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Reduction of Pollution through Sustainable and Flexible Production by Controlling By-Products

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ABSTRACT. Every manufacturing system produces toxic by-products that cause a hazardous impact on society and the environment. As a result, pollution control authorities' role has gained importance for the betterment of society and the preservation of a clean and green environment. As a result, one of the goals of this research is to develop a sustainable smart manufacturing model with less waste and controlled pollution. Here, a flexible production process is discussed under imprecise market conditions with partial backlogging and rework. Two different sustainable production models are presented here by considering pollution control costs. A sustainable production model with variable pollution costs is examined under the influence of three pollution control mechanisms to improve the model's applicability. A solution methodology, including three critical theorems, is provided to obtain the optimal production rate, length, and total cost per cycle. The paper's novelty lies in introducing pollution control costs and pollution control mechanisms together in a flexible, sustainable production system with uncertainty. In comparison to the other models, the model with a variable pollution cost appears to be more sustainable as, in this case, there is a 25.5% reduction in the pollution level compared to the other models. Implementing three pollution-controlling strategies, such as pollution cap, pollution cap and trade, and pollution tax, resulted in reductions of 34.37, 0.83, and 0.62% in pollution levels, respectively. A sensitivity analysis of the obtained results is carried out to show the model's strength and robustness.

Keywords: pollution control, imperfect quality, flexible production, rework, partial backlogging, fuzziness

1. Introduction

1.1. Background and Motivation

In industry, the production department needs to incorporate sustainable manufacturing. It is responsible for the economy, society, and the environment. It is primarily imposed by government policies and growing customer concerns about the environment. After various manufacturing stages with the desired results, some faulty products, solid waste, and polluting gases are generated in industries. These harmful by-products result in global warming. For example, various life-threatening pollutants are left untreated in the textile industry during different production processes. The boiling process emits nitrous oxides and sulphur dioxide, while the sizing process emits carbon monoxide. Bleaching discharges chlorine oxide while printing discharges hydrocarbons and ammonia, and the finishing process could release formaldehyde into the air. As a result, manufacturing models should be designed under these challenges to ensure environmental sustainability (Manna et al., 2018; Marchi et al., 2019; Sarkar and Sarkar, 2020; Bhattacharya et al., 2021). Nowadays, every government is more concerned about the environment. They established regulatory bodies to regulate these issues. Regulatory bodies have designed different pollution caps, cap and trade, and pollution tax mechanisms. These regulatory agencies also undertake an awareness program with the industry's support to educate their customers about the benefits of a clean and green environment. All of these efforts have an impact on the customer's decision. Their preferences are more environmentally friendly products.

Due to imperfect manufacturing processes, natural calamities, or damage, defective quality items are unavoidable in a production system. Nevertheless, remanufacturing of faulty products and the reuse of waste obtained from factories could reduce waste and help control pollution (Ghosh et al., 2017; Karmakar et al., 2018; Rout et al., 2020). Furthermore, separating perfect and imperfect items during the production process is critical. In the traditional production model, inventory practitioners ignore it or do it manually. However, it is necessary to shift the traditional production system into a smart one by investing in smart technologies, skilled labor, and low energy consumption. In this cutthroat competition, the indus-

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try's goodwill is critical to the market's survival. As a result, switching from manual screening to an automated screening system reduces the risk of error (Sarkar and Chung, 2019).

Inventory managers face various market uncertainties and impreciseness related to the inventory planning system's demand, setup, and cost parameters. It is better to assume these crucial parameters in a more elastic form in this situation. Implementing the fuzzy set theory could easily model the market fluctuations (Roy, 2014). Jamrus et al. (2020) studied a smart production system with uncertain production time in a coordinated supply chain. De (2020a) applied a fuzzy lock set approach to a pollution-sensitive production system. Tayyab and Sarkar (2021) applied a fuzzy set theory to cover uncertain market information for demand in an imperfect manufacturing system with rework.

1.2. Research Questions

According to the production model in literature, the impact of pollution control measures devised by regulatory agencies on the imperfect manufacturing process with flexible production and partial backlogging has rarely been examined. Keeping all these in mind, the following research questions can be raised:

(1) What should be the optimal total cost, optimal production rate, and optimal cycle length for the model with variable pollution control cost (Model 1), with ongoing pollution control cost (Model 2), and without pollution control cost (Model 3)?

(2) Which model is better and more practical for industrial managers to adopt?

(3) How do different pollution control mechanisms affect optimal policies and the amount of pollution?

(4) How do pollution control costs and mechanisms lean the system towards environmental sustainability?

(5) How does a smart and environmentally responsible production system retain its economic sustainability?

The following study is proposed to address these questions by considering an imperfect production system with partially backlogged shortages and rework. It aimed to design (i) a sustainable and smart production system with (ii) top priority on controlling pollution and (iii) to handle vagueness/uncertainty of the system.

1.3. Organization of the Study

This study is organized as follows: Section 2 presents a wealthy review of the literature, the research gap, and highlights the paper's contribution to the present literature. This section offers problem description, notation, and assumptions, which is the base of the mathematical model. In Section 3, three models with or without pollution control costs and the application of three pollution control mechanisms are presented. This Section contains the solution methodology to obtain the optimal solution. Section 4 provides a detailed numerical study. Section 5 reports a sensitivity analysis of the different results obtained. Finally, a few managerial insights and limitations and the scope of future research are concluded in Section 6.

2. Literature Review

This Section gives an overview of the literature related to the production model. Finally, a brief research gap in the existing literature is discussed.

2.1. Sustainable Manufacturing and Pollution Control

Pollution control and sustainable manufacturing strategies have become a global responsibility for all industrialists worldwide. Many production models in literature did not consider environmental issues. Bonney and Jaber (2011) developed environmentally responsible inventory models. They focused on non-traditional costs related to packaging, waste management, and transportation for promoting green production. Mukhopadhyay and Goswami (2014) worked on an imperfect production model, assuming three types of defective items with constant and variable pollution costs. Khatua and Maity (2016) presented an economic production quantity model to prevent environmental pollution by applying various policies like reliability development by the current policy. Raza and Faisal (2018) discussed two inventory models on greening effort and stock level dependent demand, together with pricing to maximize the industry's overall profit. Gautam et al. (2019) designed a vendor-buyer problem strategy considering imperfect production and shortages with carbon emissions. Rout et al. (2020) studied the impact of different emission control strategies for sustainable management of a model with deterioration and faulty production. Recently, Sarkar and Chung (2021) controlled waste and emissions to maintain product quality in a sustainable supply network. For further reference in this direction, one could see the following papers (Datta, 2017; Datta et al., 2019; Asif and Chen, 2020; Daryanto et al., 2020; Datta, 2020; Ji et al., 2020; Khanna et al., 2020; Manna et al., 2020; Ghosh et al., 2021). None of the research papers discussed above considered the two approaches to pollution control together. Hence, implementing pollution control costs and pollution control mechanisms together is a significant research gap.

2.2. Inventory Models with Smart/Flexible Manufacturing

Production systems should be adjustable to decrease any industry's defective production and waste. An automation policy can be adopted for an error-free inspection process, where a product is inspected through machines. Cárdenas-Barrón (2009) introduced defective products and reworked them with planned backorders in an economic production quantity model. Sana et al. (2007) presented two-volume flexible production models, which produce perfect and imperfect both types of items. Wee et al. (2013) developed an inventory model with screening and shortages, including unsatisfactory quality items. They applied the Renewal Reward Theorem to find the optimal profit of the system. Pal et al. (2014) studied an integrated price-dependent production model with defective products and rework. Roul et al. (2015) studied an inadequate production model, including dynamic demand and variable production rate with fuzzy budget constraints. Vandana et al. (2021) analyzed a flexible production rate that under optimum energy consumption and inflation. An inventory model was investigated by Goyal et al. (2017), considering an imperfect lot, partial backorder, and advertisement-dependent uncertain demand of customers. Manna et al. (2017) examined a model in which defective production rate depends on the production rate. They assumed that demand is advertisement-dependent. Khanna et al. (2017), Gautam and Khanna (2018), Kumar et al. (2022), and many others contributed to the field of imperfect production. Kugele et al. (2022) emphasized the smart production system to improve the reliability issue of a traditional production system. Malik and Kim (2020a) considered direct and indirect industrial emissions with flexible production in a bi-level supply chain model. Dey et al. (2021a, 2021b, 2021c) incorporated automation policies in smart production systems to optimize work in process inventory. Several other remarkable research studies (AlDurgam et al., 2017; Malik and Kim, 2020b; Sardar et al., 2021; Mahapatra et al., 2021; Sarkar et al., 2021; Sarkar and Chung, 2021; Yadav et al., 2021; Moon et al., 2022) contributed towards flexible production. However, environmentally responsible practitioners rarely touched flexible manufacturing with pollution control mechanisms such as carbon cap, cap and trade policies. It could give a new direction to smart production.

2.3. Inventory Models with Impreciseness

In industries, inventory managers usually face market impreciseness related to critical decision-making parameters. Thus, it is essential to deal with uncertainty in the right way. Zadeh (1965) first developed the fuzzy set theory, which many researchers later adopted. De and Rawat (2011) developed an expression for an imprecise inventory model with no shortages. They used real-life data to validate their model. Mahata and Goswami (2013) developed two models using trapezoidal and triangular fuzzy numbers and defuzzied the objective function using a graded mean integration method. Roy (2014) incorporated the fuzzy set theory in a model with decaying products and variable decaying rates and holding cost. They considered parameters and variables both as fuzzy in their model. Karmakar et al. (2018) analyzed a fuzzy production and remanufacturing system to control pollution and waste in the reverse logistic process. Rout et al. (2019) designed a production model considering imperfect production and deterioration with screening errors. They assumed that the deterioration rate is a type-2 fuzzy number and fully backlogged shortages. More flexible and wide studies on fuzzy techniques are discussed in these studies (De and Sana, 2018; De and Mahata, 2019a, 2019b; Bhattacharya and De, 2020; De, 2020b; Bhuniya et al., 2021a; Garai et al., 2021; Kumar et al., 2021a; Omair et al., 2021; Tayyab and Sarkar, 2021). Singh et al. (2020) studied the effect of carbon emissions on a three-stage system with deficits under uncertainty. Habib et al. (2021) recently applied a robust optimization to handle impreciseness in an animal fat-based sustainable supply chain system. Environmental sensitive models under uncertainty are rarely studied. It is a clear research gap that should be covered.

2.4. The Research Gap and Researchers Contribution

As reviewed by the literature and analyzed in Table 1, the research gap could be examined clearly. Many researchers have worked on pollution control, imperfect manufacturing, and remanufacturing. However, there is a clear research gap on sustainable and flexible production strategies for decaying products to control pollution with partially backlogged shortages and demand in fuzzy conditions. Thus, this study focuses on the following work.

- It introduces sustainable production strategies. The inventory decision-maker decides the optimal production rate, the optimal cycle length to minimize total pollution, and the total cost per cycle of the complete manufacturing system.
- Flexible production and smart inspection help control pro-

Table 1. Review and Gap Analysis of Existing and Present Research

Author’s	Pollution control cost	Imperfect quality	Flexible production	Fuzziness	Rework	Partial backlogging	Deterioration	Pollution control mechanism
Singh and Singh (2010)				✓		✓	✓	
Sarkar et al. (2022)	✓						✓	
Mukhopadhyay and Goswami (2014)	✓	✓			✓			
Pal et al. (2014)		✓			✓	✓		
Roul et al. (2015)		✓	✓	✓	✓			
Manna et al. (2017)		✓	✓		✓		✓	
Kumar and Kumar (2017)				✓		✓	✓	
Tayyab et al. (2019)		✓		✓	✓			
Karmakar et al. (2018)			✓		✓		✓	
Gautam et al. (2019)	✓	✓			✓			
Rout et al. (2019)		✓		✓	✓	✓		
Rout et al. (2020)		✓		✓	✓		✓	✓
Yadav and Khanna (2021)							✓	✓
This research	✓	✓	✓	✓	✓	✓	✓	✓

duction according to demand, allowing control defectives, holding cost of inventory, and emission. It also reduces deterioration and industrial waste. Therefore, the production rate is considered a variable.

- An attempt is made to cover market ambiguity by considering all cost parameters as fuzzy and denoted by fuzzy triangular numbers.
- Three models with product deterioration and partially backlogged shortages have been studied with (fixed and variable) pollution control costs.
- Three pollution control mechanisms: pollution cap, pollution cap and trade, as well as pollution tax are applied.

2.4.1. Problem Description

In the direction of the research gap, (i) a smart production system with deterioration and partial backlogging is designed with a controllable production rate. During production, all the products go through an automated screening system, which segregates the product into four categories: (a) perfect products used to fulfill the market demand, (b) non-reworkable defective products sold at a low price, (c) reworkable defective products which are after rework as good as perfect products, and (d) disposable waste. (ii) For making the model closer to real-world problems, the ambiguity in cost parameters is handled with fuzzy set theory. All cost parameters, including pollution control costs, are assumed as vague. (iii) Pollution control: It is done in two steps. First, the model is developed by considering variable pollution control costs. Then, three pollution control policies, i.e., pollution cap, cap and trade, and pollution tax, are applied to the model to curb the number of pollutants. Based on the three models, optimal policies, and the amount of pollution generated, a comparison among all the models and all three pollution control policies is made with the help of numerical data to analyze the appropriate environmental and economic sustainability procedure. Figure 1 depicts the involvement of various steps in the flow of the product from the production house to the market. The figure depicts that there are markets for both perfect and imperfect products. The figures show that the screening process takes place at the production house. Following the screening process, the product has been classified as follows:

(i) Perfect products sold at full price.

(ii) Imperfect products sold at a reduced cost.

(iii) Imperfect products made perfect through rework.

(iv) Waste discarded.

2.4.2. Notation and Assumptions

For the mathematical development of the proposed study, the following notation and assumptions are used. The following notation are used in mathematical modelling.

Decision Variables

T inventory cycle time (year)
P production rate (units/cycle)

Parameters

D demand rate (units/cycle)
x screening rate (units/cycle)
y lot-size (unit)
t_p production time (year)
t_s screening time (year)
t_f time when inventory level reaches zero
C_o setup /organization cost ($/setup)
C_p $\left(= a_1 + \frac{a_2}{p} + a_3 P\right)$, i.e., production cost ($/unit)
a_1 material cost ($/unit)
a_2 development cost ($/unit)
a_3 tool/die cost ($/unit)
C_h holding cost ($/unit)
C_s screening cost ($/unit)
C_r rework cost for defective items ($/unit)
C_b backlogging cost ($/unit)
C_l lost sale cost ($/unit)
B the fraction of shortage demand backordered
I_b total shortage demand (units/cycle)
p_1 probability of non-reworkable imperfect items

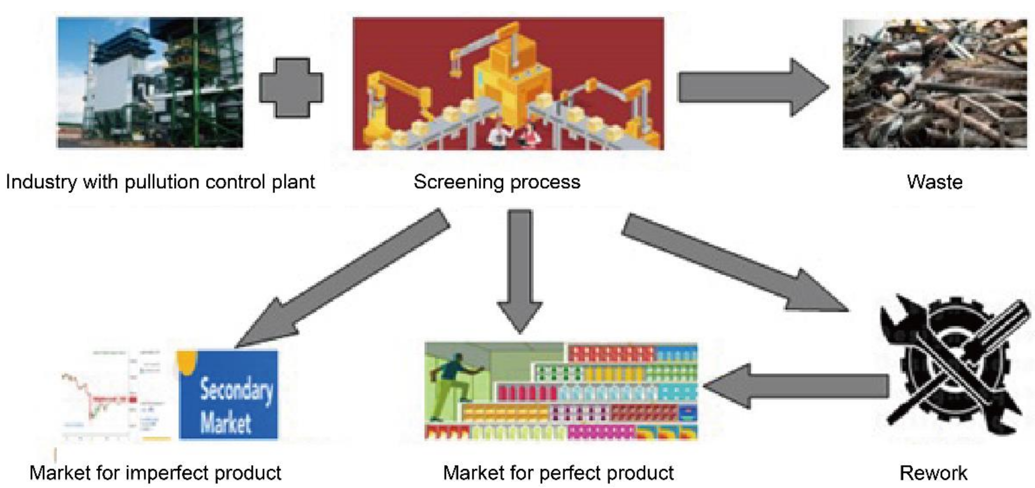

Figure 1. Flow of the product in the production system.

p_2 probability of reworkable imperfect items
p_3 probability of scrap items
θ constant rate of deterioration
μ_{Cap} pollution cap (ton/year)
$\mu(t)$ amount of pollutant at time t (ton/year)
σ the fraction of pollutant that disappears
s_1 unit purchasing price of pollution credit ($/unit)
s_2 the unit selling price of pollution credit ($/unit)
π pollution tax of emission ($/ton)

The following assumptions are taken to develop a basic mathematical model.

(1) Usually, many production processes generate defects along with non-defective or perfect ones. Irresponsible handling of which may increase pollution and unwanted landfills. Therefore, for sustainable production, a smart production system is assumed. It produces only one type of product. An automated screening is done with the help of a machine, which results in four types of products: (i) Perfect items, which are sold at full price. (ii) Non-reworkable defective items sold at a low price. (iii) Reworkable defective items are reworked, which is as good as perfect items. (iv) Disposable waste. The probability of non-reworkable imperfect things (p_1), reworkable defective items (p_2), and scrap items (p_3) follow $p_1 + p_2 + p_3 < 1$. In this way, smart management of defectives may save the environment and generate revenue.

(2) Reworked defective products are considered perfect quality items.

(3) Flexible production is a basic need for smart production. It may increase the setup cost, but it decreases holding, pollution, and waste generation during extra production. Thus, the production rate is taken as a variable, i.e., production rate taken as adjustable. This means that it can be adjusted by setting the speed of the machine. So, we can say that it varies within a prescribed interval [$P_{\min}$, $P_{\max}$]. By adjusting the production rate, the pressure of inventory in the system can be released. The cost of production is $C_p = \left(a_1 + a_2/P + a_3P\right)$, where a_1 is the material cost ($/unit), a_2 is development cost ($/unit) and a_3 is tool/die cost ($/unit).

(4) The screening process and demand proceed simultaneously. The screening rate is assumed to be higher than the demand rate, so that shortage will not occur at the time of screening.

(5) Shortages are allowed and are partially backlogged. The deterioration rate is constant.

(6) Usually, the market information is not crisp but imprecise. Hence, for enhancing the practicality of the models, all the cost parameters are considered imprecise.

3. Mathematical Formulation

In this Section, two production models with pollution control costs are presented. First, a smart production model with a flexible production rate and rework is designed with a variable pollution control scenario. Then to gain environmental sustainability, it is extended with three pollution control mechanisms. Second, a model with fixed pollution control cost is designed.

3.1. Model 1: Basic Production Model with Variable Pollution Control Cost

This study considers a controllable production system (Sarkar and Chung, 2019), enabling the manufacturer to adjust its production rate within a specific range according to the system's demand and overall need. It can handle imperfect quality items produced during the production process. Suppose a manufacturer makes a single type of product at a production rate P to satisfy its demand. The production and demand cycle starts at $t = 0$, at which inventory is zero. Figure 2 represents the inventory level of the complete process. In this figure, the slope of the curve during OP is $P – D$ which is governed by the differential Equation (1). During PQ, the slope of the curve is $–D$, which can be represented by the differential Equation (2). During QR, the slope of the curve is $–D$, which is governed by the differential Equation (3), and similarly, during RS, the slope of the curve is $–BD$, which is illustrated by differential Equation (4).

The following differential equations express the change of inventory level with time:

$$\dot{I}_1(t) + \theta I_1(t) = P - D, 0 \le t \le t_p \tag{1}$$

$$\dot{I}_2(t) + \theta I_2(t) = -D, t_p \le t < t_s \tag{2}$$

$$\dot{I}_3(t) + \theta I_3(t) = -D, t_s < t \le t_f \tag{3}$$

$$\dot{I}_4(t) = -BD, t_f \le t \le T \tag{4}$$

Using the initial condition $I_1(0) = 0$, the inventory at time t is given by:

$$I_1(t) = \left(\frac{P-D}{\theta}\right)\left(1 - e^{-\theta t}\right), 0 \le t \le t_p \tag{5}$$

Using condition $I_2(t_p) = \left[(P-D)/\theta\right]\left(1 - e^{-\theta t_p}\right)$, inventory at time t is given by:

$$I_2(t) = \left[\frac{P}{\theta}e^{\theta t_p} - \left(\frac{P-D}{\theta}\right)\right]e^{-\theta t} - \frac{D}{\theta}, t_p \le t \le t_s \tag{6}$$

The automated screening process, production process, and demand go simultaneously, and the screening process completes at the point $t = t_s$. After the screening process, all the scrap and non-reworkable items, i.e., $y(p_1 + p_3)$ units are withdrawn from the inventory. A sharp decline in the inventory level is represented in the graph through a vertical line. The graph is not continuous at $t = t_s$, whereas it is continuous in the interval [0, t_s) and (t_s, T]. Thus, it is assumed that $I_2(t_s – 0) – y(p_1 + p_3) = I_3(t_s + 0)$, where:

$$I_2(t_s - 0) = \left[\frac{P}{\theta}e^{\theta t_p} - \left(\frac{P-D}{\theta}\right)\right]e^{-\theta t_s} - \frac{D}{\theta} \tag{7}$$

The solution of Equation (3) under the condition $I_3(t_s + 0) = I_2(t_s - 0) - y(p_1 + p_3)$ is given by:

$$I_3(t) = \left[\frac{P}{\theta}e^{\theta t_p} - \left(\frac{P-D}{\theta}\right) - y(p_1 + p_3)e^{\theta t_s}\right]e^{-\theta t} - \frac{D}{\theta},$$
$$t_s < t \le t_f \tag{8}$$

Using condition $I_4(t_f) = 0$, inventory at time t is given by:

$$I_4(t) = BD(t_f - t), \text{ when } \ t_f \le t < T. \tag{9}$$

Now, t_f is obtained by using condition $I_3(t_f) = 0$:

$$t_f = \frac{1}{\theta}\ln\frac{\theta}{D}\left[\frac{P}{\theta}e^{\theta t_p} - \left(\frac{P-D}{\theta}\right) - y(p_1 + p_3)e^{\theta t_s}\right]. \tag{10}$$

At point T, from Equation (9), the total shortage is:

$$I_b = BD\left[T - \ln\frac{\theta}{D}\left(\frac{P}{\theta}e^{\theta t_p} - \left(\frac{P-D}{\theta}\right)\right.\right.$$
$$\left.\left. - y(p_1 + p_3)e^{\theta t_s}\right)\right]. \tag{11}$$

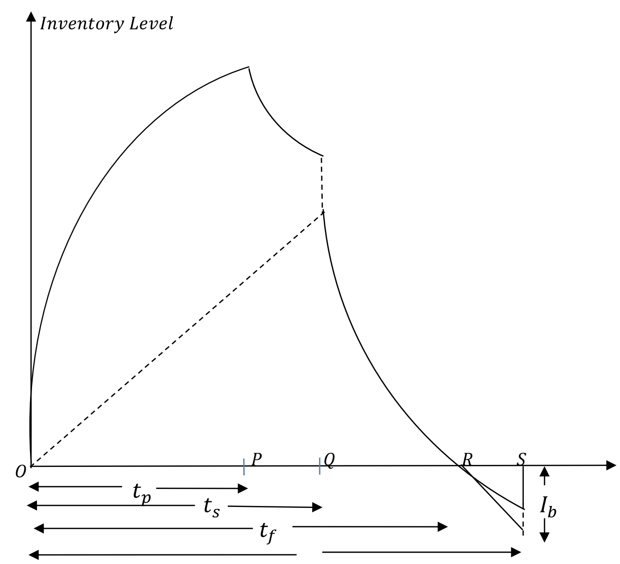

Figure 2. Representation of the inventory level of complete cycle.

The holding cost can be expressed by Equation (12):

$$\text{Holding cost} = C_h\left[\int_0^{t_p} I_1(t)dt + \int_{t_p}^{t_s} I_2(t)dt + \int_{t_s}^{t_f} I_3(t)dt\right]$$
$$= C_h\left[\frac{y(1-p_1-p_2)}{\theta} - \frac{D}{\theta^2}Log\left(\frac{P}{D}e^{\frac{y}{P}\theta}\right.\right.$$
$$\left.\left. - \left(\frac{P-D}{D}\right) - \frac{y}{D}\theta(p_1 + p_3)e^{\theta\frac{y}{x}}\right)\right] \tag{12}$$

During time $t_f \le t \le T$, shortage of products occurs in the system, partially fulfilled by backlogging rate B. Backlogging cost (shown in Equation (13)) is the cost related to an attempt to fulfill the possible part of the shortage of demand (B), and lost sale cost (shown in Equation (14)) is losing that part of the market that could not be satisfied:

$$\text{Backlogging cost} = C_b\int_{t_f}^{T} BD(t - t_f)dt = \frac{C_b}{2BD}(I_b)^2 \tag{13}$$

$$\text{Lost sale cost} = C_l\int_{t_f}^{T}(1-B)Ddt = \frac{C_l(1-B)I_b}{B} \tag{14}$$

The total percentage of perfect items can be expressed as $y(1 - p_1 - p_3)$. The setup cost is the initial cost of setting up for production, which is a constant (C_o). The production cost is essential in production systems. It is obtained by multiplying the cost of producing one unit by the number of units produced, i.e., $C_p y = (a_1 + a_2/P + a_3 P)y$. Screening cost is the cost of thoroughly checking the production output and separating the perfect and defective products. It is calculated by multiplying the screening cost of one product by the number of units produced ($C_s y$). Rework cost is the cost that occurred during the redesign of a defective product to a perfect one. Thus, rework cost is $C_r p_2 y$. Total cost is the sum of ordering, purchasing, screening, rework, inventory holding, backlogging, and lost sale costs. The total cost per cycle can be written as:

$$C_T(P,T) = \frac{1}{T}\left\{C_o + (C_p + C_s + C_r p_2)y + C_h\left[\frac{y(1-p_1-p_2)}{\theta}\right.\right.$$
$$\left. - \frac{D}{\theta^2}\ln\left(\frac{P}{D}e^{\frac{y}{P}\theta} - \left(\frac{P-D}{D}\right) - \frac{y}{D}\theta(p_1 + p_3)e^{\theta\frac{y}{x}}\right)\right]$$
$$\left. + \frac{C_b}{2BD}(I_b)^2 + \frac{C_l(1-B)I_b}{B}\right\} \tag{15}$$

where I_b is mentioned in Equation (11).

Now to find the total cost per cycle of the production model in a fuzzy environment, consider that C_o, C_p, C_s, C_r, C_h, C_b, and C_l are fuzzy and can be expressed as fuzzy triangular sets $\tilde{C}_o, \tilde{C}_p, \tilde{C}_s, \tilde{C}_r, \tilde{C}_h, \tilde{C}_b$, and $\tilde{C}_l$, respevtively. Let:

$$\tilde{C}_o = (C_o - \Delta_1, C_o, C_o + \Delta_2)$$
$$\tilde{C}_p = (C_p - \Delta_3, C_p, C_p + \Delta_4)$$
$$\tilde{C}_s = (C_s - \Delta_5, C_s, C_s + \Delta_6)$$
$$\tilde{C}_r = (C_r - \Delta_7, C_r, C_r + \Delta_8)$$
$$\tilde{C}_h = (C_h - \Delta_9, C_h, C_h + \Delta_{10})$$
$$\tilde{C}_b = (C_b - \Delta_{11}, C_b, C_b + \Delta_{12})$$
$$\tilde{C}_l = (C_l - \Delta_{13}, C_l, C_l + \Delta_{14}) \tag{16}$$

where $0 < \Delta_1 < C_o$, $0 < \Delta_3 < C_p$, $0 < \Delta_5 < C_s$, $0 < \Delta_7 < C_r$, $0 < \Delta_9$

$< C_h$, $0 < \Delta_{11} < C_b$, $0 < \Delta_{13} < C_l$, and $0 < \Delta_2, \Delta_4, \Delta_6, \Delta_8, \Delta_{10}, \Delta_{12}, \Delta_{14}$.

The fuzzy total cost per cycle can be expressed as follows:

$$\tilde{C}_T(P,T) = \frac{1}{T}\Bigg\{\tilde{C}_o + \left(\tilde{C}_p + \tilde{C}_s + \tilde{C}_r p_2\right)y + \tilde{C}_h\Bigg[\frac{y(1-p_1-p_2)}{\theta} - \frac{D}{\theta^2}\ln\left(\frac{P}{D}e^{\frac{y}{p}\theta} - \left(\frac{P-D}{D}\right) - \frac{y}{D}\theta(p_1+p_3)e^{\theta\frac{y}{x}}\right)\Bigg] + \frac{\tilde{C}_b}{2BD}(I_b)^2 + \frac{\tilde{C}_l(1-B)}{B}I_b\Bigg\}. \tag{17}$$

Now for defuzzification of $\tilde{C}_T(P,T)$ the signed distance method is used. The signed distance of $\tilde{C}_T(P,T)$ can be expressed as follows:

$$d\left(\tilde{C}_T(P,T),\tilde{0}\right) = \frac{1}{T}\Bigg\{d\left(\tilde{C}_o,\tilde{0}\right) + \left[d\left(\tilde{C}_p,\tilde{0}\right) + d\left(\tilde{C}_s,\tilde{0}\right) + d\left(\tilde{C}_r,\tilde{0}\right)p_2\right]y + d\left(\tilde{C}_h,\tilde{0}\right)\Bigg[\frac{y(1-p_1-p_2)}{\theta} - \frac{D}{\theta^2}\ln\left(\frac{P}{D}e^{\frac{y}{p}\theta} - \left(\frac{P-D}{D}\right) - \frac{y}{D}\theta(p_1+p_3)e^{\theta\frac{y}{x}}\right)\Bigg] + \frac{d\left(\tilde{C}_b,\tilde{0}\right)}{2BD}(I_b)^2 + \frac{d\left(\tilde{C}_l,\tilde{0}\right)(1-B)}{B}I_b v\Bigg\}, \tag{18}$$

or:

$$C_T(P,T)^* = \frac{1}{T}\Bigg\{C_o + \frac{(\Delta_2-\Delta_1)}{4} + \Bigg[\left(a_1 + \frac{a_2}{p} + a_3 P\right) + \frac{(\Delta_4-\Delta_3)}{4} + C_s + \frac{(\Delta_6-\Delta_5)}{4} + \left(C_r + \frac{(\Delta_8-\Delta_7)}{4}\right)p_2\Bigg]y + \left(C_h + \frac{(\Delta_{10}-\Delta_9)}{4}\right)\Bigg[\frac{y(1-p_1-p_2)}{\theta} - \frac{D}{\theta^2}\ln\left(\frac{P}{D}e^{\frac{y}{p}\theta} - \left(\frac{P-D}{D}\right) - \frac{y}{D}\theta(p_1+p_3)e^{\theta\frac{y}{x}}\right)\Bigg] + \frac{1}{2BD}\left(C_b + \frac{(\Delta_{12}-\Delta_{11})}{4}\right)(I_b)^2 + \frac{C_l + \frac{1}{4}(\Delta_{14}-\Delta_{13})(1-B)}{B}I_b\Bigg\}, \tag{19}$$

where I_b is defined in Equation (11).

The pollution factor represents the value that correlates the number of pollutants delivered in the environment during the process related to the discharge of that pollutant. It is defined as the mass of a pollutant divided by a unit mass, volume, distance, or period of action that outcomes a pollutant. According to IPCC 2006 guidelines for National Greenhouse Gas inventories (IPCC, 2008), estimates of pollutants are given by the formula: *Total emission* = (*amount of product produced*) × (*pollution factor*).

The amount of activity is calculated in tonnes/year, and the emission factor is dimensionless. The pollution control cost consists of two essential factors, i.e., capital and operating costs. The capital cost is like inventory models' setup covering the cost of space (own or rented), setting up a pollution control plant, and machinery to rework before starting production. These costs are assumed independent of the number of production cycles and invested one time. Operation and maintenance costs are related to the upkeep of treatment facilities, record keeping, and obsolescence costs.

The following notations are used for mathematical modeling of pollution control scenarios: μ_0 = pollution factor; C_1 = pollution control cost ($/unit); C_2 = capital cost for pollution control per production run ($/unit); C_3 = operating and maintenance cost for pollution control per unit of production quantity ($/unit).

The following assumptions are made to build the model.

(1) The pollution control cost is the sum of two pollution control costs: setup and maintenance. The setup cost is generally stable.

(2) The operating and maintenance cost of pollution per production unit is taken constant and further assumed to be independent of time.

In this model, it is assumed that pollution can be controlled partially. It is observed that a fraction σ of the pollutant quantity automatically becomes less intense and disappears gradually. The reason for this is evaporation, decay, chemical reaction, biological decomposition (Mukhopadhyay and Goswami, 2014). Thus, the remaining fraction of the pollutant, i.e., $(1-\sigma)$, is considered only for treatment. In this model, it is assumed that treatment policy cannot be applied to all parts of the pollutant as the quantity of the contaminants is not under control. Therefore, the treatment can only be done on the remaining portion of the contaminants. Let $\rho(t)$ denote the number of contaminants accumulated at time t. Variations in the number of pollutants with time can be represented with the following equation's help (Mukhopadhyay and Goswami, 2014):

$$\dot{\rho} = \rho_0 - \sigma\rho \text{, with the initial condition: } \rho(0) = 0 \tag{20}$$

With the help of the initial state, the solution of the above equation can be represented as:

$$\rho(t) = \frac{\rho_0}{\sigma}\left(1-e^{-\sigma t}\right) \tag{21}$$

The above equation gives the number of pollutants accumulated at time t. Thus, the amount of contaminants produced during the whole production process is $\sigma(t_p)$, where t_p is the duration of the production.

Consider C_2 and C_3 are fuzzy and expressed by triangular fuzzy numbers $\tilde{C}_2$ and $\tilde{C}_3$, respectively. Let $\tilde{C}_2 = (C_2 - \Delta_{17}, C_2, C_2 + \Delta_{18})$, $\tilde{C}_3 = (C_3 - \Delta_{19}, C_3, C_3 + \Delta_{19})$, where $0 < \Delta_{17} < C_2$, $0 < \Delta_{19} < C_3$ and $0 < \Delta_{18}, \Delta_{20}$. The fuzzy pollution cost is as follows:

$$\tilde{C}_1 = \tilde{C}_2 + \rho_0 \left(\frac{y}{P} - \frac{\sigma}{2} \left(\frac{y}{P} \right)^2 \right) \tilde{C}_3 y \tag{22}$$

On defuzzifying with the help of the Sign-distance method, the following expression is found. Now, the pollution prevention cost is given by:

$$C_1' = C_2 + \frac{(\Delta_{18} - \Delta_{17})}{4} + \rho_0 \left(C_3 + \frac{(\Delta_{20} - \Delta_{19})}{4} \right) \left(\frac{y}{P} - \frac{\sigma}{2} \left(\frac{y}{P} \right)^2 \right) \tag{23}$$

Hence, the total cost per cycle including pollution control:

$$\begin{aligned} P_1(P,T)^* &= C_T(P,T)^* + \frac{C_1'}{T} \\ &= \frac{1}{T} \Bigg\{ C_o + \frac{(\Delta_2 - \Delta_1)}{4} + \Bigg[\left(a_1 + \frac{a_2}{p} + a_3 P \right) + \frac{(\Delta_4 - \Delta_3)}{4} \\ &\quad + C_s + \frac{(\Delta_6 - \Delta_5)}{4} + \left(C_r + \frac{(\Delta_8 - \Delta_7)}{4} \right) p_2 \Bigg] y \\ &\quad + \left(C_h + \frac{(\Delta_{10} - \Delta_9)}{4} \right) \Bigg[\frac{y(1 - p_1 - p_2)}{\theta} \\ &\quad - \frac{D}{\theta^2} \ln \left(\frac{P}{D} e^{\frac{y}{p}\theta} - \left(\frac{P-D}{D} \right) - \frac{y}{D} \theta (p_1 + p_3) e^{\theta \frac{y}{x}} \right) \Bigg] \\ &\quad + \frac{1}{2BD} \left(C_b + \frac{(\Delta_{12} - \Delta_{11})}{4} \right) (I_b)^2 \\ &\quad + \frac{\left(C_l + \frac{(\Delta_{14} - \Delta_{13})}{4} \right)(1-B)}{B} I_b + C_2 + \frac{(\Delta_{18} - \Delta_{17})}{4} \\ &\quad + \rho_0 \left(C_3 + \frac{(\Delta_{20} - \Delta_{19})}{4} \right) \left(\frac{y}{P} - \frac{\sigma}{2} \left(\frac{y}{P} \right)^2 \right) \Bigg\} \end{aligned} \tag{24}$$

where I_b is given in Equation (11).

The optimization model can be defined as:

$$\begin{aligned} &\text{Min } P_1(P,T)^* \\ &\text{subject to} \\ &P_{min} \le P \le P_{max} \text{ and } T > 0 \end{aligned} \tag{25}$$

Theorem 1: $P_1(P, T)^*$ is convex if the following conditions hold:

(i) $\frac{a_3}{a_2} > \left(\alpha P + \frac{\beta}{P} \right)^2$

(ii) $\left\{ \left(C_l + \frac{(\Delta_{14} - \Delta_{13})}{4} \right)(1-B)D + \left(C_b + \frac{(\Delta_{12} - \Delta_{11})}{4} \right) T \right\} T > -\frac{1}{\theta} \ln \left[1 + \frac{\theta y}{D} - \frac{y\theta(p_1 + p_3)}{D} \left(1 + \frac{y\theta}{x} \right) \right] \left\{ \left(C_l + \frac{(\Delta_{14} - \Delta_{13})}{4} \right)(1-B)D + \frac{D}{\theta} \left(C_h + \frac{(\Delta_{10} - \Delta_9)}{4} \right) \right\}$

(iii) $\alpha = \frac{\beta}{P^2}$

Proof: See **Appendix 1**.

Now, Model 1 is extended by considering three pollution control mechanisms as described in the follow sections.

3.1.1. Implementation of Model 1 for Pollution Cap Mechanism

A pollution cap is a constraint imposed on the company through the government on certain pollutants. Its motive is to compel companies to find innovative methods of pollution control. Suppose μ_{Cap} (ton/unit time) denotes a pollution cap. The optimization model under this mechanism can be defined as:

$$\begin{aligned} &\text{Min } P_1(P,T)^* \\ &\text{subject to} \\ &P_{min} \le P \le P_{max}, T > 0 \text{ and } \mu(t_p) \le \mu_{Cap} \end{aligned} \tag{26}$$

3.1.2. Implementation of Model 1 for Pollution Cap and Trade Mechanism

Cap and trade is a top-rated government regulatory program applied to control, or cap, the number of pollutants generated as a by-product of industrial activities. Companies that surplus the cap are taxed, while companies that cut their contaminants may sell or purchase unused credits. In 2005, the European Union (EU) started the world's first international cap and trade policy to reduce pollutants (European Commission). Suppose s_1 and s_2 denotes selling price and purchasing price per unit of emission generated. The optimization model under this mechanism can be defined as:

$$\begin{aligned} &\text{Min} \begin{cases} P_1(P,T)^* + s_1 \left\{ \mu(t_p) - \mu_{Cap} \right\}, \text{ when } \mu(t_p) > \mu_{Cap} \\ P_1(P,T)^* - s_2 \left\{ \mu_{Cap} - \mu(t_p) \right\}, \text{ when } \mu(t_p) < \mu_{Cap} \end{cases} \\ &\text{subject to} \\ &P_{min} \le P \le P_{max} \text{ and } T > 0 \end{aligned} \tag{27}$$

3.1.3. Implementation of Model 1 for Pollution Tax Mechanism

A pollution tax is a fixed price imposed by the government on the number of pollutants generated in the industries' production process. It aimed to control the consumption of fossil fuels and emphasize initiatives to adopt environmentally friendly alternates. Pollution tax has been executed in various countries all over the world. The first country which implemented a pollution tax was Finland in 1990. Suppose π (\$/ton) denote a pollution tax per unit of emission generated. The optimization model under this mechanism can be defined as:

$$\text{Min } P_1\left(P,T\right)^* + \pi\mu\left(t_p\right)$$
subject to
$$P_{min} \le P \le P_{max} \text{ and } T > 0 \tag{28}$$

3.2. Model 2: Production Model with Constant Pollution Control Cost

This model assumes that the pollution removal/treatment cost is independent of time but depends on quantity like scraps, junks, and sewage. All the pollutants produced during the production process are under control and usable for the treatment process (Mukhopadhyay and Goswami, 2014). Here, the operating and maintenance cost of pollution per production unit is taken constant and further assumed to be independent of time.

The pollution control cost in this model is defined as:

$$C_1^* = C_2 + \frac{\left(\Delta_{18} - \Delta_{17}\right)}{4} + \mu_0\left(C_3 + \frac{\left(\Delta_{20} - \Delta_{19}\right)}{4}\right)y \tag{29}$$

Total cost per cycle, including pollution prevention cost, can then be

$$\begin{aligned} P_2\left(P,T\right)^* &= C_T\left(P,T\right)^* + \frac{C_1^*}{T} \\ &= \frac{1}{T}\Bigg\{C_o + \frac{\left(\Delta_2 - \Delta_1\right)}{4} + \Bigg[\left(a_1 + \frac{a_2}{P} + a_3 P\right) + \frac{\left(\Delta_4 - \Delta_3\right)}{4} \\ &\quad + C_s + \frac{\left(\Delta_6 - \Delta_5\right)}{4} + \left(C_r + \frac{\left(\Delta_8 - \Delta_7\right)}{4}\right)p_2\Bigg]y \\ &\quad + \left(C_h + \frac{\left(\Delta_{10} - \Delta_9\right)}{4}\right)\Bigg[\frac{y\left(1 - p_1 - p_2\right)}{\theta} \\ &\quad - \frac{D}{\theta^2}\ln\left(\frac{P}{D}e^{\frac{y}{P}\theta} - \left(\frac{P-D}{D}\right) - \frac{y}{D}\theta\left(p_1 + p_3\right)e^{\theta\frac{y}{x}}\right)\Bigg] \\ &\quad + \frac{1}{2BD}\left(C_b + \frac{\left(\Delta_{12} - \Delta_{11}\right)}{4}\right)\left(I_b\right)^2 \\ &\quad + \frac{\left(C_l + \frac{\left(\Delta_{14} - \Delta_{13}\right)}{4}\right)\left(1 - B\right)}{B} I_b + C_2 + \frac{\left(\Delta_{18} - \Delta_{17}\right)}{4} \\ &\quad + \mu_0\left(C_3 + \frac{\left(\Delta_{20} - \Delta_{19}\right)}{4}\right)y\Bigg\} \end{aligned} \tag{30}$$

where I_b and $C_T(P, T)^*$ are taken from Equations (11) and (19), respectively.

The optimization model can be defined as:

$$\text{Min } P_2\left(P,T\right)^*$$
subject to
$$P_{min} \le P \le P_{max} \text{ and } T > 0 \tag{31}$$

Theorem 2: $P_2(P, T)^*$ is convex if the following conditions hold:

(i) $\frac{a_3}{a_2} > \left(\alpha P + \frac{\beta}{P}\right)^2$

(ii) $\left\{\left(C_l + \frac{\left(\Delta_{14} - \Delta_{13}\right)}{4}\right)\left(1 - B\right)D + \left(C_b + \frac{\left(\Delta_{12} - \Delta_{11}\right)}{4}\right)T\right\}T > -\frac{1}{\theta}\ln\left[1 + \frac{\theta y}{D} - \frac{y\theta\left(p_1 + p_3\right)}{D}\left(1 + \frac{y\theta}{x}\right)\right] \left\{\left(C_l + \frac{\left(\Delta_{14} - \Delta_{13}\right)}{4}\right)\left(1 - B\right)D + \frac{D}{\theta}\left(C_h + \frac{\left(\Delta_{10} - \Delta_9\right)}{4}\right)\right\}$

(iii) $\alpha = \frac{\beta}{P^2}$

Proof: See **Appendix 2**.

3.2.1. Model 3 (Special Case): Production Model without Pollution Control Cost

In this case, it is assumed that there is no pollution control cost. Thus, the objective function is:

$$\text{Min } C_T\left(P,T\right)^*$$
subject to
$$P_{min} \le P \le P_{max} \text{ and } T > 0 \tag{32}$$

Theorem 3: $C_T(P, T)^*$ is convex if the following conditions hold:

(i) $\frac{a_3}{a_2} > \left(\alpha P + \frac{\beta}{P}\right)^2$

(ii) $\left\{\left(C_l + \frac{\left(\Delta_{14} - \Delta_{13}\right)}{4}\right)\left(1 - B\right)D + \left(C_b + \frac{\left(\Delta_{12} - \Delta_{11}\right)}{4}\right)T\right\}T > -\frac{1}{\theta}\ln\left[1 + \frac{\theta y}{D} - \frac{y\theta\left(p_1 + p_3\right)}{D}\left(1 + \frac{y\theta}{x}\right)\right] \left\{\left(C_l + \frac{\left(\Delta_{14} - \Delta_{13}\right)}{4}\right)\left(1 - B\right)D + \frac{D}{\theta}\left(C_h + \frac{\left(\Delta_{10} - \Delta_9\right)}{4}\right)\right\}$

(iii) $\alpha = \frac{\beta}{P^2}$

Proof: See **Appendix 3**.

3.3. Solution Methodology

It is observed that objective function, i.e., $P_1(P, T)^*$ is the non-linear function of P and T. So, the present problem is the non-linear optimization problem. In this case, a closed-form solution is not possible. Thus, the following search algorithm is applied to get the optimal solution:

Step 1: Find different first-order partial derivatives, i.e., $\partial P_1(P, T)^*/\partial T$ and $\partial P_1(P, T)^*/\partial P$.

Step 2: Set $P = 0$, and find the value of T by using Equation A1 and set it as T^i.

Step 3: Use the obtained value of T^i from step 2, in Equation A2 to get the value of P and set it as P^i.

Step 4: Repeat the above steps and get the revised values as $T^{(i+1)}$ and $P^{(i+1)}$.

Step 5: For pre-assigned value $\epsilon > 0$, if $|T^{(i+1)} - T^i| < \epsilon$ and $|P^{(i+1)} - P^i| < \epsilon$ then $T^{(i+1)}$ and $P^{(i+1)}$ are the solutions of Equations A1 and A2.

Step 6: Check the optimality conditions given in Theorem 1 at the obtained values of $T^{(i+1)}$ and $P^{(i+1)}$. If the optimality condition is satisfied, then check:

(i) If, $P_{min} \leq P^{i+1} \leq P_{max}$ then the optimal values of decision variables are T^{i+1} and P^{i+1}.

(ii) If, $P^{i+1} \leq P_{min}$ i.e., the production rate is lower than the acceptable production rate then, the optimal values of decision variables are T^{i+1} and P_{min}.

(iii) If, $P^{i+1} \geq P_{max}$ i.e., the production rate is higher than the acceptable production rate due to which pressure of inventory increases in the system then, the optimal values of decision variables are T^{i+1} and P_{max}.

4. Numerical Example

In this Section, all the models and carbon control mechanisms designed are solved to find the optimal values of P and T values and the system's optimal total cost per cycle.

Case study: A newly emerging textile industry **A** (The name is given **A** as the company does not want to publish the name and others) produces branded clothes for kids. The company invests in pollution control plants to minimize the air and water pollution caused during production and uses a rework policy to minimize solid waste. The company mentioned above provided the numerical data presented in Table 2 for better research insights. In any decision-making situation, the values of cost parameters might change due to market uncertainties. Hence, the values of cost parameters are treated as 5% uncertain with the help of a triangular fuzzy number. Proceeding in this way, all the models along with emission control policies are solved. It is given that the overall air pollution factor of the company is 0.02 and it emits $\mu_0 y = 1.8$ ton/year.

The cost functions of the three models, obtained from Equations (24), (30), and (32), are incredibly non-linear. So, it is very tedious to get the convexity with the help of calculus. Therefore, global optimal solutions for all proposed models are obtained by using MATHEMATICA 11.3. For the convexity of the cost functions corresponding to Model 1, Model 2, and Model 3, three important theorems are derived in Section 3. It is presented numerically through Table 3 and graphically by Figure 3, respectively. Here, the objective is to find the point (P, T) on the curve of the objective function where the total inventory cost is minimum, i.e., a slight change at that point increases the total inventory cost of the system. Thus, point (P^*, T^*) on the curve (Figure 3) gives the global minimum value of the total inventory cost, implying that slightly changing the value of (P^*, T^*) causes the system's total inventory cost to rise.

Table 3 presents the optimal policies for Model 1, Model 2, and Model 3 and answers the first research question. This table provides the optimal production rate, cycle time, and total pollutant quantities in different cases. It indicates that:

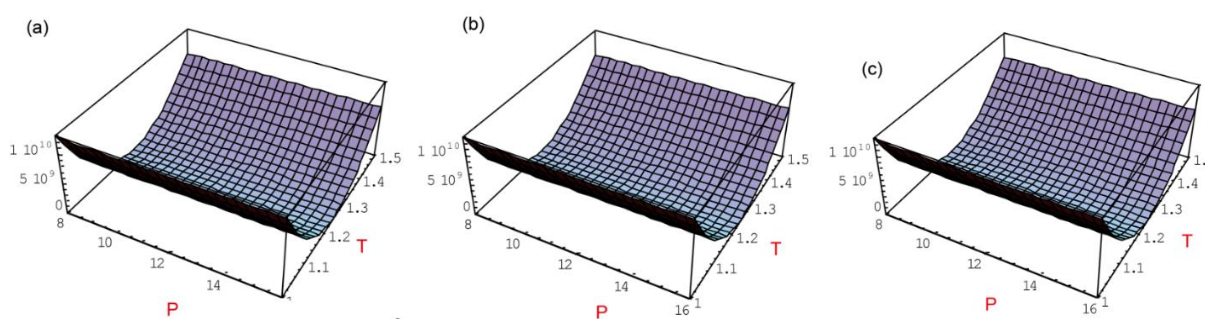

Figure 3. The convexity illustration for (a) $P_1(P,T)^*$ (Model 1), (b) $P_2(P,T)^*$ (Model 2) and (c) $C_T(P,T)^*$ (Model 3).

Table 2. Data for Numerical Analysis

$C_0 = 100$ (\$/setup)	$a_1 = 5$ (\$/unit)	$a_2 = 400$ (\$/unit)	$a_3 = 0.001$ (\$/unit)
$C_s = 0.5$ (\$/unit)	$C_h = 2$ (\$/unit)	$C_b = 5$ (\$/unit)	$C_l = 2$ (\$/unit)
$C_r = 1.5$ (\$/unit)	$s_2 = 8$ (\$/unit)	$D = 500$	$\mu_{Cap} = 1.32$
$x = 1750$ (unit/cycle)	$B = 0.40$	$\theta = 0.01$	$p_1 = 0.2$
$p_2 = 0.2$	$p_3 = 0.1$	$y = 900$ (unit)	$\rho = 0.2$
$C_2 = 30$ (\$/setup)	$C_3 = 4$ (\$/unit)	$\mu_0 = 0.002$	$\sigma = 0.02$
$\pi = 6$ (\$/ton)	$P_{\min} = 5$ (unit/cycle)	$P_{\max} = 14$ (unit/cycle)	

Table 3. Optimization Results for Models with or without Pollution Control Costs

Model	Total cost per cycle	Rate of production (P^*, unit/cycle)	Rate of production (variable)	Total cycle time (T^*, year)	Convexity of objective function	The total amount of pollutants (tonne/year)
1	6124.80	12.7121	629.576	1.24936	$\left(\frac{\partial^2 P_1(P,T)}{\partial P^2}\right)_{(P,T)} = 5.730 > 0$ $\left(\frac{\partial^2 P_1(P,T)}{\partial P^2}\frac{\partial^2 P_1(P,T)}{\partial T^2} - \left(\frac{\partial^2 P_1(P,T)}{\partial P \partial I_b}\right)^2\right)_{(P,T)}$ $= 78560.312 > 0$	1.34079
2	6133.68	12.6542	632.454	1.24936	$\left(\frac{\partial^2 P_2(P,T)}{\partial P^2}\right)_{(P,T)} = 5.681 > 0$ $\left(\frac{\partial^2 P_2(P,T)}{\partial P^2}\frac{\partial^2 P_2(P,T)}{\partial T^2} - \left(\frac{\partial^2 P_2(P,T)}{\partial P \partial T}\right)^2\right)_{(P,T)}$ $= 134158.122 > 0$	1.8
3	6081.07	12.6542	632.454	1.24936	$\left(\frac{\partial^2 C_T}{\partial P^2}\right)_{(P,T)} = 5.686 > 0$ $\left(\frac{\partial^2 C_T}{\partial P^2}\frac{\partial^2 C_T}{\partial T^2} - \left(\frac{\partial^2 C_T}{\partial T^2}\right)^2\right)_{(P,T)} = 133442.276 > 0$	-

(1) The total annual costs per cycle for Models 1 and 2 are 0.72 and 0.87%. These are than the total yearly cost per cycle for Model 3 due to extra pollution control costs in Models 1 and 2.

(2) The production rate remains the same in Models 2 and 3, but it rises 0.46% in Model 1.

(3) Optimal backlogging quantity remains almost the same in all three models.

(4) The amount of pollutants in Model 1 is 25.51% less than in Model 2, which is remarkable.

It is concluded from Table 3 that Models 1 and 2 are excellent for sustainable production and a better approach towards a green environment. Moreover, Model 1 appeals more towards the environment and fulfills all three aspects of sustainability. Thus, Model 1 is the answer to the second research question raised in the paper. Therefore, Model 1 is recommended for further application of pollution control scenarios.

Table 4 shows the effects of different pollution control mechanisms on optimal policy for Model 1 and hence answers the third research question raised in this paper. It is observed that:

(1) The amount of pollutants in the model with variable pollution control cost is comparatively (25.51%) less than in the model with constant pollution control cost. All three pollution control mechanisms: pollution cap, pollution cap and trade, and pollution tax reduce pollutants by 34.25, 1.09, and 0.67%. It proves the significance of these mechanisms and answers the fourth research question raised in the paper.

(2) All three pollution control mechanisms: pollution cap, pollution cap and trade, and pollution tax raise the production rate by 0.88, 0.66, and 55.41%, respectively, and also increase the total cost of the system.

(3) The pollution cap mechanism reduces the pollution significantly, but the system's total inventory cost per cycle and production rate increase due to this mechanism. However, this policy can be applied because customers nowadays are very responsible and ready to buy eco-friendly products.

(4) The pollution cap and trade mechanism is preferable for controlling pollution. Because in comparison to all the three pollution control policies, the inventory cost corresponding to this mechanism is lower. It reduces the number of pollutants effectively. It addresses the fifth research question asked in the paper.

Table 4. Comparison Table for Optimal Results of Different Pollution Control Mechanisms Concerning Model 1

Model	Total cost per cycle	Rate of production (P^*, unit/cycle)	Total cycle time (T^*, year)	The total amount of pollutants (tonne/year)
1	6124.80	12.7121	1.24936	1.34079
3.1.1 (cap)	6214.94	19.7557	1.24936	0.87999
3.1.2 (cap and trade)	6133.35	12.8245	1.24936	1.32969
3.1.3 (tax)	6129.29	12.7965	1.24936	1.33244

5. Sensitivity Analysis

To obtain more insights, the robustness of the proposed work is examined concerning the crucial inventory parameters of Model 1, as it is the more general form of Model 2 and Model 3.

5.1. Sensitivity Corresponding to Model 1

From Figures 4(a) to 4(g), we have:

1) When the fuzziness level increases by 30%, the total cost will increase by 6,762.99 though P decreases by 12.5372. The total cost per cycle is highly sensitive to increased impreciseness. Therefore, inventory managers should keep on making efforts to minimize market impreciseness. For this, they could do market surveys regularly or take an expert's opinions to enhance learning.

2) It is observed that change in fuzziness has no impact on T.

3) With a hike in the pollution control costs C_2 and C_3, the system's total cost per cycle increases, which is quite apparent, but this increment is significantly less. Although P downturns noticeably with increasing operating and maintenance cost of pollution control (C_3). Therefore, it is advised to apply pollution control policies to ensure cleaner production practices.

4) The rise in the value of pollution factors (μ_0) increases the total cost per cycle and a drop in P. It is because an increase in the pollution factor needs to increase pollution control cost, resulting in an increased overall cost of the organization. Hence, it is suggested that inventory managers focus on rework and waste-reducing practices.

5) As a fraction of pollutant (σ) rises over time, the total cost per cycle and P fall remarkably. It is recommended to promote some biological decomposition methods to automatically diminish the fraction of pollutants.

6) As a fraction of fulfilled shortage demand (B) increases from 0.1 to 0.3, the total cost per cycle increases promptly. As B increases from 0.3 to 0.7, the total cost per cycle rises gradually. On the other hand, the absolute shortage decreases sharply with an increase in B. Shortages result in monetary loss and may cause a company's loss of goodwill. Thus, it is beneficial to balance overproduction and lack of production.

7) When production lot size (y) rises from 500 to 1,400, the total cost per cycle rises rapidly from 5,768.98 to 6,605.82. On the other hand, P decreases sharply due to an increment in y.

8) Since total cost per cycle and production rate is highly sensitive to lot size and degree of impreciseness, inventory managers should emphasize these essential parameters while designing inventory-related strategies.

5.2. Sensitivity Corresponding to Model 3.1.1 Concerning A Pollution Tax

From Figure 4(h), we have:

1) On increasing the pollution tax (π) from 6 to 30, the total amount of pollution of the system reduces from 1.3626 to 1.30045, but due to this value, the total cost per cycle increases from 6,129.29 to 6,146.90.

2) It is noted that an increase in pollution tax (π) resulted in a rise in P with no change in T.

5.3. Sensitivity Corresponding to Model 3.1.2 Concerning the Selling Price of Pollution Credit

From Figure 4(i), we have:

1) When the selling price (s_2) of pollution credit increases from 4 to 18; the total cost per cycle slowly increases from 6,129.08 to 6,143.93. On the other hand, it significantly reduces the total amount of pollution of the system. Thus, it is observed that this mechanism shows better performance than the pollution tax mechanism.

2) It is also noted that an increase in selling price (s_2) resulted in an increase of P with no change in T.

5.4. Sensitivity Corresponding to Model 3.1.3 Concerning Pollution Cap

From Figure 4(j), we have:

1) On decreasing the pollution cap from 0.45 to 0.25, the total cost per cycle increases remarkably from 6,179.13 to 6,239.98, and P increases from 17.9193 to 20.9025. Then, it is concluded that this mechanism could lower the number of pollutants significantly, but it creates extra monetary liabilities for the organizations.

2) By comparing and analyzing three mechanisms, it is found that this mechanism reduces pollutants fast as μ_{Cap} is highly sensitive to the number of pollutants.

3) Although this mechanism increases the total cost per cycle while imposing a pollution cap, it is expected that decision-makers should be responsible enough/ethical to restrict the amount of pollution.

5.5. Managerial Insights and Industry Implication

The results present the strategy for properly managing defectives and pollution control in the manufacturing system with partial backlogging and rework. The design with minimum cost, minimum emission, and least waste generation presents the optimal solution. This study has the following insights.

Flexible manufacturing helps industrialists ensure fast production, reduced storage cost, and improved customer satisfaction.

- Implementing various pollution prevention mechanisms and pollution control costs is a step towards environmental sustainability. It helps industries get their trade license to renew easily.
- The application of fuzzy theory covers market impreciseness and helps inventory managers to decide optimal policies.
- Rework of defectives decreases energy usage and lowers the amount of waste that causes landfills. It could be beneficial to the industry economically.
- This study provides three models with or without pollution control costs. However, the model's total cost is minimal without pollution control cost. Because it has no pollution check parameter, if some industry adopts this model, it may have to face the penalty for emitting pollution. Even then, it is not economical to use.
- The model with variable pollution control cost is suggested for the industrial managers, as it lowers the emission significantly and is economical.

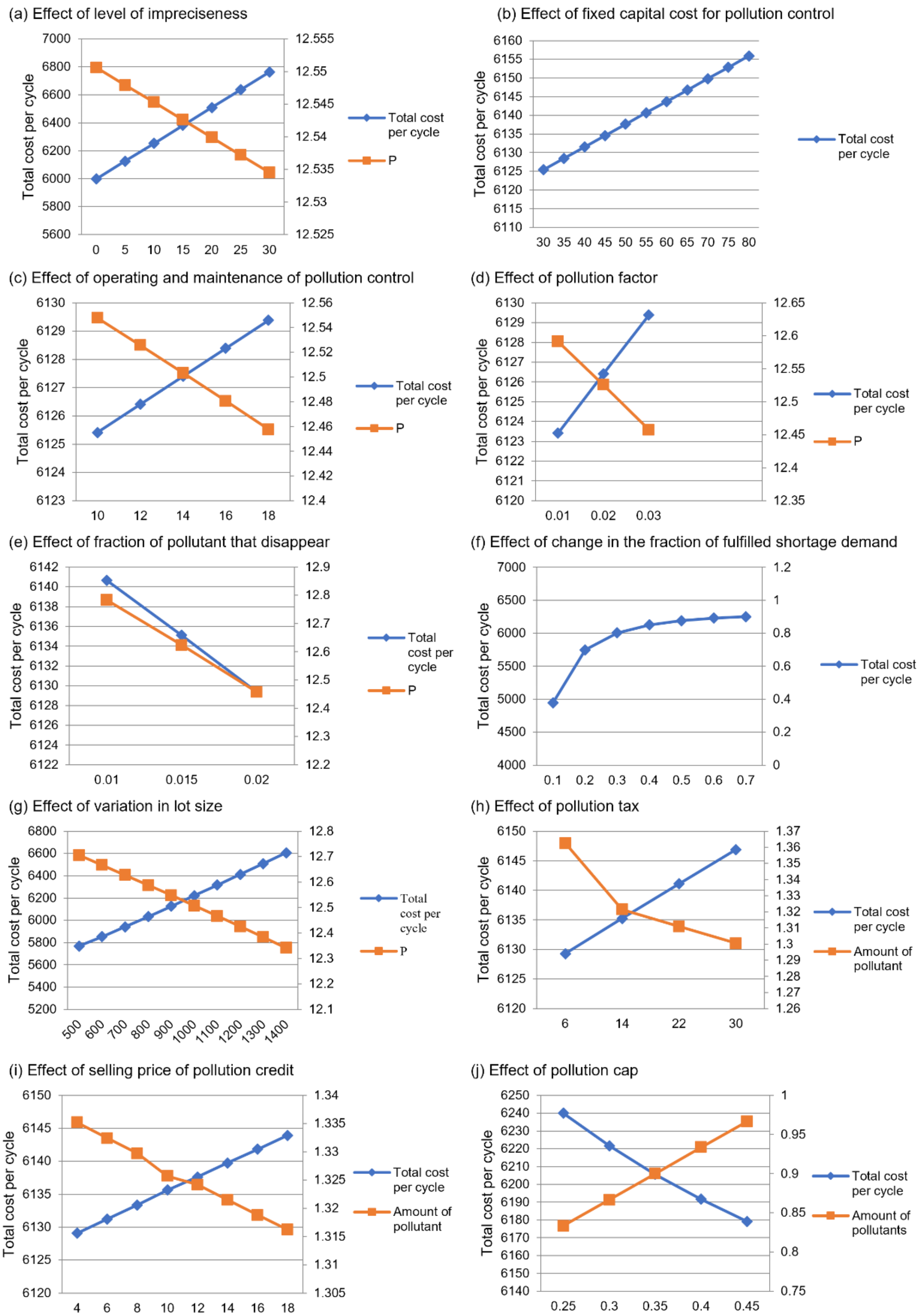

Figure 4. Effects of different key parameters on optimal solution.

- Although implementing pollution control policies raises the system's total cost, results prove that carbon cap and trade policy is best for the industry's economic and environmental sustainability.
- Since total cost per cycle and production rate is highly sensitive to lot size and degree of impreciseness, inventory managers should emphasize these essential parameters while designing inventory-related strategies.

6. Conclusions

For the production department of various industries, the management of defectives, solid waste, and air pollution is a big challenge. The government's enormous pressure and customer wakefulness forced manufacturers to adopt sustainable production practices. Keeping the thinking of sustainable manufacturing, this study aimed to provide policymakers insights to deal with pollution control scenarios and optimize the production rate compared to when pollution costs are not considered.

The numerical example and sensitivity analysis results reveal that among all three models, the models with pollution control costs are better than the traditional ones. Model 1 with variable pollution control cost is the best as it downturns the number of pollutants to 25.51%. On applying all three designed mechanisms: pollution cap, pollution cap and trade, and pollution tax, the number of contaminants decrease by 34.25, 1.09, and 0.67%, respectively, while the total cost per cycle increases by 0.0053, 0.0004, and 0.0011% respectively. Hence, it is visible that the pollution cap mechanism is the best policy to reduce pollutants. Nevertheless, the pollution cap and trade mechanism reduces pollutants and is economical to use. Therefore, it is the most favorable policy for environmental consciousness, brand reputation, and customer satisfaction. The sensitivity results highlight the application of fuzzy set theory to achieve sustainability targets. This study addressed all the research questions raised in the paper and solved the research's purpose. Further, it may be applied to chemical industries, mobile companies, textile industries, and many other industries. It could be helpful for industrial managers to decide optimal policy for their company that may ensure all the three aspects of sustainable production, i.e., economic, social, and environment. It contributes to covering the research gap in sustainable production models with the flexible production rate by incorporating pollution control cost and pollution control mechanisms together under uncertainty.

However, this study has some limitations. The effect of controllable production rate and automated screening system needs more rigorous demonstration. The results of this work would be more appealing if the industry's profit is observed in a centralized supply chain scenario under the different models presented (Ahmed et al., 2021; Gupta et al., 2021; Saha et al., 2021). The effects of inflation, learning in fuzziness, recycling, error in screening, inflation, and delay in payments could be studied in the future (Bhuniya et al., 2021b; Kumar et al., 2021b; Sepehri et al., 2021). The fuzzy lock set approach (Karmakar et al., 2017; De and Mahata, 2021) could be a better option for extending this work.

Appendix 1

Proof: First of all, different partial derivatives are found with respect to decision variables. Necessary conditions for optimality are:

$$\frac{\partial P_1(P,T)^*}{\partial P}=0 \tag{A1}$$

$$\frac{\partial P_1(P,T)^*}{\partial T}=0 \tag{A2}$$

On solving the above two equations as mentioned in Section 3.3, P and T values can be obtained. At these points, sufficient conditions for optimality are provided as follows:

$$\begin{aligned}\left(\frac{\partial^2 P_1(P,T)^*}{\partial P^2}\right)_{(P,T)} = & \frac{2y}{TP^3\left(\alpha P+\frac{\beta}{P}\right)^3}\Bigg\{\beta\left(\alpha P+\frac{\beta}{P}\right) \\ & \cdot\left[a_3\left(\alpha P+\frac{\beta}{P}\right)^2-a_2\right]+a_2P^3\left(\alpha-\frac{\beta}{P^2}\right)^2\Bigg\} \\ & +\rho_0\left(C_3+\frac{(\Delta_{20}-\Delta_{19})}{4}\right)\frac{y}{P^3} \\ & \cdot\left(2-\frac{3\sigma y}{P}\right)\end{aligned} \tag{A3}$$

if:

$$\frac{a_3}{a_2}>\left(\alpha P+\frac{\beta}{P}\right)^2 \tag{A4}$$

and:

$$\frac{3\sigma y}{P}<2\,; \tag{A5}$$

it implies that:

$$\left(\frac{\partial^2 P_1(P,T)^*}{\partial P^2}\right)_{(P,T)}>0 \tag{A6}$$

Further:

$$\frac{\partial^2 P_1(P,T)^*}{\partial T^2}=-\frac{2}{T^3}A_2+A_1=-\frac{2}{T^3}A_2+A_1>0 \tag{A7}$$

if $A_2 < 0$ and A_1 is the sum of all positive terms, where:

$$A_2=\left[\left(C_l+\frac{(\Delta_{14}-\Delta_{13})}{4}\right)(1-B)D+\left(C_b+\frac{(\Delta_{12}-\Delta_{11})}{4}\right)T\right]T$$
$$+\frac{1}{\theta}\ln\left[1+\frac{\theta y}{D}-\frac{y\theta(p_1+p_3)}{D}\left(1+\frac{y\theta}{x}\right)\right]$$
$$\cdot\left[\left(C_l+\frac{(\Delta_{14}-\Delta_{13})}{4}\right)(1-B)D\right.$$
$$\left.+\frac{D}{\theta}\left(C_h+\frac{(\Delta_{10}-\Delta_9)}{4}\right)\right]. \quad \text{(A8)}$$

Now:

$$\left(\frac{\partial^2 P_1(P,T)^*}{\partial P^2}\frac{\partial^2 P_1(P,T)^*}{\partial T^2}-\left(\frac{\partial^2 P_1(P,T)^*}{\partial P\partial T}\right)^2\right)_{(P,T)}$$
$$=\left\{\frac{2y\beta}{TP^3\left(\alpha P+\frac{\beta}{P}\right)^2}\left[a_3\left(\alpha P+\frac{\beta}{P}\right)^2-a_2\right]\right.$$
$$\left.+\frac{\rho_0}{T}\left[C_3+\frac{(\Delta_{20}-\Delta_{19})}{4}\right]\left(\frac{2y}{P^3}-\frac{3\sigma y^2}{P^4}\right)\right\}\cdot\left(-\frac{2}{T^3}A_2+A_1\right) \quad \text{(A9)}$$

if:

$$\left\{a_3\left(\alpha P+\frac{\beta}{P}\right)^2-a_2\right\}>0 \quad \text{(A10)}$$

and:

$$\frac{3\sigma y}{P}<2 \quad \text{(A11)}$$

where A_1 is the sum of positive terms and $A_2<0$. Then we have:

$$\left(\frac{\partial^2 P_1(P,T)^*}{\partial P^2}\frac{\partial^2 P_1(P,T)^*}{\partial T^2}-\left(\frac{\partial^2 P_1(P,T)^*}{\partial P\partial T}\right)^2\right)_{(P,T)}>0 \quad \text{(A12)}$$

Appendix 2

Proof: First of all, several different partial derivatives are calculated with respect to decision variables. Necessary conditions for optimality are:

$$\frac{\partial P_2(P,T)^*}{\partial P}=0 \quad \text{(A13)}$$

$$\frac{\partial P_2(P,T)^*}{\partial T}=0 \quad \text{(A14)}$$

On solving the above equations, the optimum values (P^*, T^*) can be found. Now, the sufficient condition for optimality can be verified:

$$\left(\frac{\partial^2 P_2(P,T)^*}{\partial P^2}\right)_{(P,T)}=\frac{2y}{TP^3\left(\alpha P+\frac{\beta}{P}\right)^3}\left\{\beta\left(\alpha P+\frac{\beta}{P}\right)\right.$$
$$\left.\cdot\left[a_3\left(\alpha P+\frac{\beta}{P}\right)^2-a_2\right]+a_2P^3\left(\alpha+\frac{\beta}{P^2}\right)^2\right\} \quad \text{(A15)}$$

if:

$$\frac{a_3}{a_2}>\left(\alpha P+\frac{\beta}{P}\right)^2; \quad \text{(A16)}$$

it implies that:

$$\left(\frac{\partial^2 P_2(P,T)^*}{\partial P^2}\right)_{(P,T)}>0 \quad \text{(A17)}$$

Further:

$$\frac{\partial^2 P_2(P,T)^*}{\partial T^2}=-\frac{2}{T^3}A_2+A_1=-\frac{2}{T^3}A_2+A_1>0 \quad \text{(A18)}$$

if: $A_2<0$ and A_1 is the sum of all positive terms, where:

$$A_2=\left\{\left[C_l+\frac{(\Delta_{14}-\Delta_{13})}{4}\right](1-B)D+\left[C_b+\frac{(\Delta_{12}-\Delta_{11})}{4}\right]T\right\}T$$
$$+\frac{1}{\theta}\ln\left[1+\frac{\theta y}{D}-\frac{y\theta(p_1+p_3)}{D}\left(1+\frac{y\theta}{x}\right)\right]$$
$$\cdot\left\{\left[C_l+\frac{(\Delta_{14}-\Delta_{13})}{4}\right](1-B)D\right.$$
$$\left.+\frac{D}{\theta}\left[C_h+\frac{(\Delta_{10}-\Delta_9)}{4}\right]\right\} \quad \text{(A19)}$$

Now:

$$\left(\frac{\partial^2 P_2(P,T)^*}{\partial P^2}\frac{\partial^2 P_2(P,T)^*}{\partial T^2}-\left(\frac{\partial^2 P_2(P,T)^*}{\partial P\partial T}\right)^2\right)_{(P,T)}$$
$$=\frac{2y}{TP^3\left(\alpha P+\frac{\beta}{P}\right)^3}\left\{\beta\left(\alpha P+\frac{\beta}{P}\right)\left[a_3\left(\alpha P+\frac{\beta}{P}\right)^2-a_2\right]\right.$$
$$\left.\cdot\left(-\frac{2}{T^3}A_2+A_1\right)\right\}>0 \quad \text{(A20)}$$

if:

$$\left\{a_3\left(\alpha+\frac{\beta}{P}\right)^2-a_2\right\}>0 \quad \text{(A21)}$$

where, A_1 is the sum of all positive terms and $A_2 < 0$ then:

$$\left(\frac{\partial^2 P_2(P,T)^*}{\partial P^2}\frac{\partial^2 P_2(P,T)^*}{\partial T^2}-\left(\frac{\partial^2 P_2(P,T)^*}{\partial P\partial T}\right)^2\right)_{(P,T)}>0 \quad \text{(A22)}$$

Appendix 3

Proof: First of all, one can find different partial derivatives with respect to decision variables. Necessary conditions for optimality are:

$$\frac{\partial C_T{}^*}{\partial P}=0 \quad \text{(A23)}$$

$$\frac{\partial C_T{}^*}{\partial T}=0 \quad \text{(A24)}$$

On solving the above equation, the optimum values (P^*, T^*) can be obtained. The sufficient condition of optimality is verified one by one at the point (P^*, T^*):

$$\left(\frac{\partial^2 C_T{}^*}{\partial P^2}\right)_{(P,T)}=\frac{2y}{TP^3\left(\alpha P+\frac{\beta}{P}\right)^3}\left\{\beta\left(\alpha P+\frac{\beta}{P}\right)\left[a_3\left(\alpha P+\frac{\beta}{P}\right)^2-a_2\right]\right.$$
$$\left.+a_2P^3\left(\alpha+\frac{\beta}{P^2}\right)^2\right\} \quad \text{(A25)}$$

if:

$$\frac{a_3}{a_2}>\left(\alpha P+\frac{\beta}{P}\right)^2 \quad \text{(A26)}$$

it implies that:

$$\left(\frac{\partial^2 C_T{}^*}{\partial P^2}\right)_{(P,T)}>0 \quad \text{(A27)}$$

Further:

$$\frac{\partial^2 C_T{}^*}{\partial T^2}=-\frac{2}{T^2}\left\{\left(C_l+\frac{(\Delta_{14}-\Delta_{13})}{4}\right)(1-B)D\right.$$
$$\left.+BD\left(C_b+\frac{(\Delta_{12}-\Delta_{11})}{4}\right)\right\}-\frac{2}{T^3}\left\{\left(C_l+\frac{(\Delta_{14}-\Delta_{13})}{4}\right)\right.$$
$$\cdot(1-B)D\left\{\frac{1}{\theta}\ln\left[1+\frac{\theta y}{D}-\frac{y\theta(p_1+p_3)}{D}\left(1+\frac{y\theta}{x}\right)\right]\right\}$$
$$\left.+\frac{D}{\theta^2}\ln\left[1+\frac{\theta y}{D}-\frac{y\theta(p_1+p_3)}{D}\left(1+\frac{y\theta}{x}\right)\right]\right\}+A_1 \quad \text{(A28)}$$

where A_1 is the sum of all positive terms.

$$\frac{\partial^2 C_T{}^*}{\partial T^2}=-\frac{2}{T^3}A_2+A_1=-\frac{2}{T^3}A_2+A_1>0 \quad \text{(A29)}$$

if $A_2 < 0$, where:

$$A_2=\left\{\left[C_l+\frac{(\Delta_{14}-\Delta_{13})}{4}\right](1-B)D+\left[C_b+\frac{(\Delta_{12}-\Delta_{11})}{4}\right]T\right\}T$$
$$+\frac{1}{\theta}\ln\left[1+\frac{\theta y}{D}-\frac{y\theta(p_1+p_3)}{D}\left(1+\frac{y\theta}{x}\right)\right]$$
$$\cdot\left\{\left[C_l+\frac{(\Delta_{14}-\Delta_{13})}{4}\right](1-B)D\right.$$
$$\left.+\frac{D}{\theta}\left[C_h+\frac{(\Delta_{10}-\Delta_9)}{4}\right]\right\}. \quad \text{(A30)}$$

Now:

$$\left(\frac{\partial^2 C_T{}^*}{\partial P^2}\frac{\partial^2 C_T{}^*}{\partial T^2}-\left(\frac{\partial^2 C_T{}^*}{\partial P\partial T}\right)^2\right)_{(P,T)}=\frac{2y}{TP^3\left(\alpha P+\frac{\beta}{P}\right)^3}\left\{\beta\left(\alpha P+\frac{\beta}{P}\right)\right.$$
$$\cdot\left[a_3\left(\alpha P+\frac{\beta}{P}\right)^2-a_2\right]$$
$$\left.\cdot\left(-\frac{2}{T^3}A_2+A_1\right)\right\} \quad \text{(A31)}$$

if:

$$\left\{a_3\left(\alpha P+\frac{\beta}{P}\right)^2-a_2\right\}>0 \quad \text{(A32)}$$

where A_1 is the sum of positive terms and $A_2 < 0$. Then we have:

$$\left(\frac{\partial^2 C_T{}^*}{\partial P^2}\frac{\partial^2 C_T{}^*}{\partial T^2}-\left(\frac{\partial^2 C_T{}^*}{\partial P\partial T}\right)^2\right)_{(P,T)}>0 \quad \text{(A32)}$$

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