more than two DMs, the sequence of moves taken by the opponents of a given DM is denoted as joint movement. The legal sequence of UIs by the opponents is called joint improvements.

The joint movement and improvement matrices have been constructed by Xu et al. (2009). In a graph model, the joint movement and improvement for DMs $H \subseteq N$ can be noted as M_H and M_H^+, for $H \neq \phi$. The expression of M_H and M_H^+ can be seen in Supplementary Material.

3. Stability Definitions

Stability definitions describe DMs' decision to stay or move away from a given state (Kilgour and Hipel, 2010). Four types of stabilities, Nash, SEQ, GMR, and SMR are investigated. These stabilities indicate different foresights for a DM (He et al., 2013). In Nash rationality, the focal DM perceives only moves of one step. SEQ and GMR describe the foresights of the focal DM of two steps. In SEQ, the focal DM only considers UIs by other DMs as possible sanctions to its initial move. In GMR, these possible sanctions by other DMs can even be their disimprovements. In SMR, the focal DM has the foresight one step further by considering not only sanctions from other DMs, but also its own counteractions.

An equilibrium refers to a state stable for all DMs. For example, a Nash equilibrium is a state that is Nash rational for all DMs. Equilibria in a graph model reveals possible outcomes of the conflict or courses of action for DMs to follow.

The interrelationship of these stability definitions has been studied by Kilgour and Hipel (2010). A Nash rational state for a DM is also SEQ, GMR, and SMR. A state with Nash, SEQ, and SMR is also GMR.

In this section, the logical definitions of stabilities for basic hierarchical graph model (He et al., 2013) are listed first. These definitions are then represented in matrices analogously to the theorems given by Xu et al. (2009).

3.1. Logical Definition of Stabilities

The stability definitions for a basic hierarchical graph model have been introduced by He et al. (2013) listed from Definition 2 to 5. Each stability is defined using the set of moves or the set of UIs.

Definition 2 (**Nash**): Let a DM (either CDM or LDM) in a basic hierarchical graph model, $i \in N$, and $s=(s_1,s_2)\in S$. State s is Nash for DM i if and only if $R_i^+(s)=\phi$.

Definition 3 (**SEQ**): Suppose that $R_i^+(s)$ is the set of UIs from state $s=(s_1,s_2)\in S$. for DM $i \in N$ and that $R_{N-i}^+(q)$ is the set of UIs from state $q=(q_1,q_2)\in S$ for DMs except i as a coalition.

State s is SEQ for i if and only if, for every state $q \in R_i^+(s)$, there exists at least one state $r \in R_{N-i}^+(q)$ such that $r \precsim_i s$.

Definition 4 (**GMR**): Recall that $R_i^+(s)$ is the set of UIs from state $s=(s_1,s_2)\in S$. for DM $i \in N$, and $R_{N-i}(q)$ is the set of UMs from state $q=(q_1,q_2)\in S$ for DMs except i as a coalition. State s is GMR for DM i if and only if, for every state $q \in R_i^+(s)$, there exists at least one state $r \in R_{N-i}(q)$ such that $r \precsim_i s$.

Definition 5 (**SMR**): Recall that $R_i^+(s)$ is the set of UIs from state $s=(s_1,s_2)\in S$. for DM $i \in N$ and $R_{N-i}(q)$ is the set of UMs from state $q=(q_1,q_2)\in S$ for DMs except i as a coalition. State s is SMR for DM i if and only if, for every state $q \in R_i^+(s)$, there exist at least $r \in R_{N-i}(q)$ such that $r \precsim_i s$ and $r \precsim_i s$ for all $t \in R_i(r)$.

3.2. Matrix Representation of Stabilities

In a graph model, the stability definitions can also be expressed using matrices (Xu et al., 2009). As a hierarchical graph model is a special case of graph model, the stability definitions in a hierarchical graph model can be presented using matrices from Theorems 3 to 6.

Theorem 3 (**Nash**): In a basic hierarchical graph model G consisting of G_1 and G_2, a state $s \in S$ is Nash stable for CDM, iff $e_s^T \cdot J_{CDM}^+ = 0^T$, where T denotes the transpose of matrix. The state s is Nash stable for LDM_1 iff $e_s^T \cdot J_{L_1}^{u+} = 0^T$.

For CDM, $e_s^T \cdot J_{L_1}^{u+} = 0^T$ means there is no UI from state s. This expression echoes $R_i^+(s)=\phi$ in Definition 2.

Theorem 4 (**SEQ**): A state $s \in S$ in G is SEQ for CDM, iff $M_{CDM}^{SEQ}(s,s)=0$, where

$$M_{CDM}^{SEQ} = J_{CDM}^{+} \cdot \left\{E - sign\left[M_L^+ \cdot \left(P_{CDM}^{-,=}\right)^T\right]\right\},$$

and M_L^+ is the joint improvement matrix by LDMs.

State s is SEQ for LDM_1 iff $M_{L_1}^{SEQ^u}(s,s)=0$, where

$$M_{L_1}^{SEQ^u} = J_{L_1}^{u+} \cdot \left\{E - sign\left[M_{N-L_1}^+ \cdot \left(P_{CDM}^{-,=}\right)^T\right]\right\},$$

and $M_{N-L_1}^+$ is the joint improvement matrix by CDM and LDM_2.

Note that $M_{CDM}^{SEQ}(s,s)=0$ means that the S^{th} entry on the diagonal of the matrix M_{CDM}^{SEQ} is zero. $M_{CDM}^{SEQ}(s,s)=0$ can be considered as either $J_{CDM}^+(s,s)=0$ or $\{E - sign[M_{N-L_1}^+ \cdot (P_{CDM}^{-,=})^T]\}(s,s)=0$. The first expression suggests no UI is from state s for CDM, while the latter one means the UIs from other DMs as countermoves are less preferred than state s for CDM.

In the following theorems, $M_{CDM}^{GMR}(s,s)=0$, $M_{L_1}^{GMR^u}(s,s)=0$, $M_{L_1}^{GMR^u}(s,s)=0$, $M_{CDM}^{SMR}(s,s)=0$, and $M_{L_1}^{SMR^u}(s,s)=0$, have the analogous notation.

Theorem 5 (**GMR**): A state $s \in S$ in G is GMR for CDM, iff $M_{CDM}^{GMR}(s, s) = 0$, where

$$M_{CDM}^{GMR} = J_{CDM}^{+} \cdot \left\{ E - sign\left[M_L \cdot \left(P_{CDM}^{-,=} \right)^T \right] \right\},$$

and M_L is the joint movement matrix by LDMs.

State s is GMR for LDM_1 iff $M_{L_1}^{GMR^u}(s, s)$, where

$$M_{L_1}^{GMR^u} = J_{L_1}^{u+} \cdot \left\{ E - sign\left[M_{N-L_1} \cdot \left(P_{CDM}^{-,=} \right)^T \right] \right\},$$

and M_{N-L_1} is the joint movement matrix by CDM and LDM_2.

Theorem 6 (**SMR**): A state $s \in S$ in G is SMR for CDM, iff $M_{CDM}^{SMR}(s, s) = 0$, where

$$M_{CDM}^{SMR} = J_{CDM}^{+} \cdot \left\{ E - sign\left[M_L \cdot W_{CDM} \right] \right\}$$

and

$$W_{CDM} = \left(P_{CDM}^{-,=} \right)^T \circ \left[E - sign\left(J_{CDM} \cdot \left(P_{CDM}^{+} \right)^T \right) \right],$$

and $\circ$ is the Hadamard Product of the two matrices.

State s is SMR for LDM_1 iff $M_{L_1}^{SMR^u}(s, s) = 0$, where

$$M_{L_1}^{SMR^u} = J_{L_1}^{u+} \cdot \left\{ E - sign\left[M_{N-L_1} \cdot W_{L_1} \right] \right\},$$

and

$$W_{L_1} = \left(P_{L_1}^{-,=} \right)^T \circ \left[E - sign\left(J_{L_1}^{u} \cdot \left(P_{L_1}^{u+} \right)^T \right) \right].$$

The four stability definitions are also interrelated (Kilgour and Hipel, 2010). The interrelationship among four stabilities is shown in Figure 3. A Nash stable state is also SEQ, GMR, and SMR. A state which is SEQ or SMR is also GMR.

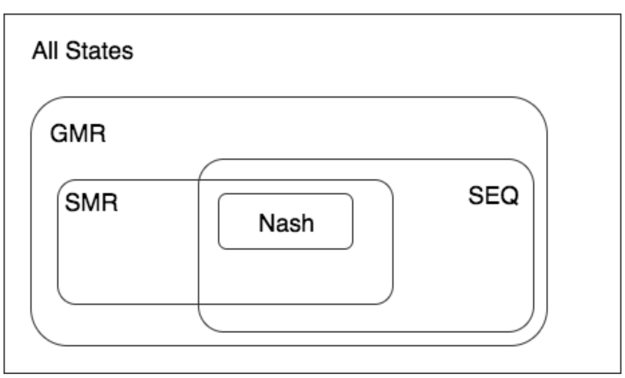

Figure 3. Interrelationship of Four Stabilities.

4. Interrelationship between Stabilities in the Hierarchical Graph and the Local Graphs

The four types solution concepts for DMs in the hierarchical model are linked with those in each local model. The interrelationship between the stabilities is investigated in theorems. Representative proofs of some theorems are listed in Appendix. Figure 4 shows the relationship of the theorems in this paper which are used to investigate the stabilities in the hierarchical graph model. As shown in Figure 4, the four types of stabilities in the basic hierarchical graph model, shown in the central part of the figure, are represented by the matrices listed in the left box. These matrices are constructed from Sections 2.4 to 2.6. Thus, the expression of a particular stability using these matrices is denoted by a directed arrow labelled with the theorem used. The connections between the central and right parts of Figure 4 show the link between the stabilities in the basic hierarchical graph model and those in the local graph models. For example, the interrelationship of Nash stability is investigated in Theorems 7.1 and 7.2, which is denoted by a two-way arrow, indicating that the stability from one side can be concluded from the other.

4.1. Nash Stability (R)

Theorem 7.1: In a weighted basic hierarchical graph model G consisting of G_1 and G_2 , a state $s = (s_1, s_2) \in S$ is Nash stable for CDM in G iff s_k (k = 1, 2) is Nash stable for CDM in G_k .

Theorem 7.1 suggests that a state in a hierarchical graph model is Nash stable for CDM if and only if both component states are Nash stable in the local graph models.

Theorem 7.2 (Nash for LDM): In a weighted basic hierarchical graph model G consisting of G_1 and G_2, a state $s = (s_1, s_2) \in S$ is Nash rational for LDM_1 in G iff s_1 is Nash rational for LDM_1 in G_1.

Theorem 7.2 indicates that a state in a hierarchical graph model is Nash stable for a LDM if and only if the component state is Nash stable for the LDM in the local graph in which it participates.

4.2. Sequential Stability (SEQ)

Theorem 8.1: In a weighted basic hierarchical graph model G consisting of G_1 and G_2, a state $s = (s_1, s_2) \in S$ is SEQ for CDM in G if s_k (k = 1, 2) is SEQ for CDM in G_k .

Theorem 8.2: Suppose G is a weighted basic hierarchical graph model consisting of G_1 and G_2, if a state $s = (s_1, s_2) \in S$ is SEQ for CDM in G, then

1) s_k is SEQ for CDM in G_k for both k = 1 and 2, or
2) when s_k is not SEQ for CDM in G_k (k = 1 or 2, but not both), there exists $r = (r_1, r_2) \in S$, such that $\Psi(r) \le \Psi(s)$ and r_i is element in $\boldsymbol{r}_i$ (i = 1, 2):

$$\boldsymbol{r}_1 = \left(J_{L_1}^{(1)+} \right)^T \cdot \boldsymbol{e}_{q_1} + \boldsymbol{e}_{q_1}$$

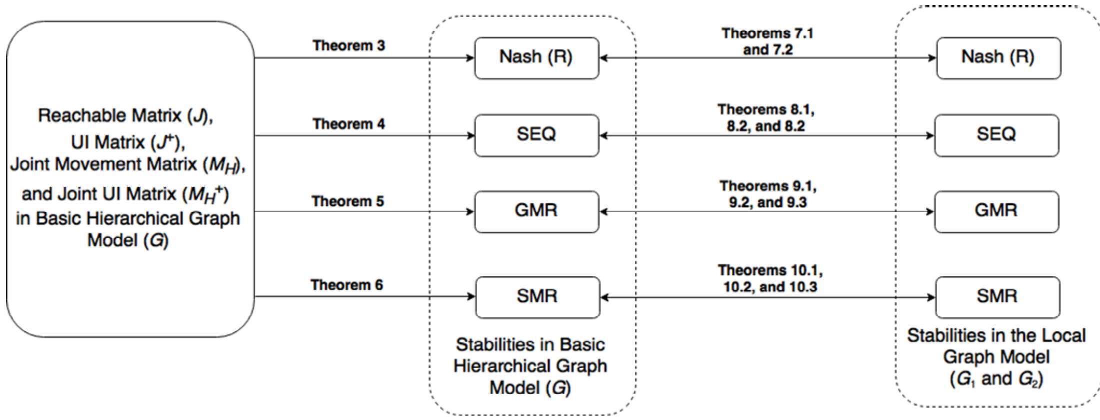

Figure 4. Relationships of Theorems on Overall and Local Stabilities in G.

and

$$\boldsymbol{r_2} = \left(J_{L_2}^{(2)+}\right)^T \cdot \boldsymbol{e}_{q_2} + \boldsymbol{e}_{q_2}$$

for all $(q_1, q_2) \in R_{CDM}^{+}(s)$ and $(r_1, r_2) \neq (q_1, q_2)$; $\boldsymbol{e}_{q_k}$ is a 0-1 vector with the $q_k{}^{th}$ entry being 1 and others 0.

Example 2: A small example makes Theorem 8.2 easier to understand. Let the sets of state in G_1 and G_2 written as $S_1 = \{1, 2, 3, 4\}$ and $S_2 = \{5, 6, 7, 8\}$, respectively. Suppose

$$J_{CDM}^{(1)+} = \begin{matrix} & 1 & 2 & 3 & 4 \\ 1 & 0 & 1 & 0 & 0 \\ 2 & 0 & 0 & 0 & 0 \\ 3 & 0 & 0 & 0 & 1 \\ 4 & 0 & 0 & 0 & 0 \end{matrix}, \quad J_{CDM}^{(2)+} = \begin{matrix} & 5 & 6 & 7 & 8 \\ 5 & 0 & 1 & 0 & 0 \\ 6 & 0 & 0 & 0 & 0 \\ 7 & 0 & 0 & 0 & 0 \\ 8 & 0 & 0 & 0 & 0 \end{matrix},$$

and

$$J_{L_1}^{(1)+} = \begin{matrix} & 1 & 2 & 3 & 4 \\ 1 & 0 & 0 & 0 & 0 \\ 2 & 0 & 0 & 1 & 0 \\ 3 & 0 & 0 & 0 & 0 \\ 4 & 0 & 0 & 0 & 0 \end{matrix}, \quad J_{L_2}^{(2)+} = \begin{matrix} & 5 & 6 & 7 & 8 \\ 5 & 0 & 0 & 1 & 0 \\ 6 & 0 & 0 & 0 & 1 \\ 7 & 0 & 0 & 0 & 0 \\ 8 & 0 & 0 & 0 & 0 \end{matrix}.$$

When state 1 is not SEQ for CDM in G_1, SEQ at state $(1, 5)$ for CDM is investigated. As can be seen from $J_{CDM}^{(1)+}$ and $J_{CDM}^{(2)+}$, (q_1, q_2) denotes all UIs from state (1, 5) in Theorem 8.2, written as:

(a) $(q_1, q_2) = (2, 6)$;

(b) $(q_1, q_2) = (2, 5)$;

(c) $(q_1, q_2) = (1, 6)$,

where (q_1, q_2), $s_1 \in S_1$, and $s_2 \in S_2$.

From the above, $q_1 = 1$ or 2; $q_2 = 5$ or 6. For q_1, the 0-1 vector $\boldsymbol{e}_{q_1}$ has the q_1th entry 1 and other entries 0. Thus, $q_1 = 1$ corresponds to $\boldsymbol{e}_{q_1} = (1\ 0\ 0\ 0)^T$.

Therefore, all 0-1 vectors corresponding to q_1 and q_2 can be written as $\boldsymbol{e}_{q_1} = (1\ 0\ 0\ 0)^T$ or $(0\ 1\ 0\ 0)^T$; $\boldsymbol{e}_{q_2} = (1\ 0\ 0\ 0)^T$ or $(0\ 1\ 0\ 0)^T$.

For (a), $\boldsymbol{e}_{q_1} = (0\ 1\ 0\ 0)^T$ and $\boldsymbol{e}_{q_2} = (0\ 1\ 0\ 0)^T$. Note that state 6 is actually the second entry of $\boldsymbol{e}_{q_2}$.

One can calculate $(J_{L_1}^{(1)+})^T \cdot \boldsymbol{e}_{q_1} = (0\ 0\ 1\ 0)^T$, which is the transpose of the second row of $J_{L_1}^{(1)+}$. Then, $\boldsymbol{r_1} = (0\ 1\ 1\ 0)^T$. Analogously, $\boldsymbol{r_2} = (0\ 1\ 0\ 1)^T$. After calculating $\boldsymbol{r}$, one can conclude that the non-zero entries in $\boldsymbol{r}$ correspond to states (2, 8), (3, 6), (3, 8). Thus, state (1, 5) is SEQ for CDM if the score of (1, 5) is no less than the score of at least one of the three states, (2, 8), (3, 6), and (3, 8).

Analogously for (b) and (c), state (1, 5) is also possible to be SEQ for CDM in G.

SEQ for CDM in G is also affected by the weights of local graphs. The relation between the weights and SEQ for CDM is indicated by the following corollary.

Corollary: State (s_1, s_2) is SEQ for CDM in G iff $w_k \in (\alpha, \beta)$ for either $\alpha = 0$ or $\beta = 1$.

According to the above corollary, a state is SEQ for CDM in G if and only if each weight w_k $(k = 1, 2)$ ranges from either side of the interval (0, 1), i.e., the range of w_k should be either $(0, \beta)$ or $(\alpha, 1)$. The proof of this corollary is demonstrated in Supplementary Material.

The stabilities for LDMs are investigated in Theorem 8.3.

Theorem 8.3: Suppose there exists a weighted basic hierarchical graph model G consisting of G_1 and G_2, and the number of states in G_1 is $|S_1| = m$ and $|S_2| = n$ in G_2. If a state $s = (s_1, s_2) \in S$ is SEQ for LDM_1 in G, then

1) s_1 is SEQ for LDM_1 in G_1 or

2) s_1 is GMR for LDM_1 in G_1, $\boldsymbol{e}_{s_2}^{\boldsymbol{T}} \cdot J_{CDM}^{(2)+} \neq 0$ and $\boldsymbol{e}_{s_2}^{\boldsymbol{T}} \cdot M_{\{C,L_2\}}^{(2)+} \neq 0$ where $M_{\{C,L_2\}}^{(2)+}$ is the joint improvement matrix for CDM and LDM_2 in G_2 and $\boldsymbol{e}_{s_2}^{\boldsymbol{T}}$ is the transpose of vector with $s_2{}^{th}$ element 1 and others 0, and $r \in S$, such that $\Psi(r) > \Psi(q)$ for CDM and the r^{th} element in 0-1 vector $\boldsymbol{r}$ is 1, where $r = (r_1, r_2)$:

$$\boldsymbol{e}_{q_1}^{\boldsymbol{T}} \cdot J_{CDM}^{(1)} \neq \boldsymbol{0}, \text{ and } \boldsymbol{e}_{s_2}^{\boldsymbol{T}} \cdot M_{\{C,L_2\}}^{(2)+} \neq \boldsymbol{0} \quad \forall q_1 \in R_{L_1}^{(1)+}(s_1),$$

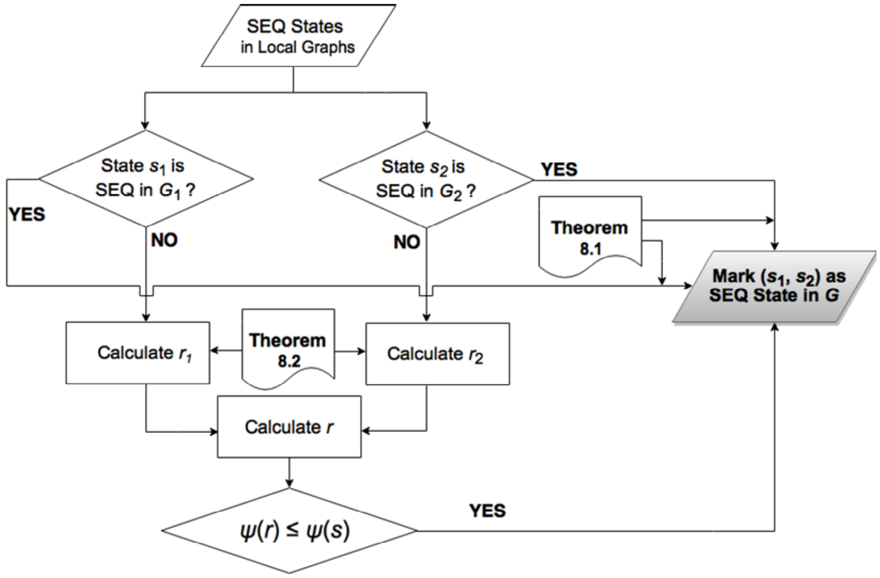

Figure 5. Algorithm to Determine SEQ States for CDM in G.

and $q=(q_1,q_2)$.

Theorem 8.2 indicates the stabilities of component states in an SEQ state for a LDM in G. The component state in the focal LDM's local graph can be SEQ. If not, some conditions should be satisfied.

4.3. General Metarationality (GMR)

Theorem 9.1: In a weighted basic hierarchical graph model G consisting of G_1 and G_2, a state $s=(s_1,s_2)\in S$ is GMR for CDM in G if s_k (k = 1, 2) is GMR for CDM in G_k.

Theorem 9.2: In a weighted basic hierarchical graph model G consisting of G_1 and G_2, if a state $s=(s_1,s_2)\in S$ is GMR for CDM in G, then

1) s_k is GMR for CDM in G_k for both k = 1 and 2, or
2) if s_k is not GMR for CDM in G_k (k = 1 or 2, but not both), there exists $r=(r_1,r_2)\in S$, such that $\Psi(r)\leq\Psi(s)$, r_i is element in $\boldsymbol{r}_i$ (i = 1, 2):

$$r_1=\left(J_{L_1}^{(1)}\right)^T\cdot\boldsymbol{e}_{q_1}+\boldsymbol{e}_{q_1}$$

and

$$\boldsymbol{r}_2=\left(J_{L_2}^{(2)}\right)^T\cdot\boldsymbol{e}_{q_2}+\boldsymbol{e}_{q_2}$$

for all $(q_1,q_2)\in R_{CDM}^{+}(s)$ and $(r_1, r_2)\neq(q_1, q_2)$; $\boldsymbol{e}_{q_k}$ is a 0-1 vector with the q_k th entry being 1 and others 0.

The proofs of Theorems 9.1 and 9.2 are analogous to the proof of Theorem 8.2. Thus, they are not included in the Appendix.

Theorem 9.3: In a weighted basic hierarchical graph model G consisting of G_1 and G_2, a state $s=(s_1,s_2)\in S$ is GMR for LDM_1 in G, iff $s_1\in S_1$ is GMR for LDM_1 in G_1.

As suggested in Theorem 9.3, a state is GMR for an LDM in G if and only if the component state in its local graph is GMR.

4.4. Symmetric Metarationality (SMR)

Theorem 10.1 (SMR): In a weighted basic hierarchical graph model G consisting of G_1 and G_2, a state $s=(s_1, s_2)\in S$ is SMR for CDM in G if s_k (k = 1, 2) is SMR for CDM in G_k.

Theorem 10.2: Suppose G is a weighted basic hierarchical graph model consisting of G_1 and G_2, if a state $s=(s_1, s_2)\in S$ is SMR for CDM in G, then

1) s_k is SMR for CDM in G_k for both k = 1 and 2, or
2) if s_k is not SMR for CDM in G_k (k = 1, 2, but not both), there exists $r=(r_1,r_2)\in S$, such that $\Psi(r)\leq\Psi(s)$ and $\Psi(t)\leq\Psi(s)$ for all $t=(t_1,t_2)\in S$,

where r and t corresponds to vectors $\boldsymbol{r}$ and $\boldsymbol{t}$, respectively, t_i is element in $\boldsymbol{t}_i$ (i = 1, 2):

$$\boldsymbol{t_1}=\left(J_{CDM}^{(1)}\right)^T\cdot\boldsymbol{e}_{r_1}+\boldsymbol{e}_{r_1}$$

and

$$\boldsymbol{t_2}=\left(J_{CDM}^{(2)}\right)^T\cdot\boldsymbol{e}_{r_2}+\boldsymbol{e}_{r_2}$$

for $\boldsymbol{e}_{r_k}$ (k = 1, 2) represents a 0-1 vector with r_k th entry being 1 and others 0, $\Psi(r)\leq\Psi(s)$, and r_i is element in $\boldsymbol{r}_i$ (i = 1, 2):

$$\boldsymbol{r}_1=\left(J_{L_1}^{(1)}\right)^T\cdot\boldsymbol{e}_{q_1}+\boldsymbol{e}_{q_1}$$

and

$$\boldsymbol{r_2}=\left(J_{L_2}^{(2)}\right)^T\cdot\boldsymbol{e}_{q_2}+\boldsymbol{e}_{q_2}$$

for all $(q_1,q_2)\in R_{CDM}^{+}(s)$, $(r_1, r_2)\neq(q_1, q_2)$, and $(t_1, t_2)\neq(r_1, r_2)$.

Note that $\boldsymbol{e}_{r_k}$ is different from $\boldsymbol{r}_k$. The entries in $\boldsymbol{e}_{r_1}$ correspond to state r_1, which is the component in (r_1, r_2). Vector $\boldsymbol{r}_1$ is related to not only state r_1, but also state q_1. Thus, $\boldsymbol{r}_1$ can be written as:

$$\boldsymbol{r}_1 = \boldsymbol{e}_{r_1} + \boldsymbol{e}_{q_1}, \text{ for } \boldsymbol{e}_{r_1} = \left(J_{L_1}^{(1)}\right)^T \cdot \boldsymbol{e}_{q_1}.$$

Theorem 10.3: In a weighted basic hierarchical graph model G consisting of G_1 and G_2, a state $s = (s_1, s_2) \in S$ is SMR for LDM_1 in G, iff $s_1 \in S_1$ is SMR for LDM_1 in G_1.

Theorems 10.1, 10.2, and 10.3 are analogous to the Theorems in Sections 4.2 and 4.3. The proofs of Theorem 10.1 can be found in Supplementary Material.

5. Algorithms for Calculating Stability

The algorithms for calculating stability results in the hierarchical model are designed according to theorems demonstrated in Section 4. Each algorithm is also described in steps.

5.1. Nash Stability (R)

According to Theorem 7.1, a state $(s_1, s_2) \in S$ is Nash stable for CDM in G if and only if $s_k \in S_k$ is Nash stable for CDM in G_k (k = 1, 2). As demonstrated in Theorem 7.2, state s is Nash stable for LDM_k in G_k if and only if s_k is Nash stable for LDM_k in G_k.

5.2. Sequential Stability (SEQ)

The SEQ states in the hierarchical graph model can be calculated according to Theorems 8.1 and 8.2. The algorithm for this calculation for CDM is shown in Figure 5. The detailed steps for the calculation are shown as follows:

Step 1: For $(s_1, s_2) \in S$, if s_k is SEQ for CDM for both k = 1 and 2, then (s_1, s_2) is SEQ for CDM in G according to Theorem 8.1, otherwise go to step 2.

Step 2: Calculate $\boldsymbol{r}$, if the non-zero element r in $\boldsymbol{r}$ satisfies $\Psi(r) \le \Psi(s)$, then (s_1, s_2) is SEQ for CDM in G according to Theorem 8.2.

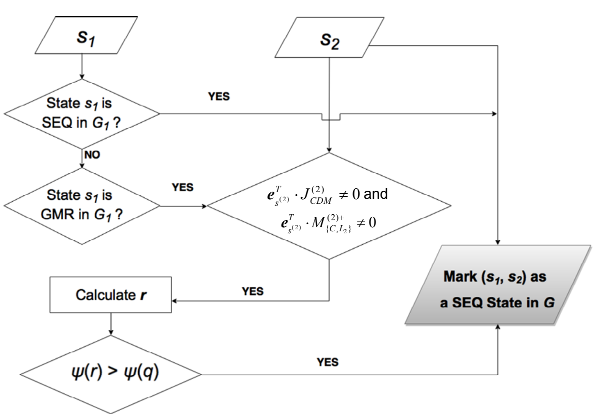

Figure 6. Algorithm to determine SEQ states for LDM_1 in G.

The algorithm to determine SEQ for LDM_1 in the hierarchical model can be depicted in Figure 6. The procedure for this calculation is demonstrated as:

Step 1: For $(s_1, s_2) \in S$, if s_1 is GMR for LDM_1 for G_1, then (s_1, s_2) is SEQ for LDM_1 in G according to Theorem 8.3, otherwise go to step 2.

Step 2: If s_1 is GMR for LDM_1 in G_1 and s_2 satisfies $\boldsymbol{e}_{s_2}^T \cdot J_{CDM}^{(2)+} \neq 0$ and $\boldsymbol{e}_{s_2}^T \cdot M_{\{C,L_2\}}^{(2)+} \neq 0$ according to Theorem 8.3, then go to Step 3.

Step 3: Calculate $\boldsymbol{r}$, if there exists non-zero element r in $\boldsymbol{r}$ such that $\Psi(r) > \Psi(q)$, then (s_1, s_2) is SEQ for LDM_1 in G.

5.3. General Metarationality (GMR)

The GMR states for CDM in the hierarchical model can be determined analogously to calculating SEQ for CDM. The algorithm is shown in Figure 7.

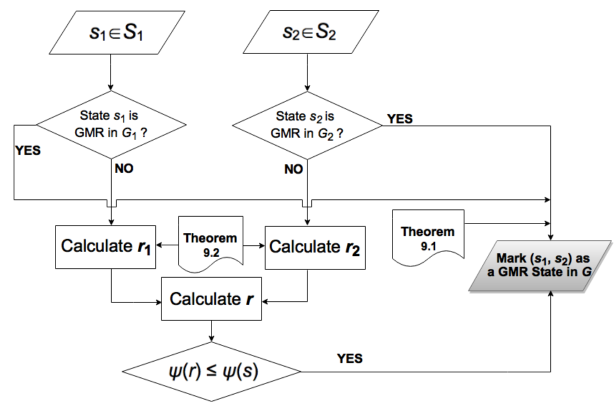

Figure 7. Algorithm to Determine GMR States for CDM in G.

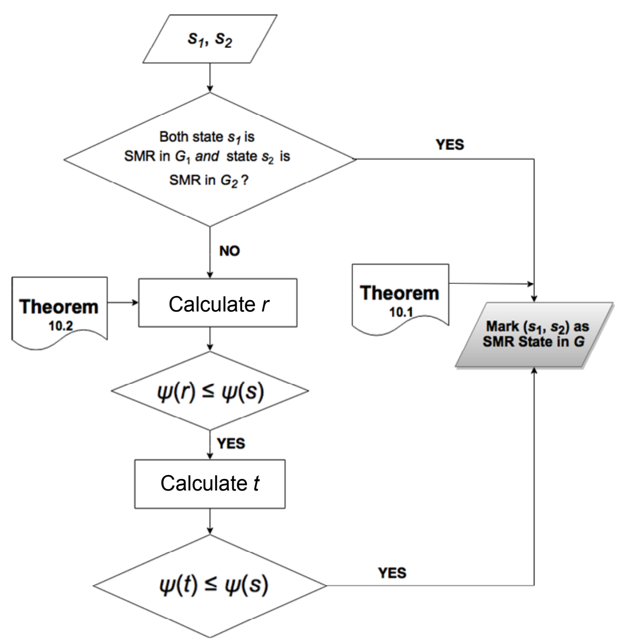

Figure 8. Algorithm to determine SMR states for CDM in G.

Table 1. DMs, Their Options, and States in the Overall Conflict

CG	1) Full Central	N	Y	N	Y	N	Y	N	Y	N	Y	N	Y	N	Y	N	Y
	2) Resume Western	N	N	Y	Y	N	N	Y	Y	N	N	Y	Y	N	N	Y	Y
LRs	3) Agree	N	N	N	N	Y	Y	Y	Y	N	N	N	N	Y	Y	Y	Y
NCs	4) Consent	N	N	N	N	N	N	N	N	Y	Y	Y	Y	Y	Y	Y	Y
State		1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
		(4, 8)	(2, 8)	(4, 6)	(2, 6)	(3, 8)	(1, 8)	(3, 6)	(1, 6)	(4, 7)	(2, 7)	(4, 5)	(2, 5)	(3, 7)	(1, 7)	(3, 5)	(1, 5)

According to Theorem 9.3, (s_1, s_2) is GMR for LDM_k in G if and only if s_k is GMR for LDM_k in G_k.

5.4. Symmetric Metarationality (SMR)

SMR states for CDM in the hierarchical model can be determined according to Theorems 10.1 and 10.2. As depicted in Figure 8, the calculation is described in the following steps:

Step 1: For $(s_1, s_2) \in S$, if s_k is SMR for CDM for both k = 1 and 2, then (s_1, s_2) is SMR for CDM in G according to Theorem 10.1, otherwise go to step 2.

Step 2: Calculate $\boldsymbol{r}$, if there exists non-zero element r in $\boldsymbol{r}$ such that $\Psi(r) \leq \Psi(s)$, then go to step 3.

Step 3: Calculate $\boldsymbol{t}$, if all elements in $\boldsymbol{t}$ are 1 and each element t satisfies $\Psi(r) \leq \Psi(s)$, then (s_1, s_2) is SMR for CDM in G according to Theorem 10.2.

According to Theorem 10.3, the SMR states for LDMs can be calculated analogously to obtaining GMR states for LDMs.

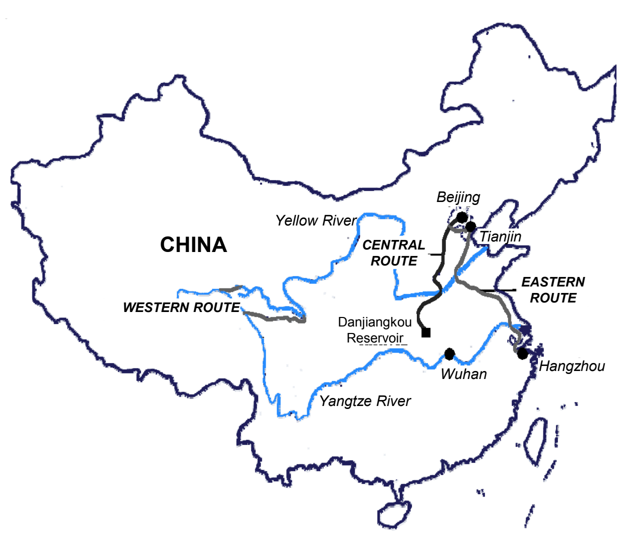

Figure 9. The Location of Three Routes of South-North Water Diversion Project (Source: http://francistopoa.edublogs.org/2011/06/03/south-north-water-diversion-project, 2011).

6. Case Study: Water Diversion Conflicts in China

The South-North Water Diversion Project (SNWDP) has been proposed by the Chinese Government (CG) to ease severe water shortage in the North China Plain. This large project is constructed at three locations. The eastern part of the project is designed to transfer water using the Grand Canal from the Yangtze Delta to Tianjin, a harbour city close to Beijing. The central route of the project starts from Hubei Province northbound to Beijing. The western part of the project connects the Yangtze River with Yellow River in Tibet Plateau. The three routes of SNWDP are shown in Figure 9. As the eastern part of the project is complete, only the central and western projects are investigated.

The conflicts in SNWDP are modelled by the weighted hierarchical graph model. Conflicts at the central and western locations are modelled by two local graphs. Local residents at the central location are forced to be relocated to make way for the construction. Although CG has initiated action plans to help these residents settle down, they still voiced dissatisfaction due to the difficulty of adjusting to the new neighbourhood. The diversion of water at the western location has caused concerns from neighbouring countries such as India and Bangladesh. These downstream countries fear that their water usages would be affected by the construction of dams and reservoirs in the Tibet Plateau. Thus, the weighted hierarchical graph model is used to formally investigate these conflicts. The results of the model suggest how DMs can act strategically in the process of reaching resolution. DMs will be provided with a comprehensive understanding of these conflicts and guidance for taking beneficial actions.

In the weighted hierarchical model, Chinese Central Government (CG) is the CDM in the hierarchal graph. The central and western subconflicts are labelled as G_1 and G_2. Local Residents (LRs) and Neighboring Countries (NCs) are two LDMs in G_1 and G_2, respectively. The DMs and their options in the hierarchical model are listed in Table 1. In the overall conflict (G), CG has two options, each of which is marked with a number followed by a half parenthesis. The selection of an option is denoted by "Y" and its negation by "N". Option 1) means CG initiates the original central project. It can also modify the projects to ease the opposition from LRs, which is represented as the negation of option 1). Similarly, option 2) denotes that CG resume the western project, the negation of which indicates the suspension of the project. The agreement of LRs is represented by option 3) while the negation means their opposition. Analogously, option 4) implies the consent from NCs and the negation their protest. The DMs and their options in the two local graph models are demonstrated in Table 2. Because the two local graph models are the components of the hierarchical graph model, the options for each DM in G_1 and G_2 are identical to those in G.

A state, represented by a selection of options for all DMs, indicates a possible outcome. The 2^4 feasible states in G are marked with decimal numbers in Table 1. The feasible states are numbered from state 1 to 4 in G_1, and from state 5 to 8 in G_2, respectively, as shown in Table 2. Thus, a state in G can also be denoted by a two-digit number contained in a parenthesis. The first and second components of the number indicate states in G_1 and G_2, respectively. For example, state 16 in Table 1 is identical to state (1, 5) which is composed by state 1 in G_1 and state 5 in G_2.

Table 2. DMs, Their Options, and States in Two Subconflicts

Central Subconflict (G_1)					
DM	Option				
CG	1) Full Central	Y	Y	N	N
LRs	3) Agree	Y	N	Y	N
States		1	2	3	4
Western Subconflict (G_2)					
DM	Option				
CG	2) Resume Western	Y	Y	N	N
NCs	4) Consent	Y	N	Y	N
States		5	6	7	8

The UI matrices for DMs in local graphs are shown below as $J_{CG}^{(1)+}$, $J_{CG}^{(2)+}$, and $J_{L_2}^{(2)+}$. Note that all moves are assumed reversible for each DM:

$$J_{CG}^{(1)+} = \begin{array}{c} \\ 1 \\ 2 \\ 3 \\ 4 \end{array}\begin{array}{c} \begin{array}{cccc} 1 & 2 & 3 & 4 \end{array} \\ \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{pmatrix} \end{array}, \quad J_{CG}^{(2)+} = \begin{array}{c} \\ 5 \\ 6 \\ 7 \\ 8 \end{array}\begin{array}{c} \begin{array}{cccc} 5 & 6 & 7 & 8 \end{array} \\ \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{pmatrix} \end{array},$$

$$J_{L_1}^{(1)+} = \begin{array}{c} \\ 1 \\ 2 \\ 3 \\ 4 \end{array}\begin{array}{c} \begin{array}{cccc} 1 & 2 & 3 & 4 \end{array} \\ \begin{pmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{pmatrix} \end{array}, \quad J_{L_2}^{(2)+} = \begin{array}{c} \\ 5 \\ 6 \\ 7 \\ 8 \end{array}\begin{array}{c} \begin{array}{cccc} 5 & 6 & 7 & 8 \end{array} \\ \begin{pmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{pmatrix} \end{array}$$

The preferences for CG in the two subconflicts can be represented using option prioritization method to calculate the score of corresponding states in algorithms, listed in Table 3. In G_1, preference statements for CG are written in the left column. Option 1) is the first statement for CG, which means that the original construction plan at the central location is the most important issue for CG. The next preference statement, option 3), is written below option 1), which denotes that the agreement from LRs has the second importance for CG. According to Equation (1), the scores for the two statements are 2^{2-1} and 2^{2-2}, respectively. As a state in G can be written as a selection of options, each state is investigated for whether the option selection matches a given preference statement. In the third column, a truth value is assigned at each state for each preference statement. For example, state 2 in G can be expressed as "YN". Because option 1) is selected and option 3) is not, the truth value of state 2 is "T" on option 1) and "F" on option 3). According to Equation (1), the score for each state is aggregated. For state 2, the score is $1 \times 2^1 + 0 \times 2^0 = 2$. Thus, the scores for states 1, 2, 3, and 4 are 3, 2, 1, 0. The four states can also be ranked according to the scores from state 1 the most preferred to state 4 the least preferred. This ranking is also consistent with the preferences for CG expressed by the UI matrix. Note that all state numbers are in bold. The preferences for CG in G_2 can be analyzed analogously.

Stability results in the local conflicts are shown in Tables 4 and 5. The stability results in the hierarchical conflict are calculated using algorithms introduced in Section 5. Taking SEQ for CDM as an example, as shown in Tables 4 and 5, states 1 and 2 are SEQ for CG in G_1 and states 5, 6, and 7 are SEQ for CG in G_2.

According to Theorem 8.1, states (1, 5), (1, 6), (2, 5), (2, 6), (1, 7), and (2, 7) are SEQ for CG in the hierarchical conflict.

Table 3. Preferences for CG in Two Subconflicts Obtained by Option Prioritization Method

G_1					
Preference Statements	Score of Each Statement	States **1**	**2**	**3**	**4**
1)	2^1	T	T	F	F
3)	2^0	T	F	T	F
	Score of Each States	3	2	1	0
	Ranking of States	**1**	**2**	**3**	**4**
G_2					
Preference Statements	Score of Each Statement	**5**	**6**	**7**	**8**
4)	2^1	T	F	T	F
2)	2^0	T	T	F	F
	Score of Each States	3	1	2	0
	Ranking of States	**5**	**7**	**6**	**8**

Table 4. Stability Results in the Central Conflict (G_1)

	Nash	SEQ	GMR	SMR
CG	1, 2	1, 2	1, 2	1, 2
LRs	2, 3	2, 3, 4	2, 3, 4	2, 3, 4

Table 5. Stability Results in the Western Conflict (G_2)

	Nash	SEQ	GMR	SMR
CG	5, 6	5, 6, 7	5, 6, 7	5, 6, 7
NCs	6, 7	6, 7, 8	6, 7, 8	6, 7, 8

To determine the rest of SEQ states for CDM in G, states 3 and 4 in G_1 and state 8 in G_2 are investigated according to Figure 4. For example, let $s_1 = 1$ and $s_2 = 8$, then $q_1 = 1$ and $q_2 = 6$, which can be written as $\boldsymbol{e}_{q_1} = (1\ 0\ 0\ 0)^{\mathrm{T}}$ and $\boldsymbol{e}_{q_2} = (0\ 1\ 0\ 0)^{\mathrm{T}}$.

Table 6. Stability Results in the Overall Conflict

	Nash	SEQ	GMR	SMR
CG	(1, 5), (1, 6), (2, 5), (2, 6)	(1, 5), (1, 6), (2, 5), (2, 6), (1, 7), (2, 7), (1, 8)	(1, 5), (1, 6), (2, 5), (2, 6), (1, 7), (2, 7), (1, 8)	(1, 5), (1, 6), (2, 5), (2, 6), (1, 7), (2, 7), (1, 8)
LRs	(2, -),(3, -)	(2, -), (3, -), (4, -)	(2, -), (3, -), (4, -)	(2, -), (3, -), (4, -)
NCs	(-, 6),(-, 7)	(-, 6), (-, 7), (-, 8)	(-, 6), (-, 7), (-, 8)	(-, 6), (-, 7), (-, 8)
Overall	(2, 6)	(2, 6), (2, 7)	(2, 6), (2, 7)	(2, 6), (2, 7)

Then, $\boldsymbol{r_1} = (1\ 1\ 0\ 0)^T$, and $\boldsymbol{r_2} = (0\ 1\ 0\ 1)^T$. In $\boldsymbol{r}$, the non-zero elements correspond to states (1, 8), (2, 6), and (2, 8). Since CG considers the central subconflict more important, the weights for the two subconflicts are assumed as $w_1 = 0.9$ and $w_2 = 0.1$. According to Equation (3) and Table 3, $\Psi(s) = 3 \times 0.9 + 0 \times 0.1 = 2.7$. When $r = (2, 6)$, $\Psi(r) = 2 \times 0.9 + 1 \times 0.1 = 1.9$. Thus, $\Psi(r) < \Psi(s)$ for $s = (1, 8)$ and $r = (2, 6)$. Then state (1, 8) is an SEQ state for CG in G. Other stable states in G can be determined analogously. The stability results for DMs in the hierarchical conflict are listed in Table 5. Note that "-" in Table 6 represents any state in S_k ($k = 1$ or 2).

States (2, 6) and (2, 7) are equilibria in the overall conflict as they are stable for all DMs. State (2, 7) is the possible outcome of this hierarchical conflict since it can be evolved from the status quo, which refers to the starting state. Equilibrium (2, 7) echoes the actual outcome (The Economic Times, 2012). Chinese Government insisted on the original plan at the central location, but suspended the western projects. NCs are satisfied with the current status.

The weighted preference structure for CDM is more flexible to express CG's assessment on the importance of each local graph. The stability results for CG can change according to different value of weights assigned to the local graphs. To carry out further analysis, these stability results with a complete range of w_1 in the water diversion conflicts are investigated.

As CG considers the central subconflict more important, the weight for the central subconflict is greater than 0.5, i.e., $w_1 > 0.5$. Because the weight is a number between 0 and 1, $w_1 > 0.5$ is equivalent to $w_1 \in [0.5, 1)$. Note that $w_1 \neq 0, 1$, as both w_1 and w_2 must be greater than 0. According to Theorem 7.1, Nash states for CG are not affected by the weights for the local graphs. The SEQ and GMR states for CG with different weights are shown in Figures 10 and 11. The relationship between the SMR and w_1 for CG are the same as that between GMR and w_1, shown in Figure 10.

In Figure 10, the horizontal axis denotes the value of w_1 ranging from 1 to 0. The 16 states in the overall conflict are listed in the vertical axis. Each bar represents the range of w_1 within which the corresponding state is SEQ for CDM. For example, state (4, 5) is SEQ for CDM when $w_1 \in (0, 1/3]$. Note that the left end of the bar is round and the right end is flat, since w_1 can be equal to 1/3 but never reach 0.

According to Figure 10, states (1, 5), (1, 6), (1, 7), (2, 5), (2, 6), (2, 7) are SEQ for CG regardless of the change of w_1. State (1, 8) is SEQ for CG when w_1 is no less than 0.5. States (3, 5) and (4, 5) are SEQ for CG if w_1 is no larger than 0.5 and 1/3 respectively. Taking state (1, 8) as an example, CG's UI is sanctioned by a LRs' UI if CG considers the central project more important, which means $w_1 \in [0.5, 1)$. Thus, the counteraction from LRs will be highly valued. As w_1 is below 0.5, the central project will be less important. This countermove from LRs would fail to sanction CG when CG has a view of both central and western projects.

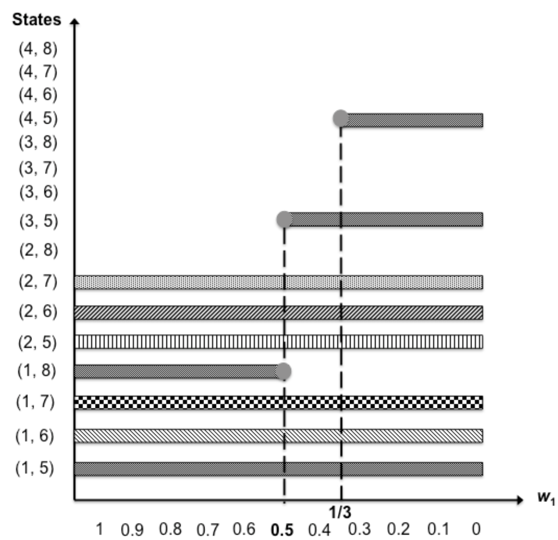

Figure 10. SEQ States for CDM in the Hierarchical Graph Changed with Weights.

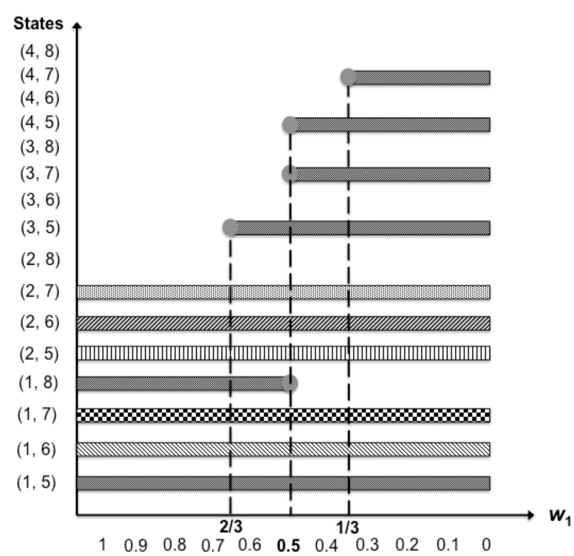

Figure 11. GMR and SMR States for CDM in the Hierarchical Graph Changed with Weights.

The SEQ states for LDMs in the hierarchical conflict are the same when the weights for local graphs are changed. For LRs, states (2, -), (3, -), (4, -) are SEQ since the corresponding component states 2, 3, and 4 are SEQ. Although the states can be SEQ for REs when the corresponding component states are not SEQ, no SEQ state is found among states (1, -) in this example.

The results calculated in the weighted hierarchical graph model indicate resolution for DMs in this conflict. Equilibrium (2, 7) suggests that the project at the central location can be successfully constructed despite opposition from LRs. CG will suspend the construction plan at the western project to appease NCs. For CG, the central projects should be constructed first. Although the dissatisfaction from LRs cannot affect the course of the construction, more action plans are advised to better accommodate LRs after relocation. As the western project is likely to cause transboundary disputes, CG should fully negotiate with NCs before resuming the project. As LRs do not have much impact in this conflict, they can still express their concerns by communicating with local governments. According to the results in Figures 10 and 11, CG can also change the results of the conflict by accordingly resetting the importance of each local graph. Therefore, CG can be provided with a wide range of resolution depending on which outcome it desires to achieve.

7. Comparison of Weighted Hierarchical Graph Model and Former Methodologies

The advantage of weighted hierarchical graph model is explained by comparing it with the former hierarchical model with lexicographical preferences (He et al., 2013).

The weighted hierarchical graph model has more flexible preference structure compared with the basic hierarchical graph model with lexicographic preferences. The lexicographic preference structure is a crisp approach of determining preference relations for CDM. A local graph can be either more important, less important, or equally important than the other local graph. In the weighted preference structure, the relative importance between the two local graphs is expressed by numbers between 0 and 1. Thus, such importance can be described more flexibly.

The calculation for stabilities in the weighted hierarchical graph model is also more effective than the former calculation method. The stabilities in the basic hierarchical model with lexicographic preferences are determined by theorems describing the interrelationship between stabilities in the hierarchical model and local models. In comparison, the stabilities in the weighted hierarchical model are obtained by matrix computation based on the algorithms. As demonstrated in the case study, the new calculation approach is more simplified and easier to implement.

The stabilities in the weighted hierarchical graph model are more sensitive to the relative importance of local graphs, compared with the stabilities in the hierarchical graph model with lexicographic preferences. As depicted in Figure 9, state (3, 5) is not GMR for CG if it values the weight of G_1 greater than 2/3. However, in the hierarchical model with lexicographic preferences, this state is still GMR for CG when it considers G_1 more important, which is interpreted as $w_1 \in (0.5, 1)$ in the weighted hierarchical model.

Compared with original graph model methodology, the hierarchical graph model can provide DMs with more meaningful insights when modelling interrelated conflicts. According to Tables 4, 5, and 6, the two SEQ states for CG in G_1 and three SEQ states for CG in G_2 are components of the six SEQ states for CG in G. Another SEQ state for CG in G, state (1, 8), contains state 8, an unstable state in G_2. This state is also GMR and SMR for CG. Thus, state (1, 8) indicates a stable outcome in the hierarchical graph model, which would be neglected by modelling the two subconflicts separately. At this state, CG initiates the original plan at both locations. LRs are pleased while NCs protest. At the western location, CG will change the status by improving from state 8 to 6, as indicated in $J_{CG}^{(2)+}$. However, CG will not initiate this UI if it considers the central and western subconflicts together, because at the central location, LRs' UI from state 1 to 2 will be less preferred by CG. The possible outcome, state (2, 6), is less desired for CG, since the loss at the central location matters more than the benefit at the western location for CG. It is rational for CG to stay at state (1, 8). Therefore, CG can take more reasonable actions based on the stabilities calculated by the hierarchical graph model.

8 Conclusions and Further Studies

A basic hierarchical graph model has been proposed with weighted preference structure for CDM. The reachable lists and UIs in this model have been expressed in matrices. Algorithms have been constructed for calculating stability results in the hierarchical model based on the theorems which link stability definitions of the hierarchical graph with the local graphs. The new model with weighted preference has been applied to water diversion conflicts in China. The calculation for stability results in this conflict has been demonstrated. The equilibrium indicates the possible resolutions for DMs in this conflict. Furthermore, the stability results in the hierarchical conflict are also investigated by changing the weights on local graphs. Compared with the previous model (He et al., 2013), the weight preference can model CDM's assessment on the importance of local graphs with more efficiency.

The hierarchical graph model can effectively solve strategic conflicts that contain several related subconflicts. Instead of focusing on one single conflict, DMs can have a comprehensive understanding of all related conflicts. CDM can achieve its goals by weighing up different subconflicts.

The algorithms designed in Section 5 have been demonstrated to calculate stability results in the hierarchical conflict. This approach can be used for calculating stability results in some hierarchical graph models consisting of a large number of local graphs. While the direct calculation methods become less efficient due to the large dimension of the model, the new approach only requires stability results in local graphs and the weights local graphs. Compared with the matrix approach (Xu et al., 2009) in which matrices are multiplied, the computation across local graphs in the weighted model is achieved by Cartesian product of vectors. Therefore, the new approach can avoid heavy calculation and require less processing time.

For further study, a decision support system based on the algorithms in the weighted model can be developed.

The structure of the hierarchical graph model can also be extended. For example, a hierarchical graph model with more than two levels can be proposed. Uncertainty and strength of preferences in the hierarchical model can also be introduced.

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References

Barbalios, N., Ioannidou, I., Tzionas, P., and Paraskeuopoulos, S. (2013). Behavioural adaptation towards efficient resource sharing under the lack of communication. *J. Environ. Inf.,* 21(2), 119-135. https://doi.org/10.3808/jei.201300239

Bashar, M.A., Kilgour, D.M., and Hipel, K.W. (2014). Fuzzy option prioritization for the graph model for conflict resolution. *Fuzzy Sets Syst.,* 246, 34-48. https://doi.org/10.1016/j.fss.2014.02.011

Brams, S.J. and Wittman, D. (1981). Nonmyopicequilibria in 2 × 2 games. *Conflict Manage. Peace Sci.,* 6(1), 39-62. https://doi.org/10.1177/073889428100600103

Fan, Y.R. and Huang, G.H. (2012). A robust two-step method for solving interval linear programming problems within an environmental management context. *J. Environ. Inf.,* 19(1), 1-9. https://doi.org/10.3808/jei.201200203

Fang, L., Hipel, K.W., and Kilgour, D.M. (1993). *Interactive Decision Making: The Graph Model for Conflict Resolution,* New York: Wiley.

Fang, L., Hipel, K.W., Kilgour, D.M., and Peng, X. (2003a). A decision support system for interactive decision making-part I: model formulation. *IEEE Trans. Syst. Man Cybernetics C. Appl. Rev.,* 33(1), 42-55.

Fang, L., Hipel, K.W., Kilgour, D.M., and Peng, X. (2003b). A decision support system for interactive decision making-part II: analysis and output interpretation. *IEEE Trans. Syst. Man Cybernetics C. Appl. Rev.*, 33(1), 56-66.

Fraser, N.M. and Hipel, K.W. (1984). *Conflict Analysis: Models and Resolutions*, New York: North-Holland.

Gilles, R.P. (2010). *The Cooperative Game Theory of Networks 1180 and Hierarchies*, volume 44. Heidelberg: Springer. https://doi.org/10.1007/978-3-642-05282-8

Gunalay, Y., Yeomans, J.S., and Huang, G.H. (2012). Modelling to generate alternative policies in highly uncertain environments: An application to municipal solid waste management planning. *J. Environ. Inf.,* 19(2), 58-69.

Gvozdeva, T., Hameed, A., and Slinko, A. (2012). Weightedness and structural characterization of hierarchical simple games. *Math. Soc. Sci.,* 65(3), 181-189. https://doi.org/10.1016/j.mathsocsci.2012.11.007

Hamouda, L., Kilgour, D.M., and Hipel, K.W. (2004). Strength of preference in the graph model for conflict resolution. *Group Decis. Negotiation,* 13(5), 449-462. https://doi.org/10.1023/B:GRUP.0000045751.21207.35

He, S., Hipel, K.W., and Kilgour, D.M. (2012). Water diversion conflict in Danjiangkou, China. *Proc. of the 2012 IEEE International Conference on Systems, Man, and Cybernetics*, pp. 2435-2440. https://doi.org/10.1109/ICSMC.2012.6378108

He, S., Hipel, K.W., and Kilgour, D.M. (2014a). Water diversion conflicts in China: A hierarchical perspective. *Water Resour. Manage.,* 28(7), 1823-1837. https://doi.org/10.1007/s11269-014-0550-1

He, S., Kilgour, D.M., and Hipel, K.W. (2014b). Matrix representation of a hierarchical water matrix representation of a hierarchical water diversion conflict in China. *The Proc. of the Joint International Conference of the INFORMS GDN Section and the EURO Working Group on DSS*, pp. 238-243. Toulouse, France.

He, S., Kilgour, D.M., Hipel, K.W., and Bashar, M.A. (2013). A basic hierarchical graph model for conflict resolution with application to water diversion conflicts in China. *INFOR Inf. Syst. Oper. Res.,* 51(3), 103-119. https://doi.org/10.3138/infor.51.3.103

Hipel, K.W., Obeidi, A., F.L., and Kilgour, D.M. (2008). Adaptive systems thinking in integrated water resources management with insights into conflicts over water exports. *INFOR Inf. Syst. Oper. Res.,* 46(1), 51-70. https://doi.org/10.3138/infor.46.1.51

Hipel, K.W. and Walker, S.B. (2011). Conflict analysis in environmental management. *Environmetrics,* 22(3), 279-293. https://doi.org/10.1002/env.1048

Howard, N. (1971). *Paradoxes of Rationality: Theory of Metagames and Political Behavior*, Cambridge: MIT Press.

Howard, N. (1999). Confrontation analysis: how to win operations other than war. Technical report, C4ISR Cooperation Research Program, The Pentagon, Washington, D.C.

Inohara, T. and Hipel, K.W. (2008a). Coalition analysis in the graph model for conflict resolution. *Syst. Eng.,* 11(4), 343-359. https://doi.org/10.1002/sys.20104

Inohara, T. and Hipel, K.W. (2008b). Interrelationships among noncooperative and coalition stability concepts. *J. Syst. Sci. Syst. Eng.,* 17(1), 1-29. https://doi.org/10.1007/s11518-008-5070-1

Kilgour, D.M. and Hipel, K.W. (2005). The graph model for conflict resolution: past, present, and future. *Group Decis. Negotiation,* 14(6), 441-460. https://doi.org/10.1007/s10726-005-9002-x

Kilgour, D.M. and Hipel, K.W. (2010). Conflict analysis methods: The graph model for conflict resolution. *Handbook of Group Decision and Negotiation*, pp. 203-222. Dordrecht: Springer. https://doi.org/10.1007/978-90-481-9097-3_13

Kilgour, D.M., Hipel, K.W., Fang, L., and Peng, X.J. (2001). Coalition analysis in group decision support. *Group Decis. Negotiation,* 10(2), 159-175. https://doi.org/10.1023/A:1008713120075

Li, K.W., Hipel, K.W., Kilgour, D.M., and Noakes, D. (2005b). Integrating uncertain preferences into status quo analysis with applications to an environmental conflict. *Group Decis. Negotiation,* 14(6), 461-479. https://doi.org/10.1007/s10726-005-9003-9

Madani, K. (2013). Modeling international climate change nego- tiations more responsibly: Can highly simplified game theory models provide reliable policy insights? *Ecol. Econ.,* 90, 68-76. https://doi.org/10.1016/j.ecolecon.2013.02.011

Madani, K. and Hipel, K.W. (2011). Non-cooperative stability definitions for strategic analysis of generic water resources conflicts. *Water Resour. Manage.,* 25(8), 1949-1977. https://doi.org/10.1007/s11269-011-9783-4

Nash, J.F. (1950). Equilibrium points in n-person games. *Proc. Natl.*

Acad. Sci. USA, 36(1), 48-49. https://doi.org/10.1073/pnas.36.1.48
Nash, J.F. (1951). Non-cooperative games. *Ann. Math.,* 54(2), 286-295. https://doi.org/10.2307/1969529
Simaan, M. and Cruz Jr, J.B. (1973). On the stackelberg strategy in nonzero-sum games. *J. Optimiz. Theory Appl.,* 11(5), 533-555. https://doi.org/10.1007/BF00935665
The Economic Times (2012). China wants to deepen strategic cooperation with India, 30 Sept, 2015. http://articles.economictimes.indiatimes.com.
Von Neumann, J. and Morgenstern, O. (1944). *Theory of Games and Economic Behavior (1st Ed)*, Princeton: Princeton University Press.
Von Stackelberg, H. (1934). *Marktform und Gleichgewicht,* Heidelberg: Springer.
Wang, L., Fang, L., and Hipel, K.W. (2003). Water resources allocation: a cooperative game theoretic approach. *J. Environ. Inf.,* 2(2), 11-22. https://doi.org/10.3808/jei.200300019
Xu, H., Hipel, K.W., and Kilgour, D.M. (2009). Matrix representation of solution concepts in multiple-decision-maker graph models. *IEEE Trans. Syst. Man Cybernetics A. Syst. Hum.,* 39(1), 96-108.
Xu, H., Kilgour, D.M., and Hipel, K.W. (2010a). Matrix representation and extension of coalition analysis in group decision support. *Comput. Math. Appl.,* 60(5), 1164-1176. https://doi.org/10.1016/j.camwa.2010.05.040
Xu, H., Hipel, K.W., Kilgour, D.M., and Fang, L., (2018). *Conflict Resolution using the Graph Model: Strategic Interactions in Competition and Cooperation*, Heidelberg: Springer.
Xu, H., Kilgour, D.M., Hipel, K.W., and Kemkes, G. (2010b). Using matrices to link conflict evolution and resolution in a graph model. *Eur. J. Oper. Res.,* 207(1), 318-329. https://doi.org/10.1016/j.ejor.2010.03.025
Xu, H., Kilgour, D.M., Hipel, K.W., and McBean, E.A. (2013). Theory and application of conflict resolution with hybrid preference in colored graphs. *Appl. Math. Model.,* 37(3), 989-1003. https://doi.org/10.1016/j.apm.2012.03.009
Xu, T.Y. and Qin, X.S. (2013). Solving water quality management problem through combined genetic algorithm and fuzzy simulation. *J. Environ. Inf.,* 22(1), 39-48. https://doi.org/10.3808/jei.201300244
Zagare, F.C. (1984). Limited-move equilibria in 2 $\times$ 2 games. *Theory Decis.,* 16(1), 1-19. https://doi.org/10.1007/BF00141672