

# A Two-Stage Stochastic Fuzzy Mixed-Integer Linear Programming Approach for Water Resource Allocation under Uncertainty in Ajabshir Qaleh Chay Dam

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**ABSTRACT.** Due to the dry climate and unsuitable distribution of rainfall in Iran, sustainable agriculture depends on the proper use of water resources. In this study, the optimal allocation of water at Ajabshir Qaleh Chay Dam in agricultural sector is investigated using an interval parameter two-stage stochastic mixed-integer linear programming approach. Indeed, interval parameters two-stage stochastic programming (ITSP) with fuzzy variables is developed based on mixed-integer programming for a water resource allocation model to retrieve the water shortage of agricultural products and to achieve the optimal allocation of Ajabshir Qaleh Chay Dam water through its river canals between different products under uncertainty conditions. In this developed method, called extended ITSP (EITSP), a number of alternatives are used to compensate for the difference between the amount of promised water allocation targets and the actual allocated water in the optimal allocation of water. Then a new solving approach based on Huang Algorithm, fuzzy chance constrained programming and Zimmermann fuzzy programming will be presented to solve the problems. Furthermore, using a case study in this dam, the results are obtained for the developed approaches to clarify the described methods and to compare these results with each other. Finally, comparing the total system profits of the models shows that in the fuzzy model, the profit and system certainty increase simultaneously. Therefore, due to the lack of water resources in the agricultural sector and the uncertainty, the agricultural authorities of Ajabshir can decrease the unsustainability of water resources using the optimal model while increasing the cost-effectiveness of the farmers.

**Keywords:** water resources management, two-stage programming, mixed-integer programming, fuzzy chance constrained programming, Ajabshir Qaleh Chay Dam

## 1. Introduction

In recent years, economic and population growth has led to an increase in demand for agricultural and industrial products, leading to more water consumption in the agricultural and industrial sectors. On the other hand, increasing demand for water resources, lack of water supply and development of agricultural, industrial and municipal sectors will require an efficient method for allocating water among consumers. Sustainable water supply plays a vital role in improving food security and socio-economic development of human societies (Singh, 2014).

However, most countries in the world face a water scarcity crisis (Garg and Dadhich, 2014). On the other hand, it can be predicted that water resources will be considered as the source of power in the coming years, and powerful countries have abundant water reserves or advanced water management. Iran has an average annual precipitation of less than 240 mm in the dry and semi-arid zone of the world. Extreme restrictions on existing water resources, and droughts and their continued anticipation

have increased the risk of farmers' livelihood and economic well-being. The optimal design of water-resource management policies will have the potential role to improve water allocations for various uses (Kang and Park, 2014).

Numerous mathematical optimization techniques have been developed to analyze the water resource allocation and environmental systems (Lin et al., 2009; Housh et al., 2013; Fan et al., 2015; Abdulbaki et al., 2017; Huang et al., 2017; Ji et al., 2017; Li et al., 2017; Kong et al., 2018; Li et al., 2018; Veintimilla-Reyes et al., 2018; Xie et al., 2018; Wang et al., 2020). For instance, an inexact two-stage stochastic partial programming method was presented by Fan et al. (2012) as an application to water resources management under uncertainty. Furthermore, an interactive two-stage stochastic fuzzy programming approach was proposed by Wang and Huang (2011) through incorporating an interactive fuzzy resolution method within an inexact two-stage stochastic programming framework. In addition, Niu et al. (2016) developed interactive fuzzy stochastic programming method for supporting crop planning and water allocation under uncertainty in China.

Because of stochastic properties of decision-making problems, the stochastic programming (SP) such as two-stage stochastic programming (TSP) has been applied in various studies. On the other hand, many real system parameters are inexact and

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are determined as interval numbers. Therefore, interval mathematical programming (IMP) is an efficient method to characterize inexact parameters without any distribution information that is always required in SP. The TSP and IMP methods were incorporated into interval-parameter two-stage stochastic programming (ITSP) framework to address random and interval information in environmental management and planning (Maqsood et al., 2005), and ITSP was presented due to these remarkable limitations of TSP.

Sometimes, the parameters of a decision-making problem have some fuzzy properties. Fuzzy mathematical programming (FMP) has been investigated in many environmental decision making problems (Lin et al., 2009; Fan et al., 2015; Li et al., 2017). Furthermore, in many practical situations, the input parameters have both possibilistic imprecision and probability uncertainty together with interval information (Lin et al., 2009; Wang and Huang, 2013; Li and Guo, 2014, 2015; Xie et al., 2018). A remarkable limitation of the aforementioned ITSP method is its incapability in considering the multiple and mixed uncertainties in the parameters of the problem, i.e., combination of interval, probability and possibility distributions. The incorporation of mixed-integer programming (MIP), ITSP and FMP methods leads to a method referred in this paper as fuzzy extended ITSP (abbreviated as fuzzy EITSP), and would be response to the remarkable limitation of ITSP on environmental decision making issues.

The TSP and ITSP are efficient methods for water management issues (Li et al., 2007; Guo and Huang, 2009; Wang and Huang, 2011, 2013; Zeng et al., 2015; Ji et al., 2017; Liu et al., 2017; Fu et al., 2018a, 2018b; Zhang et al., 2018; Zhang and Guo, 2018). In TSP, an initial decision must be made before the values of random variables are known, and then in the second stage after random variables have taken place, a corrective action can be taken in order to minimize penalties that may appear due to any infeasibility. In this approach, the uncertainties of random variables are specified with known probability distributions; however, due to uncertain nature of information, TSP has limitation (Li et al., 2010).

Interval mathematical programming deals with uncertainties expressed as intervals with definite lower and upper bounds. ITSP was presented due to insufficiencies of TSP. However, this method has remarkable limitations in handling possibilistic uncertainty that could be handled by interval-parameter two-stage stochastic fuzzy programming.

FMP is an efficient method to deal with subjective uncertainty in water resources management (Nematian, 2016), and several new methods have been developed to solve the water resources problems with fuzzy parameters. For example, Li and Guo (2014) prepared a hybrid fuzzy-stochastic programming method for planning water trading under uncertainties of randomness and fuzziness. Then they developed a random-fuzzy-variable-based inexact two-stage stochastic chance-constrained programming model for crop area optimization in more efficient and sustainable ways (Li and Guo, 2015). Furthermore, the developed model is helpful for managers in gaining insight into the trade-offs between the system benefit and the constraint-

violation risk. Fan et al. (2015) suggested a generalized fuzzy two-stage stochastic programming (GFTSP) method for planning water resources management systems under uncertainty. The developed GFTSP method can deal with uncertainties expressed as probability distributions, fuzzy sets, as well as fuzzy random variables. Furthermore, Xie et al. (2018) proposed an inexact stochastic-fuzzy programming model for irrigation water resources allocation and land resources utilization management under considering multiple uncertainties.

The water trading among users has been applied in some studies (Fu et al., 2016; Cheng et al., 2019). Recently, Huang and Wang (2019) introduced market-based stochastic optimization of water resources for improving drought resilience and economic efficiency in arid regions.

In this paper, ITSP with fuzzy variables is developed based on MIP for the problem of water allocation of Ajabshir Qaleh Chay Dam between agricultural different products in the conditions of ambiguous uncertainty to retrieve the water shortage and to achieve the optimal water allocation of this dam through its river canals. This developed method includes some alternatives such as other reservoirs to retrieve water shortages to reach the water allocation target. The main aim is to accomplish reliable methods that solve the problem in a manner that optimizes the system net benefit and gives optimal solutions, which also select the correct retrieving alternatives when the promised water allocation targets are not fully satisfied by seasonal flows. It causes irreparable harms particularly to industries, and the users need to either get water from higher-priced resources or abridge their advancement plans. The innovation of this study would be summarized as:

- Handling stochastic conditions of water flow levels of Qaleh Chay Dam by updating ITSP method with the limitation and constraints of this dam.
- Development of fuzzy programming through possibility theory as a fuzzy EITSP approach to handle combination of fuzzy conditions and interval stochastic uncertainty of the study area.
- Finding the optimal water allocation target and compensation of water shortages with the least total cost for all corps in the study region.

The remainder of this paper is organized as follows. In Section 2, an interval-parameter two-stage stochastic mixed-integer programming approach for retrieving water shortages of water resource allocation of Qaleh Chay Dam, is proposed, and then this approach is developed with fuzzy variables for the problem under uncertainty. In Section 3, our case study in Ajabshir Qaleh Chay Dam is given and analyzed to clarify the described methods. The conclusions and suggestions for further research are discussed in the final section.

## 2. Modelling Formulation

Agricultural water resources are related to uncertain variables, such as soil moisture, rainfall, temperature and market demand, which are not fully controllable, and have random and

interval information. In many practical water resource situations, fuzzily imprecision and probability uncertainty together with interval information appear in the problem parameters. Furthermore, due to the interaction between uncertainty parameters and economic variables, water resource management has been complex. The literature review of the issue of water resources management shows that the water crisis in the future is unavoidable and agriculture is vulnerable to water crisis. Considering the above complexities and uncertainties, incorporation of the MIP, SP, IMP and FMP methods would be used to manage water resources efficiently.

In this section, we introduce a developed interval-parameter two-stage stochastic mixed-integer programming approach, called extended ITSP (EITSP), for water resource allocation of agricultural products at Ajabshir Qaleh Chay Dam under stochastic uncertainty to retrieve the water shortage of users. Then for this problem under ambiguous uncertain framework, an EITSP method with fuzzy variables is introduced. These approaches include some alternatives like other reservoirs to recover water shortages to reach the water allocation target.

The water resource problem of this research is how to allocate water resource of Ajabshir Qaleh Chay Dam to different types of products under different water demands and infrastructure constraints for water transmission, with an objective of maximizing total system benefit to retrieve the water shortage and to achieve the optimal water allocation of this dam through its river canals.

These new methods deal with some issues given as:

- Taking into account the multiple uncertainties of parameters, i.e., combination of interval, probability and fuzzy distribution.
- Compensation of water shortages for all products using MIP.
- Providing an optimal water supply and transmission-programming model.

## 2.1. Interval-Parameter Two-Stage Stochastic Mixed-Integer Programming

For water resource allocation of Qaleh Chay Dam, we propose a new approach under stochastic framework. Consider the following EITSP for the problem under above consideration:

*Problem 1*

$$\begin{aligned} \max f^{\pm} = & \sum_{i=1}^m \sum_{j=1}^n q_{ij}^{\pm} NB_j^{\pm} - \sum_{i=1}^r \sum_{j=1}^n q_{ij}^{\pm} AC_{ij}^{\pm} \\ & - \sum_{i=r+1}^m \sum_{j=1}^n q_{ij}^{\pm} (TC_i^{\pm} + AC_{ij}^{\pm}) - \sum_{i=1}^m \sum_{j=1}^n \sum_{k=1}^p p_{ik}^{\pm} SC_{ij}^{\pm} S_{ijk}^{\pm} \\ & - \sum_{i=1}^m \sum_{j=1}^n \sum_{k=1}^p p_{ik}^{\pm} \sum_{l=1}^o E_{jl}^{\pm} \Delta T_{jl}^{\pm} x_{jkl}^{\pm} \end{aligned} \quad (1)$$

subject to:

Water availability constraints:

$$\sum_{j=1}^n (q_{ij}^{\pm} - S_{ijk}^{\pm})(1 + \delta^{\pm}) \leq Q_{ik}^{\pm}, \quad \forall i, k \quad (2)$$

Constraints for capacity of canal from rivers near the region:

$$q_{ij}^{\pm} - S_{ijk}^{\pm} \leq c_{ij}^{\pm}, \quad \forall i \leq r, \forall j, k \quad (3)$$

Constraints for capacity of canal from rivers far from the region:

$$\sum_{j=1}^n (q_{ij}^{\pm} - S_{ijk}^{\pm}) \leq c_i^{\pm}, \quad \forall i > r, \forall k \quad (4)$$

Allowable water allocation constraints:

$$S_{ijk}^{\pm} \leq q_{ij}^{\pm} \leq q_{ij \max}^{\pm}, \quad \forall i, j, k \quad (5)$$

Constraints for retrieving water shortages:

$$\sum_{l=1}^o \Delta T_{jl}^{\pm} x_{jkl}^{\pm} \geq \sum_{i=1}^m S_{ijk}^{\pm}, \quad \forall j, k \quad (6)$$

Constraints that prevent users to exceed capacity of alternatives:

$$\sum_{k=1}^p x_{jkl}^{\pm} \leq 1, \quad \forall j, l \quad (7)$$

Non-negativity constraints:

$$S_{ijk}^{\pm} \geq 0, \quad \forall i, j, k \quad (8)$$

Binary constraints for using alternatives:

$$x_{jkl}^{\pm} \in \{0, 1\}, \quad \forall j, k, l \quad (9)$$

where  $f^{\pm}$  = system benefit (Rial);  $NB_j^{\pm}$  = net benefit to the farm planted with crop  $j$  per  $m^3$  of water allocated (Rial/ $m^3$ );  $q_{ij}^{\pm}$  = water allocation target from river  $i$  that is promised to the farm planted with crop  $j$  ( $m^3$ ) (first-stage decision variables);  $AC_{ij}^{\pm}$  = allocating cost from river  $i$  or from the transferring station related to river  $i$  to end-user  $j$  (Rial/ $m^3$ );  $TC_i^{\pm}$  = obtain/transporting cost from river  $i$  to the transferring station (Rial/ $m^3$ ) for indirect delivery from rivers far from the region;  $SC_{ij}^{\pm}$  = shortage cost to user  $j$  per  $m^3$  of water shortage from river  $i$  (Rial/ $m^3$ );  $S_{ijk}^{\pm}$  = shortage of water to user  $j$  when the seasonal flow of river  $i$  is  $Q_{ik}^{\pm}$  ( $m^3$ ) (second-stage decision variables);  $Q_{ik}^{\pm}$  = seasonal flow of river  $i$  with probability  $p_{ik}$ ;  $q_{ij \max}^{\pm}$  = maximum allowable allocation amount from river  $i$  to the farm planted with crop  $j$  ( $m^3$ );  $c_{ij}^{\pm}$  = capacity of canal (per hectare) from river  $i$  to the farm planted with crop  $j$  when the river is near the region ( $m^3$ );  $c_i^{\pm}$  = capacity of canal (per hectare) from river  $i$  to the transferring station when the river is far from the region ( $m^3$ );  $\delta^{\pm}$  = rate of water loss during transportation;  $x_{jkl}^{\pm}$  = a binary decision variable

that takes value 1 if the farm planted with crop  $j$  uses alternative  $l$  when the seasonal flow is  $k$ ;  $E_{jl}^{\pm}$  = cost of increasing 1 m<sup>3</sup> water for user  $j$  while using alternative  $l$  (Rial /m<sup>3</sup>);  $\Delta T_{jl}^{\pm}$  = available amount of water for user  $j$  by using alternative  $l$  (m<sup>3</sup>);  $n$  = total number of crops;  $j$  = crop index;  $m$  = total number of rivers,  $i = 1$  for river 1,  $i = 2$  for river 2, and  $i = 3$  for river 3;  $p$  = total number of flow levels;  $k$  = flow level, where  $k = 1$  for low flow,  $k = 2$  for medium flow, and  $k = 3$  for high flow;  $o$  = total number of alternatives.

Indeed, total cost of delivering water to end-users will depend upon the following situations:

- Direct delivery from rivers located near the region, in which the total cost equals the obtaining/transporting cost from the river to end-users (i.e. allocating cost from the river  $i$  to end-user  $j$ ,  $AC_{ij}^{\pm}$ ).
- Indirect delivery from rivers located far from the region, in which the total cost equals the sum of the obtaining/transporting cost from river to the transferring station and the allocating cost from the transferring station to end-users (i.e. obtaining/transporting cost from the river  $i$  to the transferring station,  $TC_i^{\pm}$ , plus the allocating cost from the transferring station related to river  $i$  to end-user  $j$ ,  $AC_{ij}^{\pm}$ ).

Therefore, the total cost for delivering water from river  $i$  to the farm planted with crop  $j$ , denoted by  $CT_{ij}^{\pm}$ , is expressed as follows:

$$CT_{ij}^{\pm} = \begin{cases} AC_{ij}^{\pm} & ; \text{if river } i \text{ is near the region planted with crop } j \\ TC_i^{\pm} + AC_{ij}^{\pm} & ; \text{if river } i \text{ is far from the region planted with crop } j \end{cases}$$

According to Huang and Loucks (2000), let  $q_{ij}^{\pm} = q_{ij}^{-} + \Delta q_{ij} z_{ij}$  where  $\Delta q_{ij} = q_{ij}^{+} - q_{ij}^{-}$ ,  $z_{ij} \in [0, 1]$  and  $z_{ij}$  are defined as decision variables for identifying an optimized set of target values ( $q_{ij}^{\pm}$ ). As explained in Supplementary material A, Problem 1 will be divided into two sub-models. The first sub-model, which corresponds to  $f^{+}$ , can be formulated as follows:

#### Problem 2

$$\begin{aligned} \max f^{+} = & \sum_{i=1}^m \sum_{j=1}^n (q_{ij}^{-} + \Delta q_{ij} z_{ij}) NB_j^{+} \\ & - \sum_{i=1}^r \sum_{j=1}^n (q_{ij}^{-} + \Delta q_{ij} z_{ij}) AC_{ij}^{-} \\ & - \sum_{i=r+1}^m \sum_{j=1}^n (q_{ij}^{-} + \Delta q_{ij} z_{ij}) (TC_i^{-} + AC_{ij}^{-}) \\ & - \sum_{i=1}^m \sum_{j=1}^n \sum_{k=1}^p p_{ik} SC_{ij}^{-} S_{ijk}^{-} - \sum_{i=1}^m \sum_{j=1}^n \sum_{k=1}^p p_{ik} \sum_{l=1}^o E_{jl}^{-} \Delta T_{jl}^{-} x_{jkl}^{-} \end{aligned} \quad (10)$$

subject to:

$$\sum_{j=1}^n (q_{ij}^{-} + \Delta q_{ij} z_{ij} - S_{ijk}^{-}) (1 + \delta^{-}) \leq Q_{ik}^{+}, \quad \forall i, k \quad (11)$$

$$q_{ij}^{-} + \Delta q_{ij} z_{ij} - S_{ijk}^{-} \leq c_{ij}^{+}, \quad \forall i \leq r, \forall j, k \quad (12)$$

$$\sum_{j=1}^n (q_{ij}^{-} + \Delta q_{ij} z_{ij} - S_{ijk}^{-}) \leq c_i^{+}, \quad \forall i > r, \forall k \quad (13)$$

$$S_{ijk}^{-} \leq q_{ij}^{-} + \Delta q_{ij} z_{ij} \leq q_{ij}^{+}, \quad \forall i, j, k \quad (14)$$

$$\sum_{l=1}^o \Delta T_{jl}^{-} x_{jkl}^{-} \geq \sum_{i=1}^m S_{ijk}^{-}, \quad \forall j, k \quad (15)$$

$$\sum_{k=1}^p x_{jkl}^{-} \leq 1, \quad \forall j, l \quad (16)$$

$$S_{ijk}^{-} \geq 0, \quad \forall i, j, k \quad (17)$$

$$0 \leq z_{ij} \leq 1, \quad \forall i, j \quad (18)$$

$$x_{jkl}^{-} \in \{0, 1\}, \quad \forall j, k, l \quad (19)$$

where  $S_{ijk}^{-}$ ,  $z_{ij}$  and  $x_{jkl}^{-}$  are decision variables. Suppose that  $S_{ijkopt}^{-}$ ,  $z_{ijopt}$  and  $x_{jkl}^{-}$  are the optimal solution of the first sub-model. We can obtain the optimized water allocation target by calculating  $q_{ijopt}^{\pm} = q_{ij}^{-} + \Delta q_{ij} z_{ijopt}$ .

Furthermore, according to  $f^{-}$ , the second sub-model is formulated as:

#### Problem 3

$$\begin{aligned} \max f^{-} = & \sum_{i=1}^m \sum_{j=1}^n (q_{ij}^{-} + \Delta q_{ij} z_{ijopt}) NB_j^{-} \\ & - \sum_{i=1}^r \sum_{j=1}^n (q_{ij}^{-} + \Delta q_{ij} z_{ijopt}) AC_{ij}^{+} \\ & - \sum_{i=r+1}^m \sum_{j=1}^n (q_{ij}^{-} + \Delta q_{ij} z_{ijopt}) (TC_i^{+} + AC_{ij}^{+}) \\ & - \sum_{i=1}^m \sum_{j=1}^n \sum_{k=1}^p p_{ik} SC_{ij}^{+} S_{ijk}^{+} - \sum_{i=1}^m \sum_{j=1}^n \sum_{k=1}^p p_{ik} \sum_{l=1}^o E_{jl}^{+} \Delta T_{jl}^{+} x_{jkl}^{+} \end{aligned} \quad (20)$$

subject to:

$$\sum_{j=1}^n (q_{ij}^{-} + \Delta q_{ij} z_{ijopt} - S_{ijk}^{+}) (1 + \delta^{+}) \leq Q_{ik}^{-}, \quad \forall i, k \quad (21)$$

$$q_{ij}^{-} + \Delta q_{ij} z_{ijopt} - S_{ijk}^{+} \leq c_{ij}^{-}, \quad \forall i \leq r, \forall j, k \quad (22)$$

$$\sum_{j=1}^n (q_{ij}^{-} + \Delta q_{ij} z_{ijopt} - S_{ijk}^{+}) \leq c_i^{-}, \quad \forall i > r, \forall k \quad (23)$$

$$S_{ijk}^{+} \leq q_{ij}^{-} + \Delta q_{ij} z_{ijopt} \leq q_{ij}^{+}, \quad \forall i, j, k \quad (24)$$

$$\sum_{l=1}^o \Delta T_{jl}^{+} x_{jkl}^{+} \geq \sum_{i=1}^m S_{ijk}^{+}, \quad \forall j, k \quad (25)$$

$$\sum_{k=1}^p x_{jkl}^{+} \leq 1, \quad \forall j, l \quad (26)$$

$$x_{jkl}^+ \geq x_{jkl\ opt}^-, \quad \forall j, k, l \quad (27)$$

$$S_{ijk}^+ \geq S_{ijk\ opt}^-, \quad \forall i, j, k \quad (28)$$

$$x_{jkl}^+ \in \{0, 1\}, \quad \forall j, k, l \quad (29)$$

where  $z_{ij\ opt}^-$ ,  $S_{ijk\ opt}^-$  and  $x_{jkl\ opt}^-$  are the optimal solution of the first sub-model and  $x_{jkl\ opt}^+$ ,  $S_{ijk\ opt}^+$  are those of Problem 3. Problems 2 and 3 are deterministic MIP problems solved by one of the MIP solvers and the optimal solutions of Problem 1 are:

$$S_{ijk\ opt}^\pm = [S_{ijk\ opt}^-, S_{ijk\ opt}^+], \quad \forall i, j, k \quad (30)$$

$$x_{jkl\ opt}^\pm = [x_{jkl\ opt}^-, x_{jkl\ opt}^+], \quad \forall j, k, l \quad (31)$$

$$f^\pm = [f^-, f^+] \quad (32)$$

Therefore, we obtain the actual allocated water scheme supplied by seasonal flows as follows:

$$A_{ijk\ opt}^\pm = q_{ij\ opt}^\pm - S_{ijk\ opt}^\pm, \quad \forall i, j, k \quad (33)$$

## 2.2. Interval-Parameter Two-Stage Stochastic Mixed-Integer Programming with Fuzzy Variables

As mentioned before, a remarkable restriction of the aforementioned ITSP method is its incapability in considering the mixed and multiple uncertainties in the parameters of the problem such as combination of probability and possibility distributions. The combination of MIP and ITSP together with fuzzy programming leads to a method, which is referred to in this paper as fuzzy EITSP and can be response to the remarkable limitation of ITSP on environmental management issues.

Therefore, an interval-parameter two-stage stochastic mixed-integer programming with fuzzy variables is introduced under ambiguous uncertain framework to retrieve the water shortages, called fuzzy EITSP. Consider the following formulation of water allocation problem of Qaleh Chay Dam as an EITSP method with fuzzy variables:

### Problem 4

$$\begin{aligned} \max f^\pm &= \sum_{i=1}^m \sum_{j=1}^n q_{ij}^\pm NB_j^\pm - \sum_{i=1}^r \sum_{j=1}^n q_{ij}^\pm AC_{ij}^\pm \\ &- \sum_{i=r+1}^m \sum_{j=1}^n q_{ij}^\pm (TC_i^\pm + AC_{ij}^\pm) - \sum_{i=1}^m \sum_{j=1}^n \sum_{k=1}^p p_{ik}^\pm SC_{ij}^\pm S_{ijk}^\pm \\ &- \sum_{i=1}^m \sum_{j=1}^n \sum_{k=1}^p p_{ik}^\pm \sum_{l=1}^o \tilde{E}_{jl}^\pm \Delta T_{jl}^\pm x_{jkl}^\pm \end{aligned} \quad (34)$$

subject to:

Water availability constraints:

$$\sum_{j=1}^n (q_{ij}^\pm - S_{ijk}^\pm)(1 + \tilde{\delta}^\pm) \leq \tilde{Q}_{ik}^\pm, \quad \forall i, k \quad (35)$$

Constraints for capacity of canal from river near the region:

$$q_{ij}^\pm - S_{ijk}^\pm \leq \tilde{c}_{ij}^\pm, \quad \forall i \leq r, \quad \forall j, k \quad (36)$$

Constraints for capacity of canal from river far from the region:

$$\sum_{j=1}^n (q_{ij}^\pm - S_{ijk}^\pm) \leq \tilde{c}_i^\pm, \quad \forall i > r, \quad \forall k \quad (37)$$

Allowable water allocation constraints:

$$S_{ijk}^\pm \leq q_{ij}^\pm \leq q_{ij\ max}^\pm, \quad \forall i, j, k \quad (38)$$

Constraints for retrieving water shortages:

$$\sum_{l=1}^o \Delta T_{jl}^\pm x_{jkl}^\pm \geq \sum_{i=1}^m S_{ijk}^\pm, \quad \forall j, k \quad (39)$$

Constraints that prevent users from exceeding capacity of alternatives:

$$\sum_{k=1}^p x_{jkl}^\pm \leq 1, \quad \forall j, l \quad (40)$$

Non-negativity constraints:

$$S_{ijk}^\pm \geq 0, \quad \forall i, j, k \quad (41)$$

Binary constraints for using alternatives:

$$x_{jkl}^\pm \in \{0, 1\}, \quad \forall j, k, l \quad (42)$$

where  $NB_j^\pm = (NB_j^-, NB_j^+, \alpha_j^{NB}, \beta_j^{NB})_{LR}$ ,  $AC_{ij}^\pm = (AC_{ij}^-, AC_{ij}^+, \alpha_{ij}^{AC}, \beta_{ij}^{AC})_{LR}$ ,  $TC_i^\pm = (TC_i^-, TC_i^+, \alpha_i^{TC}, \beta_i^{TC})_{LR}$ ,  $SC_{ij}^\pm = (SC_{ij}^-, SC_{ij}^+, \alpha_{ij}^{SC}, \beta_{ij}^{SC})_{LR}$ ,  $\tilde{E}_{jl}^\pm = (E_{jl}^-, E_{jl}^+, \alpha_{jl}^E, \beta_{jl}^E)_{LR}$ ,  $\tilde{\delta}^\pm = (\delta^-, \delta^+, \alpha, \beta)_{LR}$ ,  $\tilde{Q}_{ik}^\pm = (Q_{ik}^-, Q_{ik}^+, \alpha_{ik}^Q, \beta_{ik}^Q)_{LR}$ ,  $\tilde{c}_{ij}^\pm = (c_{ij}^-, c_{ij}^+, \alpha_{ij}^c, \beta_{ij}^c)_{LR}$  and  $\tilde{c}_i^\pm = (\tilde{c}_i^-, \tilde{c}_i^+, \alpha_i^c, \beta_i^c)_{LR}$  are LR fuzzy numbers and symbol  $\lesseqgtr$  represents fuzzy inequality. Furthermore, the other variables and parameters of this problem are alike to those of the problem formulated based on the EITSP and have the same definition.

According to the method described in Supplementary Note A about inexact fuzzy interval programming, in which the FCCP approach based on possibility theory, and Zimmermann fuzzy programming method (Zimmerman, 1978) have been used, Problem 4 will be converted to the following interval MIP problem:

## Problem 5

$$\max \lambda^{\pm} \quad (43)$$

subject to:

$$\begin{aligned} & \sum_{i=1}^m \sum_{j=1}^n q_{ij}^{\pm} (NB_j^+ + \beta_j^{NB} R^*(\eta^{\pm})) \\ & - \sum_{i=1}^r \sum_{j=1}^n q_{ij}^{\pm} (AC_{ij}^- - \alpha_{ij}^{AC} R^*(\eta^{\pm})) \\ & - \sum_{i=r+1}^m \sum_{j=1}^n q_{ij}^{\pm} (TC_i^- + AC_{ij}^- - (\alpha_i^{TC} + \alpha_{ij}^{AC}) R^*(\eta^{\pm})) \\ & - \sum_{i=1}^m \sum_{j=1}^n \sum_{k=1}^p p_{ik} (SC_{ij}^- - \alpha_{ij}^{SC} R^*(\eta^{\pm})) S_{ijk}^{\pm} \\ & - \sum_{i=1}^m \sum_{j=1}^n \sum_{k=1}^p p_{ik} \sum_{l=1}^o (E_{jl}^- - \alpha_{jl}^E R^*(\eta^{\pm})) \Delta T_{jl}^{\pm} x_{jkl}^{\pm} \\ & \geq f_d^- + \lambda^{\pm} (f_d^+ - f_d^-) \end{aligned} \quad (44)$$

$$\sum_{j=1}^n (q_{ij}^{\pm} - S_{ijk}^{\pm}) (1 + \delta^- - \alpha L^*(h^{\pm})) \leq Q_{ik}^+ + \beta_{ik}^Q R^*(h^{\pm}), \quad \forall i, k \quad (45)$$

$$q_{ij}^{\pm} - S_{ijk}^{\pm} \leq c_{ij}^+ + R^*(h^{\pm}) \beta_{ij}^c, \quad \forall i \leq r, \forall j, k \quad (46)$$

$$\sum_{j=1}^n (q_{ij}^{\pm} - S_{ijk}^{\pm}) \leq c_i^+ + R^*(h^{\pm}) \beta_i^c, \quad \forall i > r, \forall k \quad (47)$$

$$S_{ijk}^{\pm} \leq q_{ij}^{\pm} \leq q_{ij\max}^- + (1 - \lambda^{\pm}) (q_{ij\max}^+ - q_{ij\max}^-), \quad \forall i, j, k \quad (48)$$

$$\sum_{l=1}^o \Delta T_{jl}^{\pm} x_{jkl}^{\pm} \geq \sum_{i=1}^m S_{ijk}^{\pm}, \quad \forall j, k \quad (49)$$

$$\sum_{k=1}^p x_{jkl}^{\pm} \leq 1, \quad \forall j, l \quad (50)$$

$$S_{ijk}^{\pm} \geq 0, \quad \forall i, j, k \quad (51)$$

$$x_{jkl}^{\pm} \in \{0, 1\}, \quad \forall j, k, l \quad (52)$$

where  $\lambda^{\pm}$  is the control variable;  $f_d^-$  and  $f_d^+$  are the lower and upper bounds of the objective function, respectively, which can be established by decision makers (DMs) for the objective function they want to achieve;  $\eta^{\pm}$  and  $h^{\pm}$  are defined as permitted possibility levels by the DMs.

Then, the obtained deterministic interval-parameter programming will be divided into two sub-model according to the method introduced by Huang (1996) and Huang & Cao (2011) which was explained with sufficient details in Supplementary material A. Furthermore, according to Huang and Loucks (2000), we convert  $q_{ij}^{\pm}$  to a deterministic value by  $q_{ij}^{\pm} = q_{ij}^- + \Delta q_{ij} z_{ij}$ , where  $\Delta q_{ij} = q_{ij}^+ - q_{ij}^-$ ,  $z_{ij} \in [0, 1]$  and  $z_{ij}$  are defined as decision variables for finding an optimized set of  $q_{ij}^{\pm}$ .

According to the upper bound of the objective function,

the first sub-model can be expressed as:

## Problem 6

$$\max \lambda^{\pm} \quad (53)$$

subject to:

$$\begin{aligned} & \sum_{i=1}^m \sum_{j=1}^n (q_{ij}^- + \Delta q_{ij} z_{ij}) (NB_j^+ + \beta_j^{NB} R^*(\eta^-)) \\ & - \sum_{i=1}^r \sum_{j=1}^n (q_{ij}^- + \Delta q_{ij} z_{ij}) (AC_{ij}^- - \alpha_{ij}^{AC} R^*(\eta^-)) \\ & - \sum_{i=r+1}^m \sum_{j=1}^n (q_{ij}^- + \Delta q_{ij} z_{ij}) (TC_i^- + AC_{ij}^- - (\alpha_i^{TC} + \alpha_{ij}^{AC}) R^*(\eta^-)) \\ & - \sum_{i=1}^m \sum_{j=1}^n \sum_{k=1}^p p_{ik} (SC_{ij}^- - \alpha_{ij}^{SC} R^*(\eta^-)) S_{ijk}^- \\ & - \sum_{i=1}^m \sum_{j=1}^n \sum_{k=1}^p p_{ik} \sum_{l=1}^o (E_{jl}^- - \alpha_{jl}^E R^*(\eta^-)) \Delta T_{jl}^- x_{jkl}^- \\ & \geq f_d^- + \lambda^+ (f_d^+ - f_d^-) \end{aligned} \quad (54)$$

$$\begin{aligned} & \sum_{j=1}^n (q_{ij}^- + \Delta q_{ij} z_{ij} - S_{ijk}^-) (1 + \delta^- - \alpha L^*(h^-)) \\ & \leq Q_{ik}^+ + \beta_{ik}^Q R^*(h^-), \quad \forall i, k \end{aligned} \quad (55)$$

$$q_{ij}^- + \Delta q_{ij} z_{ij} - S_{ijk}^- \leq c_{ij}^+ + R^*(h^-) \beta_{ij}^c, \quad \forall i \leq r, \forall j, k \quad (56)$$

$$\sum_{j=1}^n (q_{ij}^- + \Delta q_{ij} z_{ij} - S_{ijk}^-) \leq c_i^+ + R^*(h^-) \beta_i^c, \quad \forall i > r, \forall k \quad (57)$$

$$\begin{aligned} S_{ijk}^- & \leq q_{ij}^- + \Delta q_{ij} z_{ij} \leq q_{ij\max}^- \\ & + (1 - \lambda^+) (q_{ij\max}^+ - q_{ij\max}^-), \quad \forall i, j, k \end{aligned} \quad (58)$$

$$\sum_{l=1}^o \Delta T_{jl}^- x_{jkl}^- \geq \sum_{i=1}^m S_{ijk}^-, \quad \forall j, k \quad (59)$$

$$\sum_{k=1}^p x_{jkl}^- \leq 1, \quad \forall j, l \quad (60)$$

$$S_{ijk}^- \geq 0, \quad \forall i, j, k \quad (61)$$

$$x_{jkl}^- \in \{0, 1\}, \quad \forall j, k, l \quad (62)$$

$$0 \leq z_{ij} \leq 1, \quad \forall i, j \quad (63)$$

where  $S_{ijk}^-$ ,  $x_{jkl}^-$  and  $z_{ij}$  are decision variables of the first sub-model and their optimal values will be used to write the second sub-model, which corresponds to the lower bound of objective function values.

Therefore, we obtain the second sub-model as follows:

## Problem 7

$$\max \lambda^{\pm} \quad (64)$$

subject to:

$$\begin{aligned}
 & \sum_{i=1}^m \sum_{j=1}^n (q_{ij}^- + \Delta q_{ij} z_{ij \text{ opt}}) (NB_j^+ + \beta_j^{NB} R^*(\eta^+)) \\
 & - \sum_{i=1}^r \sum_{j=1}^n (q_{ij}^- + \Delta q_{ij} z_{ij \text{ opt}}) (AC_{ij}^- - \alpha_{ij}^{AC} R^*(\eta^+)) \\
 & - \sum_{i=r+1}^m \sum_{j=1}^n (q_{ij}^- + \Delta q_{ij} z_{ij \text{ opt}}) (TC_i^- + AC_{ij}^- - (\alpha_i^{TC} + \alpha_{ij}^{AC}) R^*(\eta^+)) \\
 & - \sum_{i=1}^m \sum_{j=1}^n \sum_{k=1}^p p_{ik} (SC_{ij}^- - \alpha_{ij}^{SC} R^*(\eta^+)) S_{ijk}^+ \\
 & - \sum_{i=1}^m \sum_{j=1}^n \sum_{k=1}^p p_{ik} \sum_{l=1}^o (E_{jl}^- - \alpha_{jl}^E R^*(\eta^+)) \Delta T_{jl}^+ x_{jkl}^+ \\
 & \geq f_d^- + \lambda^-(f_d^+ - f_d^-)
 \end{aligned} \quad (65)$$

$$\begin{aligned}
 & \sum_{j=1}^n (q_{ij}^- + \Delta q_{ij} z_{ij \text{ opt}} - S_{ijk}^+) (1 + \delta^- - \alpha L^*(h^+)) \\
 & \leq Q_{ik}^+ + \beta_{ik}^Q R^*(h^+), \quad \forall i, k
 \end{aligned} \quad (66)$$

$$\begin{aligned}
 & q_{ij}^- + \Delta q_{ij} z_{ij \text{ opt}} - S_{ijk}^+ \\
 & \leq c_{ij}^+ + R^*(h^+) \beta_{ij}^c, \quad \forall i \leq r, \forall j, k
 \end{aligned} \quad (67)$$

$$\begin{aligned}
 & \sum_{j=1}^n (q_{ij}^- + \Delta q_{ij} z_{ij \text{ opt}} - S_{ijk}^+) \\
 & \leq c_i^+ + R^*(h^+) \beta_i^c, \quad \forall i > r, \forall k
 \end{aligned} \quad (68)$$

$$\begin{aligned}
 & S_{ijk}^+ \leq q_{ij}^- + \Delta q_{ij} z_{ij \text{ opt}} \\
 & \leq q_{ij \text{ max}}^- + (1 - \lambda^-)(q_{ij \text{ max}}^+ - q_{ij \text{ max}}^-), \quad \forall i, j, k
 \end{aligned} \quad (69)$$

$$\sum_{l=1}^o \Delta T_{jl}^+ x_{jkl}^+ \geq \sum_{i=1}^m S_{ijk}^+, \quad \forall j, k \quad (70)$$

$$\sum_{k=1}^p x_{jkl}^+ \leq 1, \quad \forall j, l \quad (71)$$

$$x_{jkl}^+ \geq x_{jkl \text{ opt}}^-, \quad \forall j, k, l \quad (72)$$

$$S_{ijk}^+ \geq S_{ijk \text{ opt}}^-, \quad \forall i, j, k \quad (73)$$

$$x_{jkl}^+ \in \{0, 1\}, \quad \forall j, k, l \quad (74)$$

Finally, the optimal solutions of the above sub-models are used to calculate the optimal solutions of Problem 4 as a fuzzy EITSP model as follows:

$$S_{ijk \text{ opt}}^\pm = [S_{ijk \text{ opt}}^-, S_{ijk \text{ opt}}^+], \quad \forall i, j, k \quad (75)$$

$$x_{jkl \text{ opt}}^\pm = [x_{jkl \text{ opt}}^-, x_{jkl \text{ opt}}^+], \quad \forall j, k, l \quad (76)$$

$$f^\pm = [f^-, f^+] \quad (77)$$

As already mentioned, the actual allocated water scheme will be obtained as:

$$A_{ijk \text{ opt}}^\pm = q_{ij \text{ opt}}^\pm - S_{ijk \text{ opt}}^\pm, \quad \forall i, j, k. \quad (78)$$

Additionally, in our extended approach, the value of  $x_{jkl \text{ opt}}^\pm$  determines the optimal scheme for supplementary reservoirs to retrieve water shortage ( $S_{ijk \text{ opt}}^\pm$ ).

### 3. Case Study

#### 3.1. Problem Definition

Ajabshir County is one of the agricultural poles in the East Azarbaijan province, which has high quality and fertile land with major producers of wheat, barley, potatoes, onions, grapes, walnuts, almonds and apples in the province and northwest of the country. Most residents of the 40 villages of this county, and even people living in Ajabshir, are only livelihood farmers. In addition, agricultural water resources are associated with uncertain variables, such as soil moisture, rainfall, temperature and market demand, which are uncontrollable.

Accordingly, taking into account the uncertainty of the water supply flow is important in the decision making of management and allocation of water resources, and in order to reduce the damage to farmers and the adverse effects of water shortages, planning for optimal water allocation in the agricultural sector of the region is necessary.

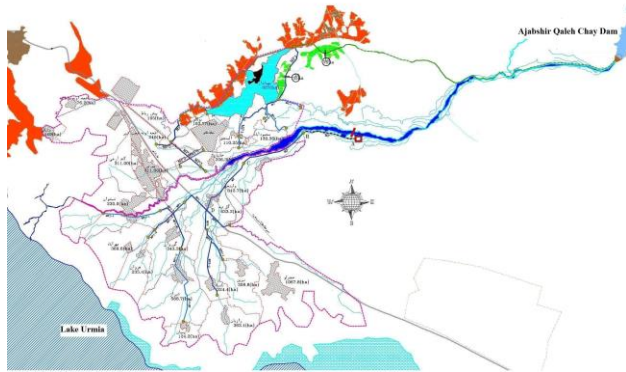
A review of the literature on water resources management shows that the future water crisis is inevitable and agriculture is vulnerable to the water crisis. As noted above, rising food demand because of population growth, climate change, and severe restrictions on water resources have created critical conditions for water resources in the country that Ajabshir County is no exception to it. Furthermore, agricultural water resources are related to uncertain variables, which are not fully controllable, and the interaction between uncertainty parameters and economic variables has complicated water resource management. Given the above complexities and ambiguities, a combination of MIP, SP, IMP and FMP approaches is used to efficiently manage water resources.

In this section, we intend to conduct a case study of Qaleh Chay Dam water allocation to study the method presented in this paper and clarify the solution method. This dam is located 100 km southwest of Tabriz in East Azarbaijan province. Ajabshir Qaleh Chay Dam with a total volume of  $40 \times 10^6 \text{ m}^3$  has a catchment area of  $250 \text{ km}^2$  and is fed through an average annual rainfall of 345 mm. The optimal water allocation of this dam is investigated in this study.

Climate change and overexploitation of groundwater and drought effects in recent years have led to a decrease in surface currents and extra-capacity pressures on the groundwater resources in the region, which has caused a sharp decrease in groundwater levels in the region and adverse effects on the water resources entering the Lake Urmia. The continuation of such a process in the exploitation of groundwater resources and the allocation of land to high water needs will increase the drying rate, the water quality reducing, undesirable water of most

wells in the region and the critical condition of Lake Urmia, and will impose additional costs on the farmers of the region.

The study area of Ajabshir Qaleh Chay Dam is an area consisting of three rivers. In this area, due to some problems and limitations, the river canal system has been used to deliver water resources to the farms and river canal planning was applied to adapt the modeling to the actual details of water resources management in the area. The rivers are divided into two groups near and far in terms of proximity to agricultural fields. The river 1 is close to the fields and the rivers 2 and 3 are located far from the regions. There are mainly 10 types of crops planted in this area. Each crop uses the water resources of all rivers, depending on the area in which it is located.



**Figure 1.** The general map of the study area of Ajabshir Qaleh Chay Dam.

The general map of the whole study area of Ajabshir Qaleh Chay Dam is shown in Figure 1 (Source: Regional Water Authority and Agriculture Jihad Organization of East Azarbaijan), from which the case study area was selected.

Within the scope of this study, Ajabshir Qaleh Chay Dam water resources are available to agricultural products by three river channels. Each crop uses water from all three rivers. When the product is in the area near river 1, it uses the water resources of this river directly; and when it is in the farther area, the water of rivers 2 and 3 reaches the transferring station near that area and it is distributed from there. The total cost of delivering water to endusers was explained with details in sub-Section 2.1.

The promised water allocation targets for products are calculated by gross irrigation requirement together with considering questionnaire and comments of experts of water resources management. All data for the selected products (wheat, barley, potatoes, onions, grapes, walnuts, almonds and apples) has been collected from Regional Water Authority and Agriculture Jihad Organization of East Azarbaijan in 2015 ~ 2016, and in some cases, has been completed by a questionnaire of experts.

The aim is to obtain the optimal amount of actual water allocated to different crops so that the constraints of water resource allocation and the capacity of water channels are satisfied. If the optimal amount of actual allocated water were equal to the promised water allocation targets for each crop, there is

no shortage of water, otherwise alternative water resources would be used at a higher price to offset the water shortage. This leads to the use of the described EITSP methods.

In other words, the system profit includes revenues from allocation of water resources, costs of transmission and supply of water resources, and costs of compensating for scarcity with alternative resources. In order to optimize the system profit so as to satisfy the water allocation constraints and the capacity of the water channels, the EITSP approach is used and then by considering the multiple uncertainties in the case study parameters such as the combination of interval, probability and probability distributions, the fuzzy EITSP approach is applied, both based on MIP.

**Table 1.** Water Supply of Rivers under Different Flow Levels, and Associated Probability in Ajabshir Qaleh Chay Dam.

| Flow level         | Probability | Available water resource for irrigation<br>( $10^3 \text{ m}^3$ ) |           |           |
|--------------------|-------------|---|-----------|-----------|
|                    |             | River   |           |           |
|                    |             | $i = 1$   | $i = 2$   | $i = 3$   |
| Low ( $k = 1$ )    | 0.31        | [3, 15]   | [2, 10]   | [1, 9]    |
| Medium ( $k = 2$ ) | 0.42        | [38, 90]  | [26, 65]  | [23, 50]  |
| High ( $k = 3$ )   | 0.27        | [117, 220]  | [79, 135] | [71, 100] |

Seasonal flows are assumed to be random variables with known probability distribution and have low, moderate, and high values that their associated interval parameters are specified in Table 1 for related probability levels.

The promised target of water allocation is calculated for different crops by gross irrigation requirement together with considering questionnaire and comments of experts, and their upper and lower bounds are calculated taking into account the upper and lower boundary of irrigation efficiency of the area. The maximum allowable water allocation to various crops from rivers has been calculated taking into account the most undesirable irrigation efficiency in the region and based on the most promised water allocation target. If the promised target of water allocation is not supplied to the farmer, he/she has two options. Whether or not to buy water from another source at a higher price or to choose a loss from the crop. The difference between the purchase price of a unit of water from other sources and the fair price was considered as a reduction in the net benefit of each product. Using income-cost data, a decrease in net profit per unit of promised target of water loss and net benefit for water allocation was calculated, and the results are shown in Table S1. Furthermore, maximum allowable allocation amount of water, allocation cost, and transportation cost from rivers are collected in Table S2.

In order to solve this water management problem, the EITSP method is used to compensate for the gap between the promised allocation and the actual allocation amount. In this model, the farmer plans to select different alternatives, such as buying a neighboring farm supply share (first alternative), the use of well water in the farm (second alternative) or the transfer of water from the neighboring farm wells (third alternative) to meet the plant's water requirement, which will obviously in-



crease production costs. Table S3 shows the cost of increasing one cubic meter of water for the farm planted with crop  $j$  and available amount of water for the farm with crop  $j$  by using these alternatives.

The rivers 2 and 3 are far from the regions and the river 1 is near the regions. The maximum capacities of canal (per hectare) from rivers 2 and 3 to the transferring station ( $c_i^\pm$ ) are equal  $[45, 50] \times 10^3 \text{ m}^3$ . The capacity of canal from river 1 to the farm planted with crop  $j$  in a crop year ( $c_{ij}^\pm$ ) are given in Table S4.

Consider this case study but assume that its parameters have a higher level of uncertainty, which is not correct for defining them with interval variables alone. In this situation, the parameters of this problem have ambiguous uncertainty determined by fuzzy variables and their values are obtained through a survey of water resource experts, which are distinguished by 4-dimensional vectors  $(m^0, m^1, \beta, \gamma)$  where  $m^0, m^1$  are the same to lower and upper bounds of interval parameters respectively and  $\beta, \gamma$  represent the left and right spread respectively. The centre (or mode) of fuzzy number is assumed as the original number of the case study. Assume that  $\phi$  is the original number in this case study, to generate right and left spread, the following relation is used:  $(r)\phi$ , where  $0 \leq r \leq 1$ . The values of  $r$  depend on level of uncertainty and could be changed by the DMs opinion. We assumed  $r = 0.1$ , so the right and left values are obtained by  $0.1\phi$ . Furthermore  $R^+(h) = L^+(h) = 1 - h$ , and the permitted possibility level is 0.7.

The deterministic problems obtained from both EITSP and fuzzy EITSP methods in this paper have been coded in GAMS v24.1.2 and run on a PC equipped with a 2.9 GHz Intel Pentium (R) CPU, 4GB of RAM and Windows 7 operating system.

### 3.2. Result Analysis

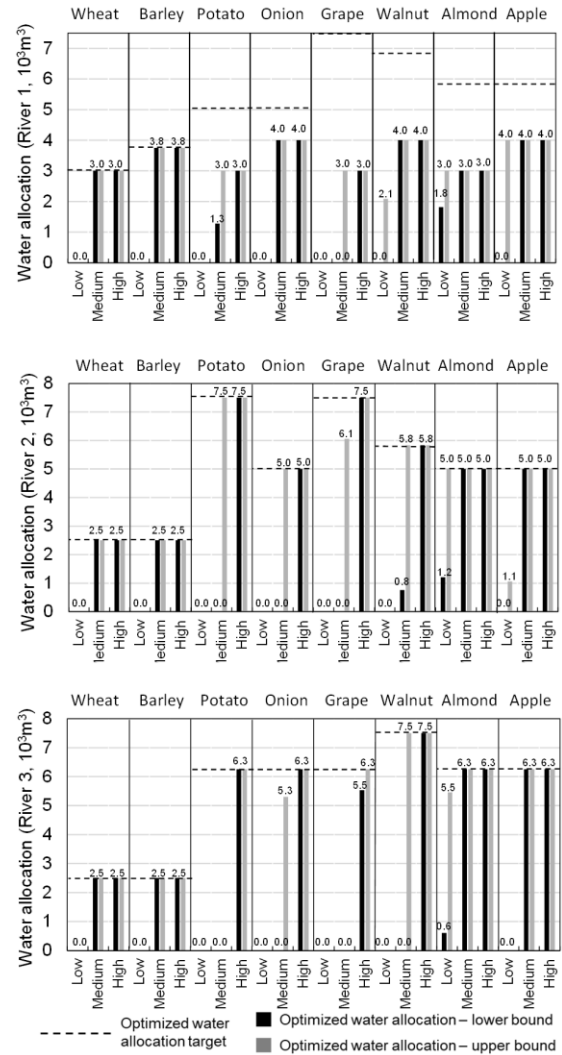
Firstly, for the described problem, we apply the general ITSP method in which no alternative is considered to retrieve users' water shortage, and then solve the obtained sub-problems by one of the MIP solvers such as GAMS v24.1.2.

The optimized water allocation targets, water shortages, and water allocation scheme are collected in Table 2. This ITSP approach cannot give optimal solution to compensate for the gap between the promised water allocation targets and the actual allocated water.

The results of the EITSP method are presented in Table 3. Water shortages at different levels of flow and for different crops are based on the promised water allocation target and the actual allocated water for different crops in each river is calculated.

For river 1, which is close to farmland, except for wheat and barley, for other crops at all levels, there is a shortage of water, as shown in Figure 2, and its values are indicated in Table 3. As the results show, there is a water shortage under low flow levels for all crops. The optimized actual water allocation would be zero, with the exception of walnut, almond and apple

fields in river 1, with actual water allocation of  $A_{161}^\pm = [0, 2.091] \times 10^3$ ,  $A_{171}^\pm = [1.818, 3] \times 10^3$  and  $A_{181}^\pm = [0, 4] \times 10^3 \text{ m}^3$ , almond and apple fields in river 2 with actual water allocation of  $A_{271}^\pm = [1.212, 5] \times 10^3$  and  $A_{281}^\pm = [0, 1.061] \times 10^3 \text{ m}^3$  and almond fields in river 3 with actual water allocation of  $A_{371}^\pm = [0.606, 5.455] \times 10^3 \text{ m}^3$ .



**Figure 2.** Optimized water allocation patterns through EITSP method under low, medium and high flows.

Under the medium flow level in all three rivers other than wheat and barley for other products, there is a shortage of water and a little water is provided for them. In addition, the product of almond does not have water shortages in rivers 2 and 3, and the actual water allocation for it is  $A_{272}^\pm = 5 \times 10^3$  and  $A_{372}^\pm = 6.25 \times 10^3 \text{ m}^3$  respectively, and the product of apple in river 2 has no shortage of water and the actual water allocation for it is  $A_{282}^\pm = 5 \times 10^3 \text{ m}^3$ .

**Table 2.** Optimized Solutions of General ITSP Method under Optimized Water Allocation Targets in Ajabshir Qaleh Chay Dam (in  $10^3 \text{ m}^3$ )

|  |              | River ( <i>i</i> ) | Wheat      | Barley     | Potato     | Onion    | Grape      | Walnut     | Almond     | Apple      |
|--|--------------|--------------------|------------|------------|------------|----------|------------|------------|------------|------------|
| Optimized water allocation targets ( $q_{ijopt}^{\pm}$ ) |              | 1                  | 3.8        | 3.8        | 5          | 5        | 7.5        | 6.7        | 5.8        | 5.8        |
|  |              | 2                  | 2.5        | 2.5        | 7.5        | 7.5      | 7.5        | 5.8        | 5          | 5          |
|  |              | 3                  | 2.5        | 2.5        | 6.3        | 6.3      | 6.3        | 7.5        | 6.3        | 6.3        |
| Shortages ( $S_{ijkopt}^{\pm}$ )                         | Low level    | 1                  | 3.8        | 3.8        | 5          | 5        | 7.5        | [4.6, 6.7] | [2.8, 4.0] | [1.8, 5.8] |
|  |              | 2                  | 2.5        | 2.5        | 7.5        | 7.5      | 7.5        | 5.8        | [0, 3.8]   | [3.9, 5]   |
|  |              | 3                  | 2.5        | 2.5        | 6.3        | 6.3      | 6.3        | 7.5        | [0.8, 5.7] | 6.3        |
|  | Medium level | 1                  | [0.8, 1.8] | [0, 3.8]   | 2          | 1        | [4.5, 7.5] | 2.7        | 2.8        | 1.8        |
|  |              | 2                  | [1.4, 2.5] | 2.5        | [0, 7.5]   | [0, 7.5] | [0, 7.5]   | [0, 5.0]   | 0          | 0          |
|  |              | 3                  | 2.5        | 2.5        | [2.2, 6.3] | [0, 6.3] | 6.3        | [0, 6.1]   | 0          | 0          |
|  | High level   | 1                  | 0.8        | 0          | 2          | 1        | 4.5        | 2.7        | 2.8        | 1.8        |
|  |              | 2                  | 0          | 0          | 0          | 0        | 0          | 0          | 0          | 0          |
|  |              | 3                  | 0          | [0, 0.7]   | 0          | 0        | 0          | 0          | 0          | 0          |
| Actual water allocation ( $A_{ijkopt}^{\pm}$ )           | Low level    | 1                  | 0          | 0          | 0          | 0        | 0          | [0, 2.09]  | [1.8, 3]   | [0, 4]     |
|  |              | 2                  | 0          | 0          | 0          | 0        | 0          | 0          | [1.2, 5]   | [0, 1.1]   |
|  |              | 3                  | 0          | 0          | 0          | 0        | 0          | 0          | [0.6, 5.5] | 0          |
|  | Medium level | 1                  | [2, 3]     | [0, 3.8]   | 3          | 4        | [0, 3]     | 4          | 3          | 4          |
|  |              | 2                  | [0, 1.1]   | 0          | [0, 7.5]   | [0, 7.5] | [0, 7.5]   | [0.8, 5.8] | 5          | 5          |
|  |              | 3                  | 0          | 0          | [0, 4.1]   | [0, 6.3] | 0          | [1.4, 7.5] | 6.3        | 6.3        |
|  | High level   | 1                  | 3          | 3.8        | 3          | 4        | 3          | 4          | 3          | 4          |
|  |              | 2                  | 2.5        | 2.5        | 7.5        | 7.5      | 7.5        | 5.8        | 5          | 5          |
|  |              | 3                  | 2.5        | [1.8, 2.5] | 6.3        | 6.3      | 6.3        | 7.5        | 6.3        | 6.3        |

**Table 3.** Optimized Solutions of EITSP Method under Optimized Water Allocation Targets in Ajabshir Qaleh Chay Dam (in  $10^3 \text{ m}^3$ )

|   |              | River ( <i>i</i> ) | Wheat | Barley | Potato   | Onion      | Grape      | Walnut     | Almond     | Apple      |
|---|--------------|--------------------|-------|--------|----------|------------|------------|------------|------------|------------|
| Optimized water allocation target ( $q_{ijopt}^{\pm}$ ) |              | 1                  | 3     | 3.8    | 5        | 5          | 7.5        | 6.7        | 5.8        | 5.8        |
|   |              | 2                  | 2.5   | 2.5    | 7.5      | 5          | 7.5        | 5.8        | 5          | 5          |
|   |              | 3                  | 2.5   | 2.5    | 6.3      | 6.3        | 6.3        | 7.5        | 6.3        | 6.3        |
| Shortages ( $S_{ijkopt}^{\pm}$ )                        | Low level    | 1                  | 3     | 3.8    | 5        | 5          | 7.5        | [4.6, 6.7] | [2.8, 5.8] | [1.8, 5.8] |
|   |              | 2                  | 2.5   | 2.5    | 7.5      | 5          | 7.5        | 5.8        | [0, 3.8]   | [3.9, 5]   |
|   |              | 3                  | 2.5   | 2.5    | 6.3      | 6.3        | 6.3        | 7.5        | [0.8, 5.7] | 6.3        |
|   | Medium level | 1                  | 0     | 0      | [2, 3.7] | 1          | [4.5, 7.5] | 2.7        | 2.8        | 1.8        |
|   |              | 2                  | 0     | 0      | [0, 7.5] | [0, 5]     | [1.4, 7.5] | [0, 5.1]   | 0          | 0          |
|   |              | 3                  | 0     | 0      | 6.3      | [1.0, 6.3] | 6.3        | [0, 7.5]   | 0          | [0, 3.6]   |
|   | High level   | 1                  | 0     | 0      | 2        | 1          | 4.5        | 2.7        | 2.8        | 1.8        |
|   |              | 2                  | 0     | 0      | 0        | 0          | 0          | 0          | 0          | 0          |
|   |              | 3                  | 0     | 0      | 0        | 0          | [0, 0.8]   | 0          | 0          | 0          |
| Actual water allocation ( $A_{ijkopt}^{\pm}$ )          | Low level    | 1                  | 0     | 0      | 0        | 0          | 0          | [0, 2.1]   | [1.8, 3]   | [0, 4]     |
|   |              | 2                  | 0     | 0      | 0        | 0          | 0          | 0          | [1.2, 5]   | [0, 1.1]   |
|   |              | 3                  | 0     | 0      | 0        | 0          | 0          | 0          | [0.6, 5.5] | 0          |
|   | Medium level | 1                  | 3     | 3.8    | [1.3, 3] | 4          | [0, 3]     | 4          | 3          | 4          |
|   |              | 2                  | 2.5   | 2.5    | [0, 7.5] | [0, 5]     | [0, 6.1]   | [0.8, 5.8] | 5          | 5          |
|   |              | 3                  | 2.5   | 2.5    | 0        | [0, 5.3]   | 0          | [0, 7.5]   | 6.3        | 6.3        |
|   | High level   | 1                  | 3     | 3.8    | 3        | 4          | 3          | 4          | 3          | 4          |
|   |              | 2                  | 2.5   | 2.5    | 7.5      | 5          | 7.5        | 5.8        | 5          | 5          |
|   |              | 3                  | 2.5   | 2.5    | 6.3      | 6.3        | [5.5, 6.3] | 7.5        | 6.3        | 6.3        |

Under high flow levels, water scarcity in all products is zero for rivers 2 and 3. In this case, the actual water allocation for products is equal to the plant's water requirement. However, at this level of flow for river 1, except for wheat and barley, there is a shortage of water for all products. These values can

be easily compared using Figure 2.

Given the scarcity of water at different levels of flow, farmers can use abundant water resources to retrieve water scarcity. The results of the EITSP method show apples, almonds, walnuts, grapes and onions would use the first alternative under a low

flow level, the third alternative under a medium flow level and the second alternative under a high flow level. Potato fields would use the first alternative under a low flow level, the third alternative under a medium flow level, and the second alternative under a high flow level.

**Table 4.** Optimal Decision to Choose Alternatives for Crops under Different Levels of Water Flows by EITSP

| Crops ( <i>j</i> ) | Flow level ( <i>k</i> ) | Alternatives |              |              |
|--------------------|-------------------------|--------------|--------------|--------------|
|                    |                         | <i>l</i> = 1 | <i>l</i> = 2 | <i>l</i> = 3 |
| Wheat              | Low ( <i>k</i> = 1)     | -            | -            | ●            |
|                    | Medium ( <i>k</i> = 2)  | -            | -            | -            |
|                    | High ( <i>k</i> = 3)    | -            | -            | -            |
| Barley             | Low ( <i>k</i> = 1)     | ●            | -            | -            |
|                    | Medium ( <i>k</i> = 2)  | -            | -            | -            |
|                    | High ( <i>k</i> = 3)    | -            | -            | -            |
| Potato             | Low ( <i>k</i> = 1)     | ●            | -            | -            |
|                    | Medium ( <i>k</i> = 2)  | -            | -            | ●            |
|                    | High ( <i>k</i> = 3)    | -            | ●            | -            |
| Onion              | Low ( <i>k</i> = 1)     | -            | -            | ●            |
|                    | Medium ( <i>k</i> = 2)  | ●            | -            | -            |
|                    | High ( <i>k</i> = 3)    | -            | ●            | -            |
| Grape              | Low ( <i>k</i> = 1)     | -            | -            | ●            |
|                    | Medium ( <i>k</i> = 2)  | ●            | -            | -            |
|                    | High ( <i>k</i> = 3)    | -            | ●            | -            |
| Walnut             | Low ( <i>k</i> = 1)     | -            | -            | ●            |
|                    | Medium ( <i>k</i> = 2)  | ●            | -            | -            |
|                    | High ( <i>k</i> = 3)    | -            | ●            | -            |
| Almond             | Low ( <i>k</i> = 1)     | ●            | -            | -            |
|                    | Medium ( <i>k</i> = 2)  | -            | -            | ●            |
|                    | High ( <i>k</i> = 3)    | -            | ●            | -            |
| Apple              | Low ( <i>k</i> = 1)     | -            | -            | ●            |
|                    | Medium ( <i>k</i> = 2)  | ●            | -            | -            |
|                    | High ( <i>k</i> = 3)    | -            | ●            | -            |

Wheat and barley are products that require auxiliary water resources only under low flow levels; accordingly, the third and first alternatives would be used for them. At medium and high flow levels, there is no need to auxiliary resources for them.

The alternatives used for crops at various levels are reported in Table 4. Furthermore, under each level of flow, these water resources are used to retrieve the shortage of water for each crop and for all rivers. For example, under a low flow level, the best alternative to wheat farming is to use the third alternative with an available value of  $\Delta T_{13}^{\pm} = [10, 13] \times 10^3 \text{ m}^3$ . Under a low flow level, the shortage of all three rivers for wheat is  $8 \times 10^3 \text{ m}^3$ , compensated by the third alternative, and the amount of water allocated is increased. At this flow level, for the barley product, the first alternative would be used with the available amount of  $\Delta T_{11}^{\pm} = [9, 12] \times 10^3 \text{ m}^3$  and the total shortage of all three rivers for the barley crop is equal to  $8.75 \times 10^3 \text{ m}^3$ , which is compensated by the first alternative.

Finally, the best decision for the apple product would be to use the first, the third and the second alternatives with available amount of  $\Delta T_{81}^{\pm} = [28, 32] \times 10^3$ ,  $\Delta T_{83}^{\pm} = [25, 29] \times 10^3$  and  $\Delta T_{82}^{\pm} = [26, 29] \times 10^3 \text{ m}^3$  under low, medium and high flow levels

respectively, to compensate for the shortage of all three rivers for this product.

Under fuzzy environments using the EITSP approach with fuzzy variables, the results are shown in Table 5 and Figure 3. The actual water allocation and water shortages at different levels of flow are calculated based on the promised water allocation target for each product in each river.

In river 1, apart from the wheat and barley products, there is a shortage of water for other products at all levels, which is much higher under a seasonal low flow level. It is noted that under a low flow level, water scarcity occurs for all products.

In rivers 2 and 3, water scarcity has occurred for all products only under a low flow level. It is found that there is a shortage of water for the grape product in both rivers 2 and 3 under a medium flow.

As the results show in Table 5, under a low flow, there are water shortages for most products and in all rivers. The optimized actual water allocation would be zero, except for apples, almonds and walnuts in river 1 with actual water allocation of  $A_{161}^{\pm} = 1.85 \times 10^3$ ,  $A_{171}^{\pm} = 3.39 \times 10^3$  and  $A_{181}^{\pm} = 4.42 \times 10^3 \text{ m}^3$  and almonds in river 2 with actual water allocation of  $A_{271}^{\pm} = 6.501 \times 10^3 \text{ m}^3$ . Under a medium flow level, there would be shortage of water in river 1 except for wheat and barley. On the other hand in river 2, there would be only water shortage of  $S_{252}^{\pm} = 7.5 \times 10^3 \text{ m}^3$  for grapes, and in river 3, there would be only water shortages of  $S_{332}^{\pm} = 3.647 \times 10^3$  and  $S_{352}^{\pm} = 6.25 \times 10^3 \text{ m}^3$  for potato and grape respectively. Finally, at a high flow level in rivers 2 and 3, water shortages in all products would be zero, but there would still be a shortage of water in river 1.

As shown in Table 6, farmers use alternative resources to deal with water scarcity. Under a low flow level, barley, potato and almond products would use the first alternative, and wheat, onion, grapes, walnuts and apples would use the third alternative. At this level, none of the products will use the second alternative. Under a medium flow level, potato and almond products would use the third alternative, and onions, grapes, walnuts and apples would use the first alternative. Based on the results of Table 6 under high and medium flow levels, wheat and barley products do not need auxiliary water resources, and eventually under a high flow level, the rest of the products would use the second alternative.

The comparison of the values of the objective function obtained with general ITSP, EITSP and fuzzy EITSP methods is given in Table 7 and Figure 4. As shown in Table 7, for these methods, the total costs without considering the cost of retrieving water shortages would be  $[188.742, 342.594] \times 10^6$ ,  $[186.508, 356.3] \times 10^6$  and  $232.692 \times 10^6$  Rials, and the total incomes would be  $[345.27, 422.03] \times 10^6$ ,  $[339.145, 414.543] \times 10^6$  and  $2972.616 \times 10^6$  Rials respectively. The results indicate that the system's final benefits have changed due to the use of alternative resources and the compensation of water shortages.

The results also show that in this case study, the fuzzy EITSP method has the optimal solutions with the highest mid value and the smallest interval among other described methods, and deterministic solution are obtained to decision variables.

**Table 5.** Optimized Solutions of Fuzzy EITSP Method under Optimized Water Allocation Targets in Ajabshir Qaleh Chay Dam (in  $10^3 \text{ m}^3$ )

|  |              | River ( <i>i</i> ) | Wheat | Barley | Potato | Onion | Grape | Walnut | Almond | Apple |
|--|--------------|--------------------|-------|--------|--------|-------|-------|--------|--------|-------|
| Optimized water allocation targets ( $q_{ijopt}^{\pm}$ ) |              | 1                  | 3.4   | 3.8    | 7.5    | 7.5   | 7.5   | 10     | 8.8    | 5.8   |
|  |              | 2                  | 2.5   | 2.5    | 7.5    | 7.5   | 7.5   | 8.8    | 7.5    | 5     |
|  |              | 3                  | 2.5   | 2.5    | 6.3    | 6.3   | 6.3   | 7.5    | 6.3    | 4.2   |
| Shortages ( $S_{ijkopt}^{\pm}$ )                         | Low level    | 1                  | 3.4   | 3.8    | 7.5    | 7.5   | 7.5   | 8.1    | 5.4    | 1.4   |
|  |              | 2                  | 2.5   | 2.5    | 7.5    | 7.5   | 7.5   | 8.8    | 1.0    | 5     |
|  |              | 3                  | 2.5   | 2.5    | 6.3    | 6.3   | 6.3   | 7.5    | 0.4    | 4.2   |
|  | Medium level | 1                  | 0     | 0      | 4.1    | 3.1   | 4.1   | 5.6    | 5.4    | 1.4   |
|  |              | 2                  | 0     | 0      | 0.0    | 0     | 7.5   | 0      | 0      | 0     |
|  |              | 3                  | 0     | 0      | 3.7    | 0     | 6.3   | 0      | 0      | 0     |
|  | High level   | 1                  | 0     | 0      | 4.1    | 3.1   | 4.1   | 5.6    | 5.4    | 1.4   |
|  |              | 2                  | 0     | 0      | 0      | 0     | 0     | 0      | 0      | 0     |
|  |              | 3                  | 0     | 0      | 0      | 0     | 0     | 0      | 0      | 0     |
| Actual water allocation ( $A_{ijkopt}^{\pm}$ )           | Low level    | 1                  | 0     | 0      | 0      | 0     | 0     | 1.9    | 3.4    | 4.4   |
|  |              | 2                  | 0     | 0      | 0      | 0     | 0     | 0      | 6.5    | 0     |
|  |              | 3                  | 0     | 0      | 0      | 0     | 0     | 0      | 5.9    | 0     |
|  | Medium level | 1                  | 3.4   | 3.8    | 3.4    | 4.4   | 3.4   | 4.4    | 3.4    | 4.4   |
|  |              | 2                  | 2.5   | 2.5    | 7.5    | 7.5   | 0     | 8.8    | 7.5    | 5     |
|  |              | 3                  | 2.5   | 2.5    | 2.6    | 6.3   | 0     | 7.5    | 6.3    | 4.2   |
|  | High level   | 1                  | 3.4   | 3.8    | 3.4    | 4.42  | 3.4   | 4.4    | 3.4    | 4.4   |
|  |              | 2                  | 2.5   | 2.5    | 7.5    | 7.5   | 7.5   | 8.8    | 7.5    | 5     |
|  |              | 3                  | 2.5   | 2.5    | 6.3    | 6.3   | 6.3   | 7.5    | 6.3    | 4.2   |

**Table 6.** Optimal Decision to Choose Alternatives for Crops under Different Levels of Water Flows by Fuzzy EITSP

| Crops ( <i>j</i> ) | Flow level ( <i>k</i> ) | Alternatives |              |              |
|--------------------|-------------------------|--------------|--------------|--------------|
|                    |                         | <i>l</i> = 1 | <i>l</i> = 2 | <i>l</i> = 3 |
| Wheat              | Low ( <i>k</i> = 1)     | -            | -            | ●            |
|                    | Medium ( <i>k</i> = 2)  | -            | -            | -            |
|                    | High ( <i>k</i> = 3)    | -            | -            | -            |
| Barley             | Low ( <i>k</i> = 1)     | ●            | -            | -            |
|                    | Medium ( <i>k</i> = 2)  | -            | -            | -            |
|                    | High ( <i>k</i> = 3)    | -            | -            | -            |
| Potato             | Low ( <i>k</i> = 1)     | ●            | -            | -            |
|                    | Medium ( <i>k</i> = 2)  | -            | -            | ●            |
|                    | High ( <i>k</i> = 3)    | -            | ●            | -            |
| Onion              | Low ( <i>k</i> = 1)     | -            | -            | ●            |
|                    | Medium ( <i>k</i> = 2)  | ●            | -            | -            |
|                    | High ( <i>k</i> = 3)    | -            | ●            | -            |
| Grape              | Low ( <i>k</i> = 1)     | -            | -            | ●            |
|                    | Medium ( <i>k</i> = 2)  | ●            | -            | -            |
|                    | High ( <i>k</i> = 3)    | -            | ●            | -            |
| Walnut             | Low ( <i>k</i> = 1)     | -            | -            | ●            |
|                    | Medium ( <i>k</i> = 2)  | ●            | -            | -            |
|                    | High ( <i>k</i> = 3)    | -            | ●            | -            |
| Almond             | Low ( <i>k</i> = 1)     | ●            | -            | -            |
|                    | Medium ( <i>k</i> = 2)  | -            | -            | ●            |
|                    | High ( <i>k</i> = 3)    | -            | ●            | -            |
| Apple              | Low ( <i>k</i> = 1)     | -            | -            | ●            |
|                    | Medium ( <i>k</i> = 2)  | ●            | -            | -            |
|                    | High ( <i>k</i> = 3)    | -            | ●            | -            |

The results of general ITSP, EITSP and fuzzy EITSP methods indicate that by using alternative resources, water shortages for products in rivers have been compensated, and the final allocation of water has increased.

In the face of fuzzy uncertainty in the parameters of the problem, a fuzzy model has been used, and in this model, better results have been obtained for water deficit compensation and final allocation of water for the corps. The fuzzy EITSP method presented in this paper has a potential advantage in coping with water resource management issues under uncertain parameters to compensate for water scarcity.

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**Table 7.** Comparison of Objective Values ( $10^6$  Rials) through Described Methods

|              | Total Income     | Total Cost*      | Cost of retrieving water shortages |
|--------------|------------------|------------------|------------------------------------|
| General ITSP | [345.27, 422.03] | [188.74, 342.59] | ---                                |
| EITSP        | [339.15, 414.54] | [186.51, 356.30] | [1468.95, 1985.63]                 |
| Fuzzy EITSP  | 2972.62          | 232.69           | [1424.73, 1582.63]                 |

\* Without the cost of retrieving water shortages.

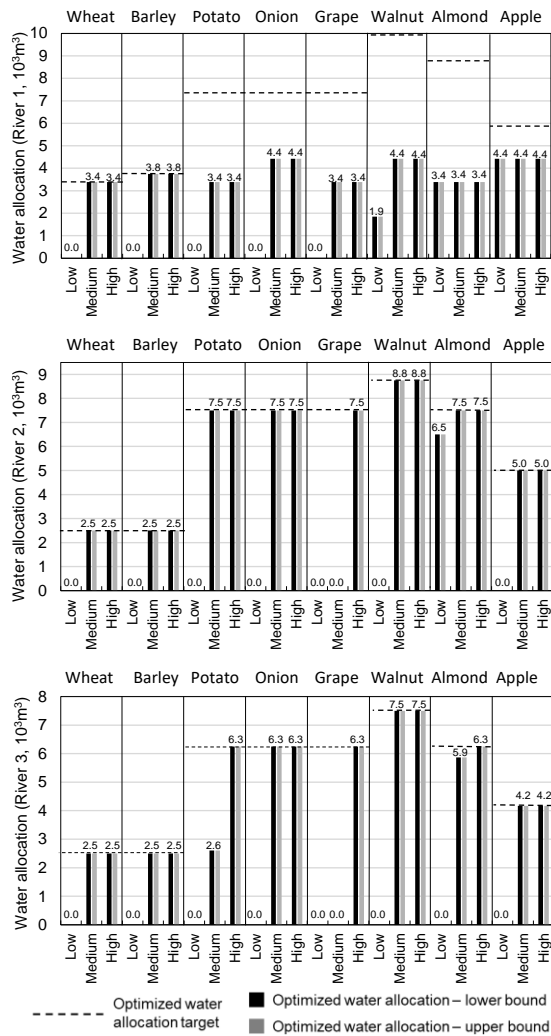


Figure 3. Optimized water allocation chart patterns through fuzzy EITSP method under low, medium and high flows.

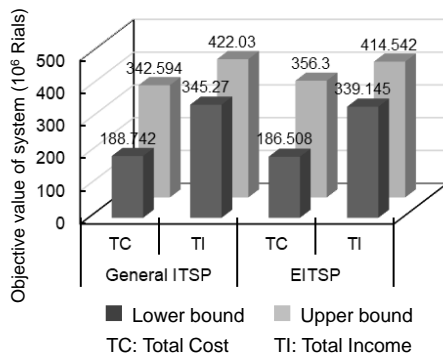


Figure 4. Comparison of upper and lower bounds of objective values through described methods.

As the conclusion, we proposed two methods for water allocation of Ajabshir Qaleh Chay Dam between agricultural products. The EITSP and then EITSP with fuzzy parameters were proposed based on MIP, and their results were compared. The advantage of these extended approaches over the conventional type is considering some alternatives for retrieving water shortages when the water demands of users are not completely satisfied by seasonal flows, which would cause irreparable damages especially to farmers. In this case, they have to either obtain water from higher-priced resources by considering our described approaches (i.e. EITSP and Fuzzy EITSP) or curtail their development plans. The main purpose of these models is to provide a way for farmers to use alternative resources at the lowest cost that may not be a positive benefit to the system because the objective function would be a combination of system revenue and costs.

DMs would choose proper alternatives for retrieving water shortages according to the optimal results of described methods. Accordingly, they can reduce the unsustainability of water resources using the optimal results of these extended approaches for irrigation while decreasing the system cost with the best possible decision.

In this section (i.e. result analysis), the optimal water allocation target was reported for all crops. Furthermore, the water shortages and optimal retrieving alternatives related to shortage were collected in Tables 2 ~ 7 and Figures 2 and 3. In accordance with the optimal results of the described methods, DMs would make their best irrigation scheme and would use other reservoirs water resources for retrieving water shortages.

Finally, by studying the conditions of the region, its water resources status and optimal results of the described methods, using these results would allow farmers to make the best possible decision and to increase the sustainability of water resources.

#### 4. Conclusion

In this paper, EITSP and fuzzy EITSP methods for water resource allocation under uncertainty were introduced in Ajabshir Qaleh Chay Dam to retrieve the water shortage of agricultural products and to achieve the optimal allocation of Qaleh Chay Dam water through its river canals between different agricultural products under uncertainty conditions. Then a new solving approach based on Huang Algorithm, FCCP method based on possibility theory and Zimmermann fuzzy programming was presented to solve the problems.

Finally, using a case study in Ajabshir Qaleh Chay Dam, the results are obtained for general ITSP, EITSP and fuzzy EITSP methods to compare these results with each other and indicated what the difference between these methods was. Furthermore, under different levels of flow and for different crops, water shortages based on the promised water allocation target and the actual allocated water for each river from Qaleh Chay Dam, were obtained.

The farmer would use auxiliary resources to compensate for water scarcity. The results showed that at all levels of water flow; shortage of water resources has reduced compared to the previous model and final water allocation values have increased, so that under a high flow level, water deficit in all products was zero, except for the first river, where there is some water shortage for some garden products.

Using the alternative resources of water, the possibility distribution function of parameters and fuzzy programming in some constraints, it is clear that our fuzzy extended method for water resource management has led to optimal solutions with high mid values and smaller intervals than the ITSP and EITSP approaches, and can be used for real cases under uncertainty conditions. Furthermore, our fuzzy extended approach obtained deterministic solutions to decision variables in the case study, and the water shortages were retrieved by our extended approaches in comparison with the general ITSP method.

By assessing the condition of this region, the status of its water resources, and according to the obtained results, providing the optimal model will allow farmers to make the best possible decision for the most profit; and policymakers of the water resources management systems in critical conditions will use water resources in an optimal manner. There will thus be the least instability in the use of water resources.

In this study, we treated the possibility level of  $\eta = 0.7$  as a sample level to obtain optimal solutions of the proposed water resources allocation models; therefore, there is no limitation for DMs to choose it. Any DM would choose other levels between the ranges of (0, 1] based on the circumstances or any other constraints to find his/her optimal solutions cause the related function of possibility level in the obtained deterministic relations, i.e.  $R^*(\eta)$ ,  $L^*(\eta)$  have been defined explicitly.

The main results of this study would be listed as:

- Deterministic solutions were obtained to decision variables by the fuzzy EITSP approach.
- Optimal decisions to choose supplementary water reservoirs for water shortages of crops under different flow levels, would be made by EITSP and fuzzy EITSP respectively.
- For this case study, the fuzzy EITSP approach has led to deterministic optimal solutions with higher mid values and smaller intervals for objective function values.
- Our proposed approaches would be used for real practical situations under stochastic, interval and possibilistic conditions simultaneously.

For future research, our developed method can be extended under other conditions of uncertainty, such as uncertain and rough variables, and new ways of resolving water resource management issues under these uncertain conditions will be presented. The developed method of this study can be used for other water resource management issues, such as flood diversion planning, river management, and environmental and energy management issues. Furthermore, other issues that exist for water resource systems, such as multi-objective, bi-level and non-linear issues, can be investigated in future research.

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