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Effects of Mitigation Options on the Control of Methane Emissions Caused by Rice Paddies and Livestock Populations to Reduce Global Warming: A Modeling Study and Comparison with Environmental Data

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ABSTRACT. In this paper, a non-linear mathematical model is proposed and analyzed to study the effects of mitigation options on the control of methane emissions in the atmosphere caused by rice paddies and livestock populations to reduce global warming. In the modeling process, it is assumed that the cumulative biomass density of rice paddies and the density of livestock populations follow logistic models with their respective growth rates and carrying capacities. The growth rate of concentration of methane in the atmosphere is assumed to be directly proportional to the cumulative density of various processes involved in the production of rice paddies as well as the cumulative density of various processes used in the farming of livestock populations. This growth rate is also assumed to increase with natural factors such as wetlands but it decreases with the cumulative density of mitigation options, considered to be proportional to the increased level of methane concentration in the atmosphere. The non-linear model is analyzed by using the stability theory of differential equations and computer simulation. The analysis shows that mitigation options can control the methane emissions in the atmosphere caused by rice paddies and livestock populations considerably. The computer simulation of the model confirms this analytical result. The data from model prediction is compared with actual methane data in the atmosphere and found to be very satisfactory.

Keywords: mathematical model, methane, rice paddies, livestock populations, mitigation strategies, stability analysis

1. Introduction

During past several decades, scientists have found that the average temperature of the earth's environment is increasing due to emission of global warming gases such as carbon dioxide (CO_2), methane (CH_4) and nitrous oxide (N_2O). Methane is the most prevalent anthropogenic greenhouse gas after carbon dioxide constituting 16% of the global anthropogenic gas emissions (Cao et al., 1996). Methane, though less abundant than carbon dioxide, its global warming potential is 25-times more than that of carbon dioxide (Zhang et al., 2011). In addition, CH_4 acts as a precursor to tropospheric ozone (O_3) which is another greenhouse gas. Therefore, it is crucial to control the enhanced level of atmospheric methane by using various mitigation options so that global warming can be reduced.

The Rice Paddies are the largest anthropogenic source of methane emissions. It is estimated that the worldwide rice production is responsible for nearly 20% of global anthropogenic methane emission (Schutz et al., 1990; Sass et al., 1994). The emission of methane is widely affected by the method of cultivation, water management, cultivar selection, cropping and fertilization patterns. It is also affected by physical and chemical properties of soil (Tyagi et al., 2010; Khosa et al., 2011). Water is considered as a key factor of methane emission from rice paddies. If soil redox potential can be adjusted by changing the irrigation processes, then the methane emissions can be controlled by water regime. Soil drainage in the early stage of residue incorporation has been found to lower the methane emissions by 45 ~ 74% (Tariq et al., 2017). The methane emission from paddy fields can be reduced significantly by taking into account the appropriate water management system like mid season drainage, non-flooding irrigation techniques. (Singh et al., 2003; Eckard et al., 2010; Shibata and Terada, 2010; Linquist et al., 2012). Several studies have confirmed that emissions of methane depend upon the use of rice cultivars and it can be controlled by using low emitting cultivars (Shalini et al., 1997; Mitra et al., 1999). The methane emission by rice paddies can also be controlled by effective use of fertilizers, proper use of irrigation management techniques (Johnson and Johnson, 1995; Boadi et al., 2004; Lassey, 2008; Patra, 2012).

The farming of livestock populations is also the largest anthropogenic source of methane emissions. It is estimated that nearly 33% of global anthropogenic methane emissions are from livestock populations, such as buffalo, cattle, goat, and sheep. Livestock populations produce significant amount of

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methane directly with enteric fermentation during the digestive process of ruminants and indirectly with manure (excreta) management (Moss et al., 2000; Alemu et al., 2011; Priano et al., 2014). The methane emissions in livestock population is affected by various factors including the health of animal, age, growth rate, physical and chemical characteristics of feeds and environmental temperature. (Johnson and Johnson, 1995; Shibata and Terada, 2010).

It is important to reduce CH_4 emissions from the livestock populations, because methanogenesis corresponds to 2 ~ 12% of dietary energy loss as well as contributing to global warming. Enteric CH_4 emissions represent an economic loss to the farmer where feed is converted to CH_4 rather than to product output. Various CH_4 mitigation options have been studied to control methane emission from livestock populations (Benchaar et al., 2001; Boadi et al., 2004; Patra et al., 2012; Knapp et al., 2014). Supplementation of traditional diets with lipids is one of the most promising mitigation options due to its effectiveness in reducing the emission of CH_4 (Hristov et al., 2013). Decreasing fiber (neutral detergent fiber) proportion, while increasing the amount of crude fat (either extract) in dairy diet reduces enteric CH_4 emissions (Johnson and Johnson, 1995; Jordan et al., 2006; Granger et al., 2008). Further, methane emission from enteric fermentation can be reduced by increasing dry matter intake, increasing the proportion of concentrate in the diet, using legume rather than grass forage, and upgrading the of poor quality forages (Alemu et al., 2011; Jing et al., 2016; Huang and Qin, 2017; Shen et al., 2018).

The methane emissions from rice paddies as well as from livestock populations causing global warming have both direct and indirect economic losses. The direct losses are mostly related to farmers in both the above mentioned cases. For example, in the case of rice paddies lot of water is misused for irrigation causing wasteful expenditure. In the case of livestock populations, methane emissions represent an economic loss to farmers when feed is converted to CH_4 rather than product output. Also indirect losses are many to people including farmers, due to global warming related climate change causing floods, hurricanes droughts having both economic and political consequences.

It is noted here, as mentioned above, that in the emissions of CH_4 by rice paddies and livestock populations, various kinds of processes are involved. To study this problem, a simple approach is needed, where these processes for emissions of CH_4 can be combined together in the form of two separate variables, one dependent on the cumulative biomass density of rice paddies and the other dependent on the cumulative density of livestock populations. This is an innovative idea used in the modeling process.

Thus, in this paper, a non-linear mathematical model is proposed and analysed by considering the following six variables.

1. The cumulative biomass density of rice paddies.

2. The cumulative density of livestock population.

3. The cumulative density of CH_4 formed by various processes involved in the production of rice paddies such as ebullition, transport through rice paddies.

4. The cumulative density of CH_4 formed by various processes in the farming of livestock populations such as enteric fermentation, manure management.

5. The concentration of CH_4 in the atmosphere. The examples are, water irrigation management, method of cultivation, cultivar selection, fertilization method for rice paddies and substitution of traditional diet with lipids, increasing dry matter intake using legume in place of grass for farming of livestock populations.

6. The cumulative density of mitigation options to control CH_4.

In the modeling process we consider six variables as mentioned above to take care of various processes that are involved in the production of rice paddies and farming of livestock populations [see points 3 and 4 above]. We could have used only four variables in the model as listed above in points 1, 2, 5 and 6 but then the corresponding model would have not captured other processes noted above in points 3 and 4.

The main objectives of the paper are the following:

(i) to study the increase of cumulative concentration of methane causing global warming by rice paddies and livestock population.

(ii) to study the effects of cumulative density of various mitigation options in the reduction of methane emissions.

2. Mathematical Model

To model the above mentioned problem, let $B(t)$ be the cumulative density of rice paddies, $C_a(t)$ be the cumulative density of livestock populations, $P_B(t)$ be the cumulative density of CH_4 formed by various processes involved in the production of rice paddies, $P_a(t)$ be the cumulative density of CH_4 formed by various processes in the farming of livestock populations, $C(t)$ be the atmospheric concentration of methane caused by $P_B(t)$, $P_a(t)$ and some natural factors. Let $M(t)$ be the cumulative density of various mitigation options, which is applied to reduce methane emissions in the atmosphere to reduce global warming.

Keeping the above points in view, the problem is proposed to the governed by the following system of non-linear ordinary differential equations (Misra and Verma, 2013, 2017):

$$\frac{dB}{dt} = s\left(B - \frac{B^2}{L}\right) - s_1 B C_a \tag{1}$$

$$\frac{dC_a}{dt} = r_a\left(C_a - \frac{{C_a}^2}{K_a}\right) + r_{a1} B C_a \tag{2}$$

$$\frac{dP_B}{dt} = \alpha B - \alpha_0 P_B \tag{3}$$

$$\frac{dP_a}{dt} = \beta C_a - \beta_0 P_a \tag{4}$$

$$\frac{dC}{dt} = Q_0 + \lambda_B P_B + \lambda_a P_a - \lambda_0 C - \lambda_1 C(M - M_0) \tag{5}$$

$$\frac{dM}{dt} = \varphi(C - C_0) - \varphi_0(M - M_0) \tag{6}$$

where $C_0 = Q_0 / \lambda_0$, $B(0) \geq 0, C_a(0) \geq 0, P_B(0) \geq 0, P_a(0) \geq 0, C(0) \geq 0, M(0) \geq 0$.

In Equation 1, the growth rate of the cumulative biomass density B of rice paddies is assumed to be governed by a logistic equation, where s and L are its intrinsic growth rate and the carrying capacity respectively. The constant s_1 is the depletion rate coefficient of the cumulative biomass density of rice paddies used by livestock populations (i.e., s_1BC_a).

In Equation 2, the growth rate of cumulative density of livestock populations C_a is also assumed to be governed by a logistic equation, where r_a and K_a are its intrinsic growth rate and the carrying capacity respectively. The constant ra_1 is the growth rate coefficient of livestock population density due to use of rice paddies (i.e., $r_{a1}BC_a$).

In Equation 3, the rate of cumulative density of CH_4 formed by various processes involved in the production of rice paddies (i.e., P_B), is assumed to be proportional to the cumulative density of rice paddies, where the constant α is its growth rate coefficient and the constant α_0 is its depletion rate coefficient due to natural factors.

In Equation 4, the rate of cumulative density of CH_4, formed by various processes used in the farming of livestock populations (i.e., P_a), is assumed to be proportional to the cumulitive density of livestock populations, where the constant β is its growth rate coefficient and the constant β_0 is its depletion rate coefficient due to natural factors.

In Equation 5, the emission rate of methane from natural sources is assumed to be a constant Q_0. The constants, λ_B and λ_a are the growth rate coefficient of atmospheric methane due to cumulative density of CH_4 formed by various processes involved in the production of rice paddies and the farming of livestock populations respectively. The constant λ_0 is the natural depletion rate coefficient of atmospheric methane due to natural factors. The constant C_0 is input methane concentration in the atmosphere from natural sources. The constant λ_1 is the depletion rate coefficient of atmospheric methane due to the effectiveness of mitigation options. The constant M_0 is the basic level of mitigation options applied at all time in order to maintain the methane concentration at the level C_0. This implies that $C = C_0$ when $M = M_0$.

In Equation 6, the rate of increased level of mitigation options is assumed to be proportional to the net emissions of methane ($C - C_0$) from of rice paddies and livestock populations. The constant φ is the implementation rate coefficient of the mitigation options and the constant φ_0 is its natural depletion rate coefficient caused by ineffectiveness.

A particular case of the system of Equations 1 ~ 6 can be obtained by assuming that the processes forming methane are instantaneous, i.e., $dPB / dt = 0$ and $dPa / dt = 0$.

In such a case the system of Equations 1 ~ 6 reduces to the following form:

$$\frac{dB}{dt} = s\left(B - \frac{B^2}{L}\right) - s_1 BC_a \tag{7}$$

$$\frac{dC_a}{dt} = r_a\left(C_a - \frac{{C_a}^2}{K_a}\right) + r_{a1}BC_a \tag{8}$$

$$P_B = \frac{\alpha}{\alpha_0}B \tag{9}$$

$$P_a = \frac{\beta}{\beta_0}C_a \tag{10}$$

$$\frac{dC}{dt} = Q_0 + \lambda_B\frac{\alpha}{\alpha_0}B + \lambda_a\frac{\beta}{\beta_0}C_a - \lambda_0 C - \lambda_1 C(M - M_0) \tag{11}$$

$$\frac{dM}{dt} = \varphi(C - C_0) - \varphi_0(M - M_0) \tag{12}$$

It is pointed out here that the six dimensional system of Equations 1 ~ 6 involving six variables capturing various processes involved in the formation of methane as described by Equations 3 and 4 but the four dimensional model involving only four variables described by Equations 7, 8, 11 and 12 does not take care of the various processes involved in the formation of methane from rice paddies and livestock populations [see Equations 3 and 4].

As this model is non-linear, it is to be analyzed by using the stability theory of differential equations (Shukla et al., 2015; Goyal and Shukla, 2018).

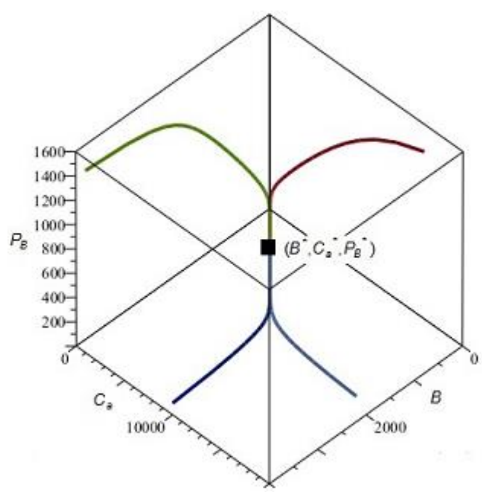

Figure 1. Nonlinear stability in $B - C_a - P_B$ plane.

2.1. Positivity and Boundedness of the Solutions

The positivity and boundedness of the solutions governing the system of Equations 1 ~ 6 is given in the form of following lemma:

The set $\Omega = \{(B, C_a, P_B, P_a, C, M) \in R_+^6 : 0 \le B \le L, 0 \le C_a \le C_{a\,max}, 0 \le P_B \le P_{B\,\max}, 0 \le P_a \le P_{a\,\max}, C_0 \le C \le C_{\max}, M_0 \le M \le M_{\max}\}$, where $C_{a\,\max} = K_a(r_a + r_{a1}L) / r_a$, $P_{B\,\max} = (\alpha / \alpha_0)L$, $P_{a\,\max} = (\beta / \beta_0)C_{a\,max}$, $C_0 = Q_0 / \lambda_0$, $C_{\max} = (Q_0 + \lambda_B P_{B\,\max} + \lambda_1 P_{a\,\max}) / \lambda_0$, and $M_{\max} = M_0 + \varphi / \varphi_0 (C_{\max} - C_0)$, attracts all solutions initiating in the interior of positive octant.

3. Equilibrium Analysis

The system of Equations 1 ~ 6 has the following four equilibria:

1. $E_0(0,\ 0,\ 0,\ 0,\ C_0,\ M_0)$
2. $E_1(L,\ 0,\ \bar{P}_B,\ 0,\ \bar{C},\ \bar{M})$
3. $E_2\ (0,\ K_a,\ 0,\ \tilde{P}_a,\ \tilde{C},\ \tilde{M})$
4. $E^*(B^*,\ C_a^*,\ 0,\ P_B^*,\ C^*,\ M^*)$

The equilibrium $E_0(0, 0, 0, 0, C_0, M_0)$ always exists. This equilibrium implies that when rice paddies and livestock populations are absent and not contributing to the methane emissions in the atmosphere, then the atmospheric methane is at its natural level C_0.

The equilibrium $E_1(L, 0, \bar{P}_B, 0, \bar{C}, \bar{M})$ always exists, where

$$\bar{P}_B = \frac{\alpha}{\alpha_0} L \ ,$$

$$\bar{C} = \frac{\left(Q_0 \frac{\lambda_1 \varphi}{\lambda_0 \varphi_0} - \lambda_0\right) + \sqrt{\left(Q_0 \frac{\lambda_1 \varphi}{\lambda_0 \varphi_0} - \lambda_0\right)^2 + 4\lambda_1 \frac{\varphi}{\varphi_0}\left(Q_0 + \lambda_B \frac{\alpha}{\alpha_0} L\right)}}{2\lambda_1 \frac{\varphi}{\varphi_0}}$$

and

$$\bar{M} = M_0 + \frac{\varphi}{\varphi_0}(\bar{C} - C_0).$$

This equilibrium implies that rice paddies are present but livestock populations are absent. Then in this case, the concentration of methane is more than its level of natural emissions from various processes involved in the production of rice paddies.

The equilibrium $E_2(0, K_a, 0, \tilde{P}_a, \tilde{C}, \tilde{M})$ also always exists, where

$$\tilde{P}_a = \frac{\beta}{\beta_0} K_a \ ,$$

$$\tilde{C} = \frac{\left(Q_0 \frac{\lambda_1 \varphi}{\lambda_0 \varphi_0} - \lambda_0\right) + \sqrt{\left(Q_0 \frac{\lambda_1 \varphi}{\lambda_0 \varphi_0} - \lambda_0\right)^2 + 4\lambda_1 \frac{\varphi}{\varphi_0}\left(Q_0 + \lambda_a \frac{\beta}{\beta_0} K_a\right)}}{2\lambda_1 \frac{\varphi}{\varphi_0}}$$

and

$$\tilde{M} = M_0 + \frac{\varphi}{\varphi_0}(\tilde{C} - C_0).$$

This equilibrium corresponds to the case, when livestock populations are present but rice paddies are absent. In this case, the concentration of methane will be more than its level of natural emissions from various processes in the farming of livestock populations.

3.1. Existence of $E^*(B^*, C_a^*, P_B^*, P_a^*, C^*, M^*)$

The existence and uniqueness of nontrivial equilibrium E^* is carried out as follows.

The variables in $E^*(B^*,\ C_a^*,\ 0,\ P_B^*,\ C^*,\ M^*)$ are given by the following algebraic equations:

$$s\left(B - \frac{B^2}{L}\right) - s_1 B C_a = 0 \tag{13}$$

$$r_a\left(C_a - \frac{{C_a}^2}{K_a}\right) + r_{a1} B C_a = 0 \tag{14}$$

$$\alpha B - \alpha_0 P_B = 0 \tag{15}$$

$$\beta C_a - \beta_0 P_a = 0 \tag{16}$$

$$Q_0 + \lambda_B P_B + \lambda_a P_a - \lambda_0 C - \lambda_1 C(M - M_0) = 0 \tag{17}$$

$$\varphi(C - C_0) - \varphi_0(M - M_0) = 0 \tag{18}$$

From Equations 13 and 14, we get:

$$B = \frac{L r_a (s - s_1 K_a)}{s r_a + s_1 r_{a1} L K_a} = B^* \ , \tag{19}$$

$$C_a = \frac{K_a s (r_a + r_{a1} L)}{s r_a + s_1 r_{a1} L K_a} = {C_a}^* \tag{20}$$

Using Equations 19 and 20 in Equations 15 and 16, we get:

$$P_B = \frac{\alpha}{\alpha_0} B^* \tag{21}$$

$$P_a = \frac{\beta}{\beta_0} {C_a}^* \tag{22}$$

From Equation 18, we get:

$$M - M_0 = \frac{\varphi}{\varphi_0}(C - C_0) \tag{23}$$

Using Equations 21 ~ 23 in Equation 17, we define $F(C)$ as as follows:

$$F(C) = Q_0 + \lambda_B \frac{\alpha}{\alpha_0} B^* + \lambda_a \frac{\beta}{\beta_0} C_a^* - \lambda_0 C - \lambda_1 C \frac{\varphi}{\varphi_0}(C - C_0) = 0 \quad (24)$$

From Equation 24, we note that:

(i) $$F(C_0) = \lambda_B \frac{\alpha}{\alpha_0} B^* + \lambda_a \frac{\beta}{\beta_0} C_a^* > 0, \, as \; B^* > 0, C_a^* > 0 \quad (25)$$

(ii) $$F(C_{\max}) = -\lambda_B \frac{\alpha}{\alpha_0}(L - B^*) - \lambda_a \frac{\beta}{\beta_0}(C_{a\max} - C_a^*) - [(\lambda_1 \varphi \left(Q_0 + \lambda_B L \frac{\alpha}{\alpha_0} + \lambda_a C_{a\max} \frac{\beta}{\beta_0} \right) \left(\lambda_B L \frac{\alpha}{\alpha_0} + \lambda_a C_{a\max} \frac{\beta}{\beta_0} \right)) / \varphi_0 \lambda_0^2] < 0 \quad (26)$$

(iii) $$F'(C) = -\lambda_0 - \lambda_1 \frac{\varphi}{\varphi_0}(2C - C_0) < 0 \quad (27)$$

Thus, $F(C) = 0$ has a unique root (let $C = C^*$) in $C_0 < C \leq C_{\max}$ within the region of attraction Ω.

3.2. Variations of *C* with Different Parameters

Using Equations 19 and 20 in Equation 24, we have:

$$F(C) = Q_0 + \lambda_B \frac{\alpha}{\alpha_0} \left(\frac{L r_a (s - s_1 K_a)}{s r_a + s_1 r_{a1} L K_a} \right) + \lambda_a \frac{\beta}{\beta_0} \left(\frac{(r_a + r_{a1} L) s K_a}{s r_a + s_1 r_{a1} L K_a} \right) - \lambda_0 C - \lambda_1 C \frac{\varphi}{\varphi_0}(C - C_0) = 0 \quad (28)$$

3.3. Variation of *C* with φ

Differentiating Equation 28 with respect to φ, we have:

$$\frac{dC}{d\varphi} = -\frac{\lambda_1 C(C - C_0)}{\lambda_0 \varphi_0 + \lambda_1 \varphi(2C - C_0)} < 0, \, as \; C > C_0 \quad (29)$$

This implies that the atmospheric concentration of methane decreases as the growth rate coefficient of the cumulative density of mitigation strategies φ increases.

3.4. Variation of *C* with λ_1

Differentiating Equation 28 with respect to λ_1, we have:

$$\frac{dC}{d\lambda_1} = -\frac{\varphi C(C - C_0)}{\lambda_0 \varphi_0 + \lambda_1 \varphi(2C - C_0)} < 0, \, as \; C > C_0 \quad (30)$$

This implies that the atmospheric concentration of methane decreases as the depletion rate coefficient λ_1 increases.

4. Stability Analysis

4.1. Local Stability of the Equilibria

The local stability of the equilibria can be investigated by determining the sign of the eigenvalues of Jacobian matrix evaluated at each equilibrium. The Jacobian matrix for the system of Equations 1 ~ 6 is given as follows:

$$J = \begin{bmatrix} s\left(1 - \frac{2B}{L}\right) - s_1 C_a & -s_1 B & 0 & 0 & 0 & 0 \\ r_{a1} C_a & r_a \left(1 - \frac{2C_a}{K_a}\right) + r_{a1} B & 0 & 0 & 0 & 0 \\ \alpha & 0 & -\alpha_0 & 0 & 0 & 0 \\ 0 & \beta & 0 & -\beta_0 & 0 & 0 \\ 0 & 0 & \lambda_B & \lambda_a & -[\lambda_0 + \lambda_1(M - M_0)] & -\lambda_1 C \\ 0 & 0 & 0 & 0 & \varphi & -\varphi_0 \end{bmatrix}$$

Let J_0, J_1 and J_2 be the Jacobian matrices evaluated at equilibria $E_0(0, 0, 0, 0, C_0, M_0)$, $E_1(L, 0, \bar{P}_B, 0, \bar{C}, \bar{M})$ and $E_2(0, K_a, 0, \tilde{P}_a, \tilde{C}, \tilde{M})$ respectively. It can easily be checked that E_0, E_1 and E_2 are unstable.

To check the local stability of $E^*(B^*, C_a^*, P_B^*, P_a^*, C^*, M^*)$, we consider the following positive definite function:

$$V = \frac{1}{2} k_1 B_1^2 + \frac{1}{2} k_2 C_{a1}^2 + \frac{1}{2} k_3 P_{B1}^2 + \frac{1}{2} k_4 P_{a1}^2 + \frac{1}{2} k_5 C_1^2 + \frac{1}{2} k_6 M_1^2 \quad (31)$$

where $B_1 = B - B^*$, $C_{a1} = C_a - C_a^*$, $P_{B1} = P_B - P_B^*$, $P_{a1} = P_a - P_a^*$, $C_1 = C - C^*$ and $M_1 = M - M^*$; k_1, k_2, k_3, k_4, k_5, and k_6 are positive constants chosen suitably.

Differentiating above equation with respect to '*t*', we get:

$$\frac{dV}{dt} = k_1 B_1 \frac{dB_1}{dt} + k_2 C_{a1} \frac{dC_{a1}}{dt} + k_3 P_{B1} \frac{dP_{B1}}{dt} + k_4 P_{a1} \frac{dP_{a1}}{dt} + k_5 C_1 \frac{dC_1}{dt} + k_6 M_1 \frac{dM_1}{dt} \quad (32)$$

Putting the values of the linearized form of derivatives and simplifying, we get:

$$\begin{aligned} \frac{dV}{dt} = & -k_1 \frac{s}{L} B^* B_1^2 - k_2 \frac{r_a}{K_a} C_a^* C_{a1}^2 - k_3 \alpha_0 P_{B1}^2 - k_4 \beta_0 P_{a1}^2 \\ & - k_5 \{\lambda_0 + \lambda_1 (M^* - M_0)\} C_1^2 - k_6 \varphi_0 M_1^2 \\ & + (k_2 r_{a1} C_a^* - k_1 s_1 B^*) B_1 C_{a1} + k_3 \alpha B_1 P_{B1} + k_4 \beta C_{a1} P_{a1} \\ & + k_5 \lambda_B P_{B1} C_1 + k_5 \lambda_a P_{a1} C_1 + (k_6 \varphi - k_5 \lambda_1 C^*) C_1 M_1 \end{aligned} \quad (33)$$

After some algebraic manipulations and by choosing:

$$k_1 = \frac{1}{B^*}, \, k_2 = \frac{s_1}{r_{a1} C_a^*}, \, k_3 < \frac{2}{3}\left(\frac{s\alpha_0}{\alpha^2 L}\right), \, k_4 < \frac{\beta_0 s_1 r_a}{\beta^2 K_a r_{a1}}, \, k_6 = 1$$

$$\frac{\varphi}{\lambda_1 C^*} < \left[\lambda_0 + \lambda_1 (M^* - M_0)\right] \min\left[\frac{2}{3} \frac{s\alpha_0^2}{\lambda_B^2 \alpha^2 L}, \frac{s_1 r_a \beta_0^2}{\lambda_a^2 \beta^2 K_a r_{a1}}\right] \quad (34)$$

The dV/dt will be negative definite and hence E^* is locally asymptotically stable provided the Condition 34 is satisfied. These results are stated in the following theorem.

Theorem 1. The equilibria E_0, E_1 and E_2 are unstable but the equilibrium E^* is locally asymptotically stable, provided the following condition is satisfied in the neighborhood of E^*:

$$\left[\lambda_0+\lambda_1(M^*-M_0)\right]\min\left[\frac{2}{3}\frac{s\alpha_0^{\,2}}{\lambda_B^{\,2}\alpha^2 L}, \frac{s_1 r_a\beta_0^{\,2}}{\lambda_a^{\,2}\beta^2 K_a r_{a1}}\right]-\frac{\varphi}{\lambda_1 C^*}>0 \quad (35)$$

4.2. Global Stability of the Equilibria

To establish nonlinear stability, we consider the following positive definite function:

$$\begin{aligned} U &= m_1\left(B-B^*-B^*\log\frac{B}{B^*}\right)+m_2\left(C_a-C_a^{\,*}-C_a^{\,*}\log\frac{C_a}{C_a^{\,*}}\right) \\ &+\frac{1}{2}m_3(P_B-P_B^{\,*})^2+\frac{1}{2}m_4(P_a-P_a^{\,*})^2 \\ &+\frac{1}{2}m_5(C-C^*)^2+\frac{1}{2}m_6(M-M^*)^2 \end{aligned} \quad (36)$$

Differentiating with respect to 't' we get:

$$\begin{aligned} \frac{dU}{dt} &= \frac{m_1}{B}(B-B^*)\frac{dB}{dt}+\frac{m_2}{C_a}(C_a-C_a^{\,*})\frac{dC_a}{dt} \\ &+m_3(P_B-P_B^{\,*})\frac{dP_B}{dt}+m_4(P_a-P_a^{\,*})\frac{dP_a}{dt} \\ &+m_5(C-C^*)\frac{dC}{dt}+m_6(M-M^*)\frac{dM}{dt} \end{aligned} \quad (37)$$

Putting the values of derivatives from the system Equations 1 ~ 6 and simplifying, we get:

$$\begin{aligned} \frac{dU}{dt} &= -m_1\frac{s}{L}(B-B^*)^2-m_2\frac{r_a}{K_a}(C_a-C_a^{\,*})^2-m_3\alpha_0(P_B-P_B^{\,*})^2 \\ &-m_4\beta_0(P_a-P_a^{\,*})^2-m_5\{\lambda_0+\lambda_1(M-M_0)\}(C-C^*)^2 \\ &-m_6\varphi_0(M-M^*)^2+(m_2 r_{a1}-m_1 s_1)(B-B^*)(C_a-C_a^{\,*}) \\ &+m_3\alpha(B-B^*)(P_B-P_B^{\,*})+m_4\beta(C_a-C_a^{\,*})(P_a-P_a^{\,*}) \\ &+m_5\lambda_B(P_B-P_B^{\,*})(C-C^*)+m_5\lambda_a(P_a-P_a^{\,*})(C-C^*) \\ &+(m_6\varphi-m_5\lambda_1 C^*)(C-C^*)(M-M^*) \end{aligned} \quad (38)$$

After some algebraic manipulations and by choosing:

$$\begin{aligned} & m_1=1,\ m_2=\frac{s_1}{r_{a1}},\ m_3<\frac{2}{3}\left(\frac{s\alpha_0}{\alpha^2 L}\right),\ m_4<\frac{\beta_0 s_1 r_a}{\beta^2 K_a r_{a1}},\ m_6=1 \\ & \frac{\varphi}{\lambda_1 C^*}<\lambda_0\min\left[\frac{2}{3}\frac{s\alpha_0^{\,2}}{\lambda_B^{\,2}\alpha^2 L}, \frac{s_1 r_a\beta_0^{\,2}}{\lambda_a^{\,2}\beta^2 K_a r_{a1}}\right] \end{aligned} \quad (39)$$

The dV/dt is negative definite and hence E^* is globally asymptotically stable, provided the Condition 39 is satisfied inside the region of attraction Ω. This result is stated in the following theorem.

Theorem 2. The equilibrium E^* is globally asymptotically stable, provided the following condition is satisfied inside the region of attraction Ω:

$$S=\lambda_0\min\left[\frac{2}{3}\frac{s\alpha_0^{\,2}}{\lambda_B^{\,2}\alpha^2 L}, \frac{s_1 r_a\beta_0^{\,2}}{\lambda_a^{\,2}\beta^2 K_a r_{a1}}\right]-\frac{\varphi}{\lambda_1 C^*}>0 \quad (40)$$

It is noted that the Condition 40 is stronger than Condition 35 as expected.

5. Numerical Simulation

In this section, numerical simulation is performed to check the feasibility of analytical results for the system of Equations 1 ~ 6 by taking into account the following set of parameter values, some have been taken from (Alemu et al., 2011 and references therein; Liu et al., 2017) [See Table 1].

The equilibrium values corresponding to E^* are as follows:

$B^*=1991.4115,\ C_a^{\,*}=10019.9141,\ P_B^{\,*}=796.5646,$
$P_a^{\,*}=1669.9856,\ C^*=2315.2266,\ M^*=281.5226.$

The eigenvalues of the Jacobian matrix J corresponding to equilibrium point E^* are –0.696978, –0.501010, –0.093375, –0.5, –0.0127768 and –0.06. Since all the eigenvalues are negative and hence equilibrium E^* is locally asymptotically stable. For the given parameter values, the nonlinear stability conditions, corresponding to E^*, are also satisfied.

To present nonlinear stability of E^* for the system of Equations 1 ~ 6, trajectories with different initial starts have been plotted in $B-C_a-P_B$ plane as shown in Figure 1. It is apparent from this figure that all trajectories approach the equilibrium E^* showing that the equilibrium E^* is nonlinearly stable.

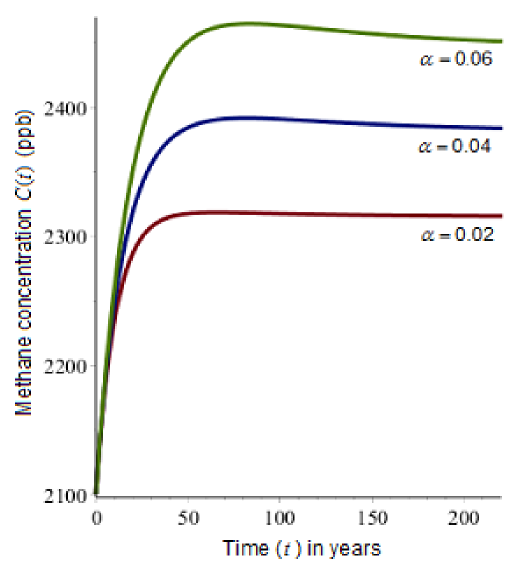

Figure 2. Variation of methane concentration $C(t)$ with time t for different values of the growth rate coefficient of P_B (i.e., α).

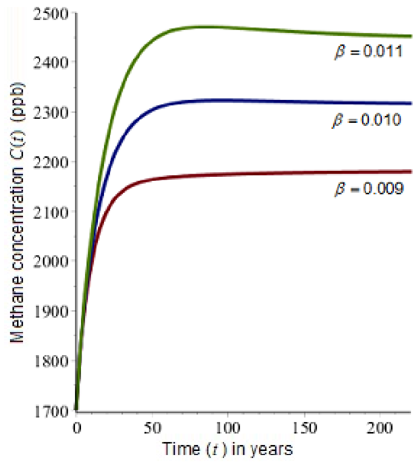

Figure 3. Variation of methane concentration $C(t)$ with time t for different values of the growth rate coefficient of P_a (i.e., β).

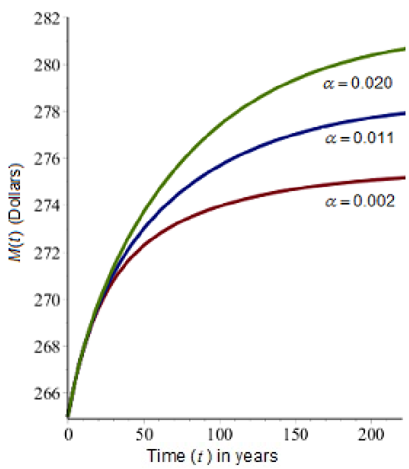

Figure 4. Variation of the cumulative density of various mitigation options $M(t)$ with time t for different values of the growth rate coefficient of P_B (i.e., α).

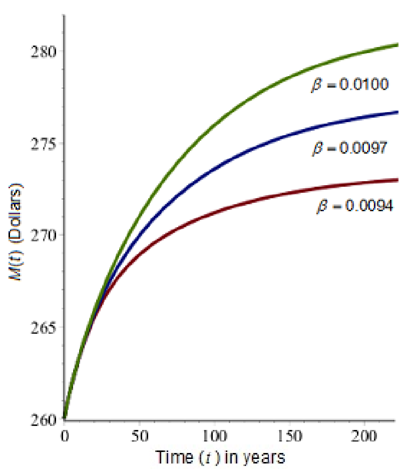

Figure 5. Variation of measure of mitigation options $M(t)$ with time t for different values of the growth rate coefficient of P_a (i.e., β).

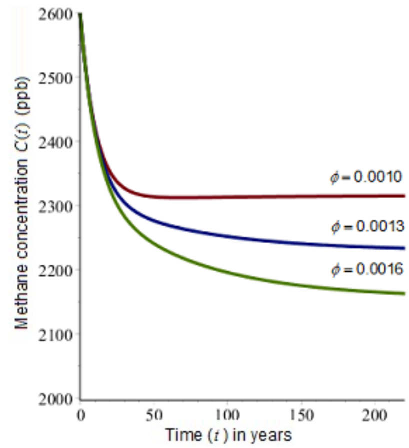

Figure 6. Variation of methane concentration $C(t)$ with time t for different values of the implementation rate coefficient of the various mitigation strategies (i.e., ϕ).

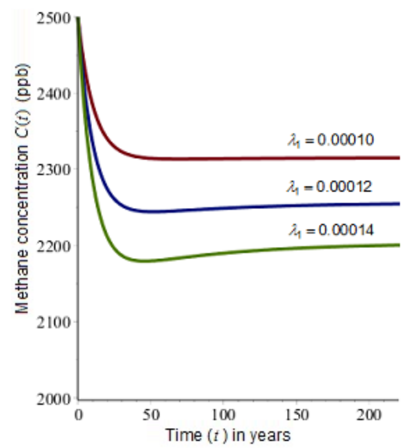

Figure 7. Variation of methane concentration $C(t)$ with time t for different values of depletion rate coefficient of atmospheric methane (i.e., λ_1).

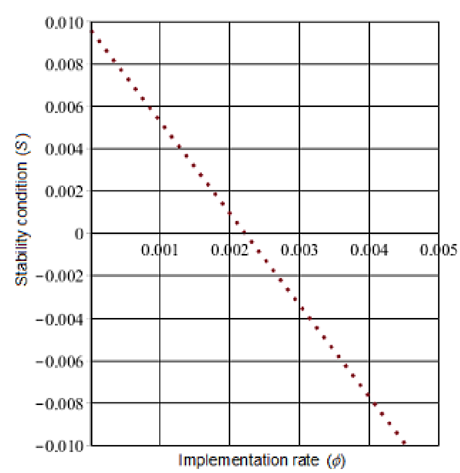

Figure 8. Variation of stability condition S with the implementation rate coefficient of the various mitigation strategies (i.e., ϕ).

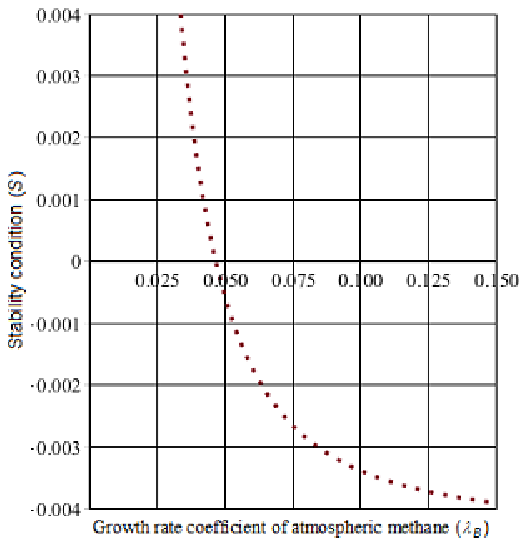

Figure 9. Variation of stability condition S with the emission rate coefficient of methane from various processes involved in the production of rice paddies (i.e., λ_B).

To visualize the variations of various variables with time for different values of parameters, these variables are plotted with time as shown in Figures 2 ~ 9. In Figures 2 and 3, the variations of the concentration (C) of methane with time 't' are shown for different values of growth rate coefficient of the cumulative density of CH_4 formed by various processes involved in the production of rice paddies (i.e., α = 0.02, 0.04, 0.06) and the growth rate coefficient of the cumulative density of CH_4 formed by various processes in the farming of livestock populations (i.e., β = 0.009, 0.010, 0.011) respectively. From these figures, it is seen that the concentration of methane increases as the growth rate of the cumulative density of CH_4 formed by various processes involved in the production of rice paddies (α) or the growth rate of the cumulative density of CH_4 formed by various processes in the farming of livestock populations (β) increases. In Figures 4 and 5, the variation of the cumulative density of various mitigation options (M) with time 't' are shown for different values of growth rate of the cumulative density of CH_4 formed by various processes involved in the production of rice paddies (i.e., α = 0.02, 0.011, 0.020) or the growth rate coefficient of the cumulative density of CH_4 formed by various processes in the farming of livestock populations (i.e., β = 0.0094, 0.0097, 0.0100) respectively. From these figures, it is seen that the cumulative density of mitigation options increases as the growth rate of the cumulative density of CH_4 formed by various processes involved in the production of rice paddies (α) or the growth rate of the cumulative density of CH_4 formed by various processes in the farming of livestock populations (β) increases.

In Figure 6, the variation of the concentration (C) of methane with time 't' is shown for different values of the implementation rate coefficient of the various mitigation strategies (i.e., φ = 0.0010, 0.0013, 0.0016) respectively. From this figure, it is observed that the concentration of methane decreases as the implementation rate coefficient of the various mitigation strategies (φ) increases (also see Table 2). In Figure 7, the variation of the concentration (C) of methane with time 't' is shown for different values of depletion rate coefficient of atmospheric methane due to net effectiveness mitigation (i.e., λ_1 = 0.00010, 0.00012, 0.00014) respectively. From this figure, it can be noted that the concentration of methane decreases as the depletion rate coefficient of atmospheric methane due to net effectiveness mitigation (λ_1) increases (also see Table 3).

The stability condition (40) is also plotted with respect to parameters φ, λ_B in Figures 8 and 9 respectively. It is apparent from Figure 8 that S remains positive for φ < 0.002215, it becomes zero at φ = 0.002215 and negative for φ > 0.002215.

This implies that the stability condition is satisfied for 0 ≤ φ < 0.002215 and for higher values of φ it will not be satisfied. Hence, φ has destabilizing effect on the model system. Likewise, from Figure 9, we note that λ_R has destabilizing effect on the model system.

The stability condition (40) is also plotted with respect to parameters φ, λ_B in Figures 8 and 9 respectively. It is apparent from Figure 8 that S remains positive for φ < 0.002215, it becomes zero at φ = 0.002215 and negative for φ > 0.002215. This implies that the stability condition is satisfied for 0 ≤ φ < 0.002215 and for higher values of φ it will not be satisfied. Hence, φ has destabilizing effect on the model system. Likewise, from Figure 9, we note that λ_R has destabilizing effect on the model system.

Table 1. Parameter Values of the System of Equations 1 ~ 6

Parameter	Value
Q_0	56
λ_0	0.08
s	0.7
s_1	3×10^{-7}
L	2000
r_a	0.5
r_{a1}	5×10^{-7}
K_a	10000
α	0.02
α_0	0.05
β	0.01
β_0	0.06
λ_1	0.0001
λ_B	0.01
λ_a	0.095
ϕ	0.001
ϕ_0	0.01
C_0	700
M_0	120

The concentration of methane in the atmosphere with model data is compared with actual data taken from European Environmental Agency (EEA, 2018) as shown in Figure 10. It is noted from the figure that the trend of increase of the atmospheric concentration of methane is quite similar from model prediction as well as from actual data.

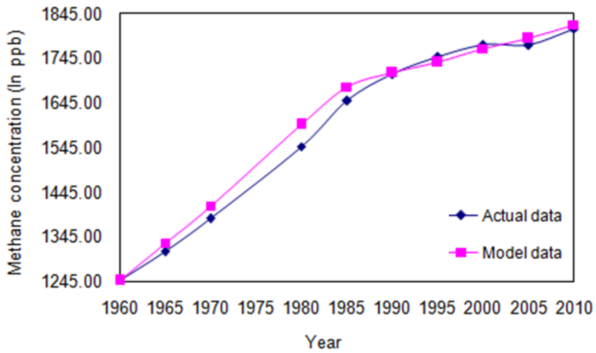

Figure 10. Comparison between actual data and model data of atmospheric methane.

Table 2. Variation of Methane Concentration (C) for Different Values of ϕ

ϕ	C
0.0010	2315.2266
0.0013	2228.9103
0.0016	2155.3330
0.0019	2091.4874
0.0022	2035.3005

Table 3. Variation of Methane Concentration (C) for Different Values of λ_1

ϕ	C
0.0010	2315.2266
0.0011	2284.7985
0.0012	2256.0815
0.0013	2228.9103
0.0014	2203.1422

6. Conclusions

In this paper, a non-linear model has been proposed and analyzed to control the concentration of methane emissions caused by rice paddies and livestock populations using various mitigation options. The non-linear mathematical model under consideration is assumed to be governed by a system of ordinary differential equations with six non-linearly dependent variables, namely, the cumulative density of rice paddies, the cumulative density of livestock populations, the cumulative density of CH_4 formed by various processes involved in the production of rice paddies such as ebullition, transport through rice paddies, the cumulative density of CH_4 formed by various processes in the farming of livestock populations such as enteric fermentation, manure management, the atmospheric concentration of methane, and the cumulative density of various mitigation options. The example for mitigation options are cultivar selection, method of cultivation, fertilizer patters, water drainage for rice paddies and substitution of traditional diet by legume, grass by lipids, etc. for livestock population. The model has been analyzed by using stability theory of ordinary differential equations. The local and the global stability conditions have been established [see Conditions 35 and 40]. The model analysis has shown that as the cumulative density of mitigation options increases, the concentration of CH_4 decreases in the atmosphere (see Figures 6 and 7). Further, the model data for methane concentration is compared with actual data, taken from European Environmental Agency (EEA, 2018), which shows that the model prediction is very satisfactory when it is compared with actual data (Figure 10).

It is concluded here that the present paper provides a basic framework to study the impact of cumulative density of mitigation options to control methane emissions from various processes for production of rice paddies and farming of livestock populations and more research is required in this area.

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