

## Mixed Integer Linear Programming for Oil Sands Production Planning and Tailings Management

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**ABSTRACT.** In oil sands open-pit mining, further processing of the extracted oil sands generates massive volumes of tailings. To save space, the tailings are deposited in in-pit tailings containments constructed by internal dykes using mine waste material. In this paper, an integrated mine planning framework is proposed and implemented using mixed-integer linear programming to optimize the production schedule with respect to dyke construction and in-pit tailings deposition. A case study is carried out to verify the performance of the proposed optimization model. The results demonstrate how the produced tailings are deposited in the excavated mining pits as the mining operation proceeds and the in-pit dykes are constructed using mine waste material. The framework facilitates sustainable oil sands mining through a reduced environmental footprint.

**Keywords:** dyke construction, mine planning, mixed integer linear programming, oil sands mining, production planning, tailings management optimization

### 1. Introduction

An oil sands deposit is a mixture of bitumen and water in sands and clay. Oil sands mining comprises of the removal of overburden material and the mining of the oil-bearing McMurray formation. It is one of the fastest growing industries in North America. Though in recent times oil prices are relatively low, there have been considerable investments in the past that can keep this industry vibrant for some decades. It is also more relevant now that further research aimed at improving the profitability and sustainability of these operations in the long-term is pursued through robust mine planning.

An efficient mine plan determines the best schedule for extraction and the destination of the extracted material, in a way that maximizes the net present value of the mining project. In oil sands operations, the material mined is sent to the processing plant for extraction of bitumen through hot water extraction process, which produces tailings. About 80% of the material sent to the processing plant ends up in the tailings dam. These tailings facilities require large amounts of waste materials for their construction.

Oil sands mining operations generate considerable volumes of solid waste mostly as overburden and interburden (OI)

to access the mineralized zone. The current practice is to dump the waste material for later use mainly in dyke construction and reclamation. The dykes may be constructed either in-pit or expit depending on the waste management strategy in place at the time. The main source of the required material for dyke construction is OI material coming from the mining operations, and the tailings coarse sand (TCS) coming from the processing plant (Fauquier et al., 2009; Ben-Awuah, 2013). Ben-Awuah et al. (2012) provide a detailed description of an integrated oil sands mining operation including material flows and solid waste management. Hence, waste disposal, reclamation planning and dyke construction planning can be well integrated with the mine planning framework. In the literature, few works have addressed such integration, but none of them has covered the mentioned domains completely (Ben-Awuah and Askari-Nasab, 2011; Ben-Awuah et al., 2012; Badiozamani and Askari-Nasab, 2012a; Badiozamani and Askari-Nasab, 2012b; Ben-Awuah, 2013; Badiozamani and Askari-Nasab, 2013a; Ben-Awuah et al., 2015).

Mathematical programming models such as goal programming and mixed integer programming have frequently been used for solving industrial and municipal waste management problems (Ahluwalia and Nema, 2007; Xu et al., 2014). Since the 1960s, operations research techniques in the form of linear programming, integer programming, mixed-integer linear programming (Johnson, 1969) and dynamic programming (Tan and Ramani, 1992) have been used to find the optimized pattern of extraction and to determine a destination for the extracted material in open-pit mining and block caving (Newman et al.,

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2010). The most common way to control the precedence order of extraction for mining blocks is to define integer variables, which makes the mine planning problem a non-deterministic polynomial-time hard for large-scale problems (Gleixner, 2008). Due to the large number of integer variables corresponding to mining blocks over large number of periods, it takes considerably a long time for the current solvers to generate results.

Mixed-integer linear programming (MILP) is a powerful tool extensively used in the literature for mine planning optimization. Typical mine planning models maximize the NPV over the mine-life, with respect to the mining and processing capacities, ore blending constraints, and spatial precedence among mining blocks (Johnson, 1969; Askari-Nasab and Awuah-Offei, 2009; Askari-Nasab et al., 2011). Further than the pure long-term mine planning models, few works are published addressing the linkage between mine planning and tailings production (Kalantari et al., 2013). Solid waste disposal management and dyke construction planning in oil sands are also integrated into the long-term mine planning framework by Ben-Awuah (2013) (Ben-Awuah and Askari-Nasab, 2011; Ben-Awuah et al., 2012; Ben-Awuah, 2013).

Badiozamani and Askari-Nasab (2014) proposed an integrated model for long-term mine planning, with respect to reclamation material handling and tailings capacity constraints. The concept of directional mining is used in modeling to provide capacity for in-pit tailings facility. The model determines the destination for each extracted parcel (dynamic cut-off grade) in such a way to maximize the NPV over the mine-life. Mining aggregates are used in the model to follow the selective mining units. The authors reach an integer solution within 2% optimality gap in less than 10 minutes for cases with more than 98,000 mining-blocks aggregated to 535 mining-cuts. The optimality gap refers to the absolute tolerance on the gap between the best integer objective and the objective of the best node remaining in the branch and cut algorithm. The resulting schedule generates the maximum NPV, minimizes the material handling cost of reclamation, and the tailings volume produced downstream meets the tailings capacity constraints in each period. The authors take a further step in integrated mine planning, by including tailings management in terms of composite tailings (CT) production and deposition in the mine planning optimization framework (Badiozamani and Askari-Nasab, 2013b; Badiozamani, 2014; Badiozamani and Askari-Nasab, 2016). This integrated framework serves as the starting point for the current research.

The gap in current literature is the integration of all these areas: maximization of profit in pure mine planning, minimization of dyke construction costs and minimization of tailings disposal costs. The main contribution of this work includes the direct scheduling and precedence of dyke construction. The proposed MILP model maximizes the net present value (NPV) and at the same time minimizes the costs of dyke construction and CT deposition. The optimization is subject to a number of constraints, including the mining, processing, tailings storage capacities and extraction precedence constraints. This integrated model will reduce the rehandling cost of dyke construction and reclamation material. The model schedules these ma-

terial types when they are needed both in quality and quantity directly to the appropriate destination.

There are different aspects involved in long-term mine planning for oil sands, such as tailings management, reclamation planning, solid waste management and dyke construction planning. The authors have investigated some of the environmental management issues that are triggered by oil sands mining operations and mineral processing and have included them in decision making. A common approach to address the environmental impacts is to audit the mining sites in accordance with environmental codes and regulations, investigate violations from environmental regulations and report the violations to stakeholders (Badiozamani, 2014). Audit reports will raise the awareness about environmental impacts and are essential in sustainable mining practice. Such reports are either mandatory, i.e. required by governments or regional authorities, or voluntary in which the company aims to show its differences from others in the market in terms of environmentally clean practices. However, in order to consider the environmental impacts practically in mine planning and mine design, review of reports will not be sufficient. The better strategy is to consider the environmental impacts as part of mine design and mine planning. That means to take into account the environmental costs in designing the final mine pit-limits and consider mine site reclamation in the mine planning phase (Badiozamani and Askari-Nasab, 2014b). Among the environmental impacts of oil sands production, two issues seem to be the most important ones: (1) tailings slurry and its dewatering, and (2) the remaining footprint from mining operations and tailings ponds. In this research, the focus is on addressing these two issues in terms of integrating them with the long-term mine planning framework. The capacity of tailings facility and the deposition of produced CT are considered explicitly in the long-term mine planning model.

The next section of this paper presents a conceptual model of the integrated mine planning framework as well as a MILP model for the integrated mine planning optimization. Section 3 covers a case study that highlights the strategy used to incorporate dyke construction and composite tailings deposition scheduling into oil sands production planning. The paper concludes in section 4.

## 2. Data and Methods

### 2.1. The Integrated Mine Planning Framework: Coupling CT Deposition and Dyke Construction Scheduling

In the proposed model, it is assumed that the produced CT will be deposited mainly in a number of in-pit CT cells shaped by internal dykes and pit walls. The external tailings facility (ETF) acts as a buffer and since it has a limited capacity, the in-pit CT cells must be prepared in time for CT storage to start. The internal dykes ensure that both mining and tailings deposition can occur simultaneously in the pit during the mine life. In order to meet such a requirement, the OI and TCS material must be produced and used for the construction of in-pit dykes.

Before raising the internal dyke walls, the first step is to choose the dykes' footprints. To guarantee a feasible schedule, dyke footprints are selected from among pushback footprints.

This selection is made based on the volume of material in the pushbacks and the potential volume of CT to be produced from processing the extracted material. Since the pushbacks are extracted following a precedence order, no material will be left behind before constructing a dyke after the dyke's footprint has been cleared. Figure 1 illustrates a plan view of the dykes' footprints and the schematic ETF used in a conceptual mine design. The in-pit colors represent the mining panels used to control material extraction on a level. Figure 2 shows a typical dyke construction schedule for Dyke A and CT deposition schedule in Cell C1 after initial deposition in the ETF. The highlighted squares with values of null/one demonstrate that whereas no dyke construction is required for CT deposition in the ETF, Dyke A is required for CT deposition in cell C1 (Figure 2). These waste management operations must be incorporated into the mine planning framework to ensure an optimal integrated global sustainable mine plan.

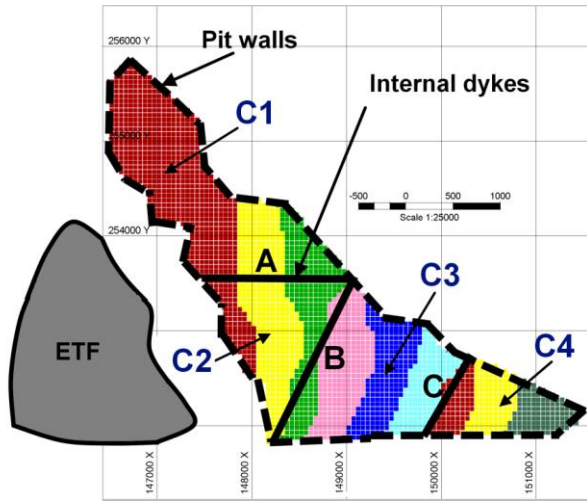


Figure 1. Dykes' footprints (A to C), CT cells (C1 to C4) and the ETF for a conceptual mine design.

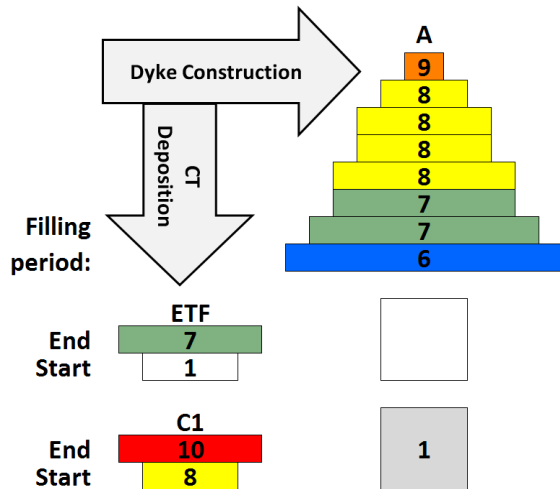


Figure 2. Construction of Dyke A and CT deposition in the ETF and cell C1.

## 2.2. The Mathematical Model

The proposed mathematical model includes both tailings management in terms of CT deposition, and waste management in terms of dyke construction planning. The objective function includes three parts as: (1) maximization of NPV, (2) minimization of Dyke construction costs, and (3) minimization of CT deposition costs. Mining-panels (intersections of bench faces and pushbacks) are used as the units for mining operations, while mining-cuts (aggregated blocks) are used for processing. The definitions of notations used in formulating the mathematical model can be found in the Appendix section. The detailed structure of the MILP model is as follows:

### 2.2.1. Objective Function

The objective function maximizes the net present value of the profit gained from processing of each mining-panel. The revenue from each mining-panel consists of two terms: the revenue from selling each tonne of bitumen and a summation of operational costs. The operational costs include the material extraction costs, the extra costs for mining ore material, and the cost of selling the ore. The other two operational costs are the extra costs of mining and preparing material for dyke construction, and the cost of CT deposition in the CT cells. The economic value of mining-panels is calculated through Equations (1) to (5):

$$d_p^{a,u,t} = \sum_{k \in P_a} (r_k^{u,t} - n_k^{u,t} - m_k^{u,t}) - q_p^{a,t} \quad (1)$$

$$\forall t \in \mathbf{T}, u \in \mathbf{U}, p \in \mathbf{P}, a \in \mathbf{A}$$

where,

$$r_k^{u,t} = \sum_{e=1}^E o_k \times g_k^e \times r^{u,e} \times (p^{e,t} - cs^{e,t}) - \sum_{e=1}^E o_k \times cp^{u,e,t} \quad (2)$$

$$\forall t \in \mathbf{T}, u \in \mathbf{U}, k \in \mathbf{K}$$

$$q_p^{a,t} = \sum_{k \in p} (o_k + d_k + w_k) \times cm^{a,t}, \forall t \in \mathbf{T}, p \in \mathbf{P}, a \in \mathbf{A} \quad (3)$$

$$n_k^{u,t} = d_k \times cl^{u,t}, \forall t \in \mathbf{T}, u \in \mathbf{U}, k \in \mathbf{K} \quad (4)$$

$$m_k^{u,t} = l_k \times cu^{u,t}, \forall t \in \mathbf{T}, u \in \mathbf{U}, k \in \mathbf{K} \quad (5)$$

And the cost of CT deposition is calculated as in Equation (6):

$$i_c^t = h_c \times ct^{c,t}, \forall t \in \mathbf{T}, c \in \mathbf{C} \quad (6)$$

The objective function is defined as in Equation (7):

$$\begin{aligned} \text{Max} [ & \sum_{t=1}^T \sum_{u=1}^U \sum_{a=1}^A \sum_{j=1}^J \sum_{\substack{p \in B_j \\ k \in B_p}} (r_k^{u,t} \times x_k^{u,t} - q_p^{a,t} \times y_p^{a,t}) \\ & - \sum_{t=1}^T \sum_{u=1}^U \sum_{a=1}^A \sum_{j=1}^J \sum_{\substack{p \in B_j \\ k \in B_p}} (n_k^{u,t} \times w_k^{u,t} + m_k^{u,t} \times v_k^{u,t}) - \sum_{t=1}^T \sum_{c=1}^C i_c^t \times z_c^t ] \end{aligned} \quad (7)$$

2.2.2. Constraints

The objective function is subject to the constraints stated by Equations (8) to (45). Equations (8) and (9) present the mining and processing capacity constraints. These equations ensure that the total material mined, including ore, waste and dyke material; and the total ore sent to the processing plant do not exceed the specified operational capacities in each period. Equations (10) and (11) ensure that the material sent for dyke construction (OI and TCS) are within the range of minimum and maximum requirements. These targets enable the mine planner to have good control over dyke material and provide a robust platform for effective dyke construction planning and tailings storage management. With these controls, movement of dyke material and dyke construction scheduling can be well integrated with the mining fleet management plan. Equations (12) to (14) are mass-balance constraints that control the proportions of material tonnages extracted and used for different purposes. Equation (12) ensures that the total mined material in each period for all destinations does not exceed the summation of the ore and OI material mined. As the TCS is part of the extracted ore material, Equation (13) ensures that the total tonnage of tailings sand cannot exceed the total ore tonnage. Equation (14) makes the amount of TCS and OI dyke material available for constructing all sections of the dykes. The blending constraints for ore and OI material are presented in Equations (15) to (17). These equations monitor the quality of ore and OI dyke material in terms of bitumen and fines. Equations (18) to (21) add up the total tonnage of different components of tailings and ensure that the tonnages are not exceeding the corresponding capacity ranges. This includes tailings fines, sand and water. Equations (22) to (24) ensure that the total CT produced and deposited in CT cells does not exceed the capacity of CT containments available in each period. Mining precedence constraints are presented in Equations (25) to (31). These equations control the vertical and horizontal block precedence relations. They ensure that mining proceeds in the specified mining direction as the mine goes deeper. The precedence order of CT cells construction and CT deposition is controlled through Equations (32) to (39). Finally, Equations (40) to (45) ensure that the summation of decision variables adds up to one.

$$T_{Ml}^{a,t} \leq \sum_{j=1}^J \left( \sum_{p \in B_j} \sum_{k \in B_p} (o_k + w_k + d_k) \times y_p^{a,t} \right) \leq T_{Mu}^{a,t}, \quad (8)$$

$$\forall t \in \mathbf{T}, \forall a \in \mathbf{A}$$

$$T_{Pl}^{u,t} \leq \sum_{j=1}^J \left( \sum_{k \in B_j} (o_k \times x_k^{u,t}) \right) \leq T_{Pu}^{u,t}, \quad \forall t \in \mathbf{T}, u \in \mathbf{U} \quad (9)$$

$$T_{Cl}^{u,t} \leq \sum_{j=1}^J \left( \sum_{k \in B_j} (d_k \times w_k^{u,t}) \right) \leq T_{Cu}^{u,t}, \quad \forall t \in \mathbf{T}, u \in \mathbf{U} \quad (10)$$

$$T_{Nl}^{u,t} \leq \sum_{j=1}^J \left( \sum_{k \in B_j} (l_k \times v_k^{u,t}) \right) \leq T_{Nu}^{u,t}, \quad \forall t \in \mathbf{T}, u \in \mathbf{U} \quad (11)$$

$$\sum_{u=1}^U \sum_{k \in B_p} (o_k \times x_k^{u,t} + d_k \times w_k^{u,t}) \leq \sum_{a=1}^A \sum_{k \in B_p} (o_k + d_k) \times y_p^{a,t}, \quad (12)$$

$$\forall t \in \mathbf{T}, p \in \mathbf{P}$$

$$\sum_{u=1}^U (l_k \times v_k^{u,t}) \leq \sum_{u=1}^U (o_k \times x_k^{u,t}), \quad \forall t \in \mathbf{T}, k \in \mathbf{K} \quad (13)$$

$$\sum_{d=1}^D (k_d \times u_d^t) \leq \sum_{u=1}^U \sum_{j=1}^J \left( \sum_{k \in B_j} (d_k \times w_k^{u,t} + l_k \times v_k^{u,t}) \right), \quad (14)$$

$$\forall t \in \mathbf{T}, u \in \mathbf{U}$$

$$\underline{g}^{u,t,e} \leq \sum_{j=1}^J \left( \sum_{k \in B_j} g_k^e \times o_k \times x_k^{u,t} / \sum_{k \in B_j} o_k \times x_k^{u,t} \right) \leq \bar{g}^{u,t,e}, \quad (15)$$

$$\forall t \in \mathbf{T}, u \in \mathbf{U}, e \in \mathbf{E}$$

$$\underline{f}^{u,t,o} \leq \sum_{j=1}^J \left( \sum_{k \in B_j} f_k^o \times o_k \times x_k^{u,t} / \sum_{k \in B_j} o_k \times x_k^{u,t} \right) \leq \bar{f}^{u,t,o}, \quad (16)$$

$$\forall t \in \mathbf{T}, u \in \mathbf{U}$$

$$\underline{f}^{u,t,c} \leq \sum_{j=1}^J \left( \sum_{k \in B_j} f_k^c \times d_k \times w_k^{u,t} / \sum_{k \in B_j} d_k \times w_k^{u,t} \right) \leq \bar{f}^{u,t,c}, \quad (17)$$

$$\forall t \in \mathbf{T}, u \in \mathbf{U}$$

$$T_{Tl}^{u,t} \leq \sum_{j=1}^J \left( \sum_{k \in B_j} (t_k \times x_k^{u,t}) \right) \leq T_{Tu}^{u,t}, \quad \forall t \in \mathbf{T}, u \in \mathbf{U} \quad (18)$$

$$T_{Fl}^{u,t} \leq \sum_{j=1}^J \left( \sum_{k \in B_j} (f_k \times x_k^{u,t}) \right) \leq T_{Fu}^{u,t}, \quad \forall t \in \mathbf{T}, u \in \mathbf{U} \quad (19)$$

$$T_{Sl}^{u,t} \leq \sum_{j=1}^J \left( \sum_{k \in B_j} (s_k \times x_k^{u,t}) \right) \leq T_{Su}^{u,t}, \quad \forall t \in \mathbf{T}, u \in \mathbf{U} \quad (20)$$

$$T_{Wl}^{u,t} \leq \sum_{j=1}^J \left( \sum_{k \in B_j} (r_k \times x_k^{u,t}) \right) \leq T_{Wu}^{u,t}, \quad \forall t \in \mathbf{T}, u \in \mathbf{U} \quad (21)$$

$$T_{Xl}^{u,t} \leq \sum_{j=1}^J \left( \sum_{k \in B_j} (h_k \times x_k^{u,t}) \right) \leq T_{Xu}^{u,t}, \quad \forall t \in \mathbf{T}, u \in \mathbf{U} \quad (22)$$

$$T_{Yl}^{u,t} \leq \sum_{j=1}^J \left( \sum_{k \in B_j} (p_k \times x_k^{u,t}) \right) \leq T_{Yu}^{u,t}, \quad \forall t \in \mathbf{T}, u \in \mathbf{U} \quad (23)$$

$$\sum_{c=1}^C (h_c \times z_c^t) \leq \sum_{j=1}^J \sum_{k \in B_j} \sum_{u=1}^U (p_k \times x_k^{u,t}), \quad \forall t \in \mathbf{T} \quad (24)$$

$$b_p^t - \sum_{a=1}^A \sum_{i=1}^t y_s^{a,i} \leq 0, \quad \forall t \in \mathbf{T}, p \in \mathbf{P}, s \in N_p(L) \quad (25)$$

$$b_p^t - \sum_{a=1}^A \sum_{i=1}^t y_r^{a,i} \leq 0, \forall t \in \mathbf{T}, p \in \mathbf{P}, r \in O_p(L) \quad (26)$$

$$\sum_{a=1}^A \sum_{i=1}^t y_p^{a,i} - b_p^t \leq 0, \forall t \in \mathbf{T}, p \in \mathbf{P} \quad (27)$$

$$b_p^t - b_p^{t+1} \leq 0, \forall t \in \{1, \dots, T-1\}, p \in \mathbf{P} \quad (28)$$

$$H \times c_j^t - \sum_{a=1}^A \sum_{i=1}^t y_h^{a,i} \leq 0, \forall t \in \mathbf{T}, j \in \mathbf{J}, h \in B_j(H) \quad (29)$$

$$\sum_{a=1}^A \sum_{i=1}^t y_h^{a,i} - H \times c_j^t \leq 0, \forall t \in \mathbf{T}, j \in \mathbf{J}, h \in B_{j+1}(H) \quad (30)$$

$$c_j^t - c_j^{t+1} \leq 0, \forall t \in \{1, \dots, T-1\}, j \in \mathbf{J} \quad (31)$$

$$a_c^t - \sum_{i=1}^t z_r^i \leq 0, \forall t \in \mathbf{T}, c \in \mathbf{C}, r \in Q_c(R) \quad (32)$$

$$\sum_{i=1}^t z_c^i - a_c^t \leq 0, \forall t \in \mathbf{T}, c \in \mathbf{C} \quad (33)$$

$$a_c^t - a_c^{t+1} \leq 0, \forall t \in \{1, \dots, T-1\}, c \in \mathbf{C} \quad (34)$$

$$q_d^t - \sum_{i=1}^t u_m^i \leq 0, \forall t \in \mathbf{T}, d \in \mathbf{D}, m \in S_d(G) \quad (35)$$

$$\sum_{i=1}^t u_d^i - q_d^t \leq 0, \forall t \in \mathbf{T}, d \in \mathbf{D} \quad (36)$$

$$q_d^t - q_d^{t+1} \leq 0, \forall t \in \{1, \dots, T-1\}, d \in \mathbf{D} \quad (37)$$

$$q_d^t - \sum_{a=1}^A \sum_{i=1}^t y_f^{a,i} \leq 0, \forall t \in \mathbf{T}, d \in \mathbf{D}, f \in X_d(P) \quad (38)$$

$$a_c^t - \sum_{i=1}^t u_n^i \leq 0, \forall t \in \mathbf{T}, c \in \mathbf{C}, n \in T_c(D) \quad (39)$$

$$\sum_{u=1}^U \sum_{t=1}^T x_k^{u,t} \leq 1, \forall k \in \mathbf{K} \quad (40)$$

$$\sum_{u=1}^U \sum_{t=1}^T w_k^{u,t} \leq 1, \forall k \in \mathbf{K} \quad (41)$$

$$\sum_{u=1}^U \sum_{t=1}^T v_k^{u,t} \leq 1, \forall k \in \mathbf{K} \quad (42)$$

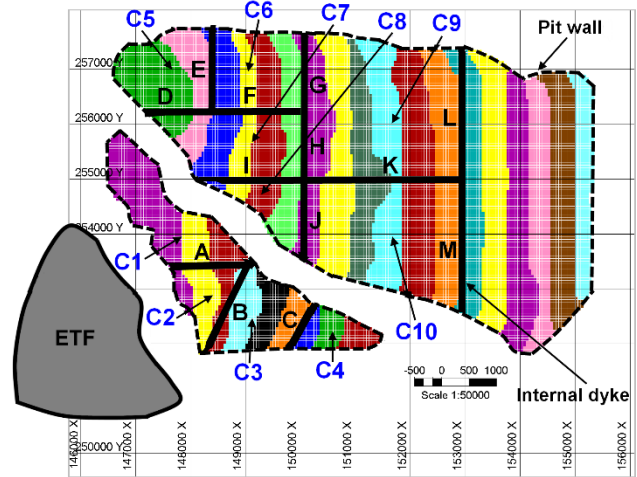
$$\sum_{t=1}^T z_c^t \leq 1, \forall c \in \mathbf{C} \quad (43)$$

$$\sum_{t=1}^T u_d^t \leq 1, \forall d \in \mathbf{D} \quad (44)$$

$$\sum_{t=1}^T y_p^{a,t} \leq 1, \forall p \in \mathbf{P}, a \in \mathbf{A} \quad (45)$$

### 2.2.3. Case Study: Incorporating Dyke Construction and Composite Tailings (CT) Deposition into Oil Sands Mine Planning

This case study is designed to show how tailings management, in terms of CT production and solid waste management, in terms of dyke construction can be integrated into the ore production schedule. The oil sands dataset used has two final pits. Mining-panels are used as the mining units and mining-cuts are used for processing. Preliminary investigations based on NPV showed the best mining direction as W-E. The model was implemented on a Dual Quad-Core Dell Precision T7500 computer at 2.8 GHz, with 24 GB of RAM. To execute the model, the required matrices for objective function, constraints and variable bounds are constructed in MATLAB. A commercial optimization solver TOMLAB/CPLEX (Holmström et al., 2009) is deployed in solving the MILP formulation. A plan view from the mine site, including pits one and two is illustrated in Figure 3. It shows the mining-panels resulting from 27 pushbacks designed in the W-E direction on a level. Dyke footprints (A to M), in-pit CT cells (C1 to C10), and a schematic view of the ETF are also illustrated on the plan view.



**Figure 3.** Plan view of the pits with dykes A to M and CT cells C1 to C10.

Table 1 shows how the pushbacks provide the space required for CT deposition when the pushbacks are completely extracted. The total volume of each CT cell equals the volume of corresponding pushbacks minus the occupied volume of the dykes that are constructed for each CT cell. Since each dyke divides two CT cells, half of a dyke's volume is deducted from the volume of each cell. The dykes' volumes are calculated by multiplying the cross-section area of the dyke to the dyke length. For simplicity, all the dykes are assumed to be orthogonal along the N-S and W-E directions except dykes B and C in pit one. The specifications of the dataset and parameters used for this case study are presented in Table 2. The tonnage of different material types in mining pits one and two are compared in Table 3. The mineralized material is defined by a regulatory cut-off grade of 7% bitumen content; and the cut-off size between fines and coarse sand is 44 μm (Masliyah, 2010).

**Table 1.** Mapping of Pushbacks to Dykes and CT Cells

Pushbacks	Volume (Mm <sup>3</sup> )	Dividing dyke	Volume (Mm <sup>3</sup> )	CT cell	Volume (Mm <sup>3</sup> )
-	-	-	-	ETF	148
1,2,3	387	A	84	C1	152
				C2	103
4,5,6	173	B	98	C3	97
7,8,9	73	C	53	C4	47
10,11,12	762	D	7	C5	182
13,14,15		E	9	C6	147
		F	7	C7	105
		I	86	C8	112
16,17,18	1,221	G	71	C9	411
19,20,21		H	71		
		J	71	C10	447
		K	133		
22,23,24	1,063	L	123	-	-
25,26,27		M	123		

**Table 2.** Input Parameters for Integrated Mine Planning MILP Model

Parameter	Value
Recovered barrel of bitumen per tonne of Bit.	0.65
Ore price (\$/t of Bitumen)	450
Mining cost (\$/t)	4.60
Processing cost (\$/t)	5.03
Extra OI dyke mining cost (\$/t)	0.92
Extra TCS dyke mining cost (\$/t)	1.38
CT deposition cost (\$/m <sup>3</sup> )	0.50
Ore cut-off grade (%)	7
Upper bound (fines grade in ore) (%)	18
Upper bound (fines grade in OI) (%)	30
OI density (t/m <sup>3</sup> )	2.03
TCS density (t/m <sup>3</sup> )	1.72
Interest rate (%)	10
Recovery (%)	90

**Table 3.** Material Tonnages and Number of Mining-blocks, Mining-cuts and Mining-panels in Pits One and Two

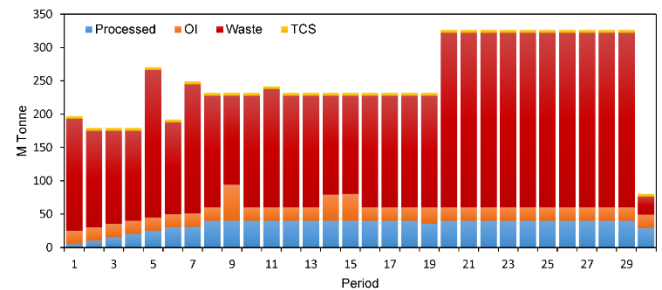
Parameter	Pit 1	Pit 2	Total
Total material (Mt)	1,237	6,217	7,454
Mineralized material (Mt)	374	1,819	2,193
OI material (Mt)	597	3,620	4,217
TCS material (Mt)	278	1,253	1,531
# of mining-blocks	16,878	81,193	98,071
# of mining-cuts	972	4,588	5,560
# of mining-panels	70	144	214

### 3. Results and Discussions

#### 3.1. Solving the MILP Problem

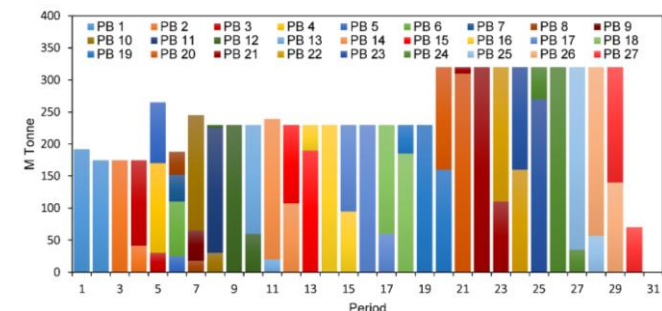
In the experiment, the proposed MILP model is directly solved to generate an optimized schedule for mine production, dyke construction and CT deposition over 30 periods. In order

to control the fluctuations of the generated schedule, a minimum and maximum are set for mining and processing capacities in each period. The maximum tonnage of oil sands material that can be sent to the processing plant ramps up from 5 Mt in period one to 40 Mt in period 8, and is fixed at 40 Mt over the next periods up to year 30. The minimum and maximum of mining capacities per period are set in a way to avoid steep jumps from one period to another through two increases in the minimum mining capacity; in period 8 (from 175 Mt to 228 Mt) and period 20 (from 228 Mt to 322 Mt). The resulting schedule is illustrated in Figure 4. The accumulated tonnage of mined material has a fairly constant slope, meaning that the operation progresses under a constant rate over 30 periods. This is a preferred mine operations scheduling result when it comes to planning for mining equipment.



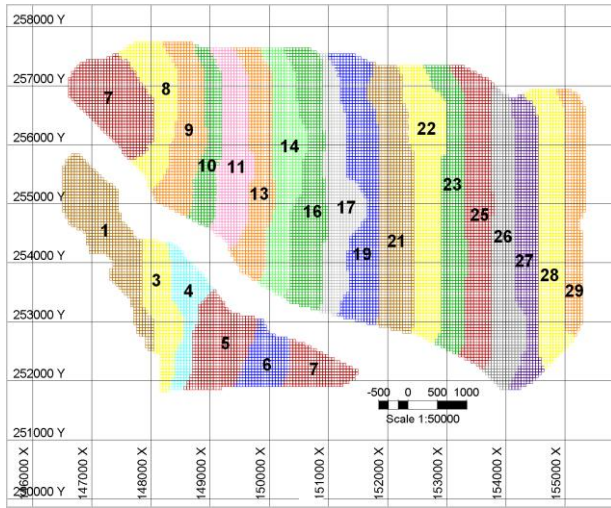
**Figure 4.** Production schedule for 30 periods equivalent to 30 years.

Figure 4 contains the following information: the total material mined; the high grade oil sands material with an average grade higher than 7% that is sent to the processing plant; the material extracted as OI for dyke construction; the low grade material extracted as waste; and the tonnage of TCS material produced in cyclone underflow at the end point of the oil sands hot water extraction process. TCS is generated from ore material sent for processing. To integrate dyke material production with mine planning, it is essential to define a separate variable to control its production to ensure that the required dyke material would be generated in each period. Because TCS plays a key role, it is illustrated at the top of each bar in Figure 4. Further details are provided regarding the connection between CT deposition and dyke construction when the corresponding schedules are discussed in Section 3.2.



**Figure 5.** Production schedule for 30 periods equivalent to 30 years.

Figure 5 illustrates how the pushbacks are extracted over 30 years. Pushbacks one to nine are in pit one and are extracted in order from west to east over periods one to seven. The extraction of pushbacks in the second pit (pushbacks 10 to 27) starts in period seven, after complete extraction of pit one, and continues up to period 30. Figure 6 illustrates the production schedule in a sample bench, showing the period numbers in which the portions are scheduled to be extracted. It shows the extraction progressing in the west-east direction.



**Figure 6.** Production schedule in W-E direction (numbers: period of extraction).

A summary of numerical results of the production schedule is provided in Table 4. The total material tonnage in pits one and two is 7,454 Mt, which has been extracted completely to clear the in-pit space for dyke construction and CT deposition. From the total material tonnage, 2,193 Mt is the mineralized tonnage with a bitumen grade of more than 7%. Based on mining and processing capacities, 1,041 Mt from the mineralized tonnage has been processed as ore, resulting in 129 Mt of recovered bitumen, equivalent to 84 million barrels of bitumen. The total tonnage of extracted OI and TCS dyke material is 681 Mt and 128 Mt respectively. The integer solution is optimized (0% optimality gap), resulting in an NPV of \$2,212 M.

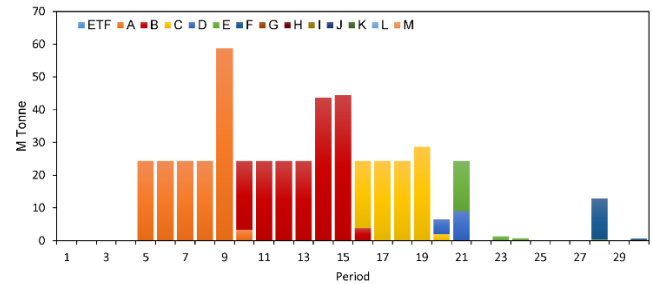
**Table 4.** Summary of the Results

Total material extracted (Mt)	7,454
Mineralized material (Mt)	2,193
Processed ore (Mt)	1,041
Recovered bitumen (Mt)	129
Recovered bitumen (M barrel)	84
Extracted OI (Mt)	681
Produced TCS (Mt)	128
# of continuous variables	1,010,700
# of integer variables	9,900
# of constraints	214,878
Optimality gap	0%
Run time (h)	29.5
NPV (M\$)	2,212

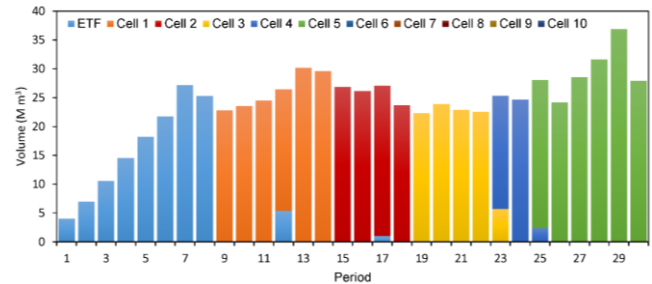
### 3.2. Dykes Construction Schedules and CT Deposition

The generated production schedule includes corresponding schedules for dyke construction and CT deposition. For a clearer picture of these schedules, the progress of dykes construction and CT deposition are illustrated in Figure 7 and Figure 8.

Dyke construction starts in period five with Dyke A. During the first four periods no in-pit dyke will be constructed, because the space required to construct Dyke A will only be available after the complete extraction of pushback three in period five (Figure 5). Dyke A will be constructed over periods 5 to 10. Dyke construction continues with Dyke B (periods 10 to 16), Dyke C (periods 16 to 20), Dyke D (periods 20 and 21), Dyke E (periods 21, 23 and 24) and finally part of Dyke F (over periods 28 and 30). The generated schedule follows the dyke precedence in the west-east direction. The volumes of dykes are different, following the length of the specified footprints for dykes (Figure 3). Therefore, they have been constructed in different time periods. As an example, Dyke C is 1,010 m long, less than 55% of Dyke B’s length (1,880 m). Because of this difference, it takes less time to construct Dyke C than Dyke B. The generated dyke construction schedule that shows the material tonnages used for each dyke in each period is illustrated in Figure 7.



**Figure 7.** Dyke construction schedule for 30 periods equivalent to 30 years.

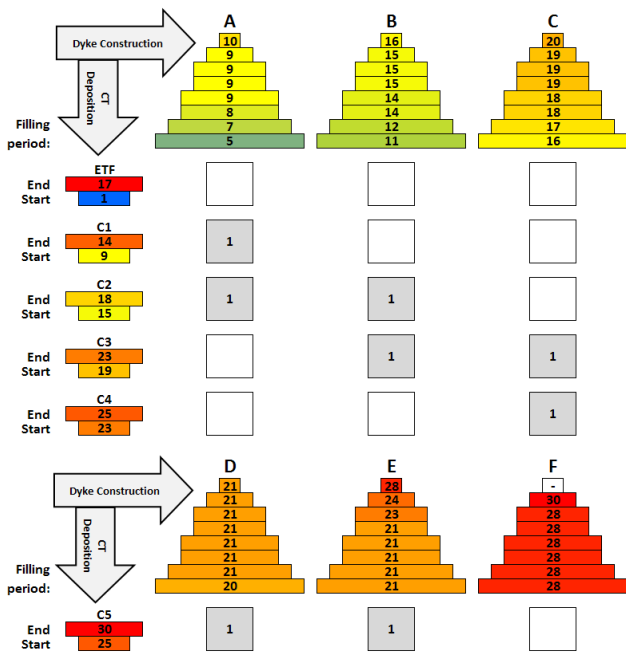


**Figure 8.** CT deposition schedule for 30 periods equivalent to 30 years.

CT deposition starts in the first period by sending the produced CT to the ETF (Figure 8) until period eight. In period nine and when the main portion of Dyke A is constructed, CT cell 1 starts receiving CT and continues to accommodate it until period 14 when maximum capacity is reached. CT cells are filled in the following order: cell 1 (periods nine to 14), cell 2 (periods 15 to 18), cell 3 (periods 19 to 23), cell 4 (periods 23

to 25) and cell 5 (periods 25 to 30). In addition to the CT cells' precedence order, the other constraint that determines when CT can be deposited in a cell is the completion of dyke walls that shape the CT cell. Figure 7 and Figure 8 show that this constraint has been generally followed. However, there are some overlaps in the construction of cells and deposition of CT in them. That is because of an assumption in modeling the construction of CT cells and CT deposition: of the eight lifts involved in constructing any dyke, when the lower seven lifts are completely constructed, the cell can receive CT and the eighth lift will be constructed at the same time by depositing sand over the dyke walls. This assumption justifies the overlaps between CT cell construction and CT deposition. The role of the ETF is also well illustrated in this case study. The ETF has worked as a buffer to accommodate the produced CT in the early periods one to nine, when the in-pit space is not readily available for dyke construction.

The volumes of CT cells are different, following the volumes of extracted pushbacks and pit dimensions for each cell (Figure 3). This difference has resulted in different capacity of cells and therefore the cells have been filled with CT over a different number of time periods. The generated CT deposition schedule, showing the volume of CT deposited in each cell over each period is illustrated in Figure 8.



**Figure 9.** A schematic cross-sectional view of dykes' lifts construction & CT deposition.

The schematic cross-sectional view in Figure 9 illustrates the construction schedule for dykes' lifts and the period number in which CT deposition has been started and finished in each CT cell. The highlighted squares with a value of one demonstrate which dykes (in columns) are pre-requisites for each CT cell. For example, CT deposition in CT cell 3 can only be started when Dyke B and the 7<sup>th</sup> lift of Dyke C are already completed. These dykes are marked as the precedence for CT cell 3.

The start deposition period for cell 3 is 19, which is the earliest possible period that CT can be sent to cell 3 after completion of the 7<sup>th</sup> lift of Dyke C.

### 3.3. Summary

The results of the case study show that dyke construction and CT deposition can be well integrated with the long-term mine planning framework. The resulting schedule is practically mineable through a fairly uniform production rate and generates a uniform feed to the processing plant. By following a chosen mining direction, dyke footprints are cleared and the dyke becomes ready for construction. The OI and TCS material required for dyke construction are produced and used to raise the dyke walls for in-pit CT cells. The produced CT is sent to the ETF during the early periods when the in-pit cells are not yet ready. The in-pit cells accommodate the CT over 21 periods; from periods nine to 30. In conclusion, the proposed integrated model has successfully optimized (0% gap) the long-term mine production, dyke construction, and CT deposition scheduling problem within an integrated framework in a reasonable solution time.

## 4. Conclusions

The current literature related to mine planning and waste management optimization shows a lack of integration in terms of in-pit deposition of solid waste material and tailings disposal management. The implemented framework is a novel technique that fills the current literature gap in strategic open-pit mine planning. An integrated long-term mine production model has been developed to generate the optimal production schedule, with respect to dyke construction and tailings deposition. The model is verified through a case study on a real oil sands data set. The generated schedule is practically mineable, follows the chosen direction, provides a smooth feed for the oil sands processing plant, provides the material required to construct in-pit dykes, and accommodates the produced CT in the ETF and in-pit CT cells. The value of this model to the mining industry can be quantified directly from the savings made by avoiding re-handling of the dyke construction material and indirectly from the reduced mining footprint. The main contribution of this work includes the direct scheduling and precedence of dyke construction.

It is recommended to consider efficient methods to reduce the problem size for large-scale problems, through preprocessing and period aggregation technique. The other area for development of this research is to consider other means of tailings dewatering, such as atmospheric fine drying (AFD) used in ETFs, or non-segregated tailings technology (NST) for in-pit impoundment of tailings products.

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